Condensed Matter Physics I
Final written test
academic year 2011/2012
January 23, 2012

(*Time: 3 hours)*

Exercise 1: *Free electrons in hcp and triangular lattices*

1. The Zn atom has 2 valence electrons. Zn atoms aggregate giving a metal with hcp structure, ABAB... is the sequence of the close-packed planes. The nearest neighbor interatomic distance is 2.66 Å and the distance between two neighboring planes in the ABAB... sequence is $c/2$ with c=4.95 Å.
   Write the primitive translation vectors and the basis of Zinc.

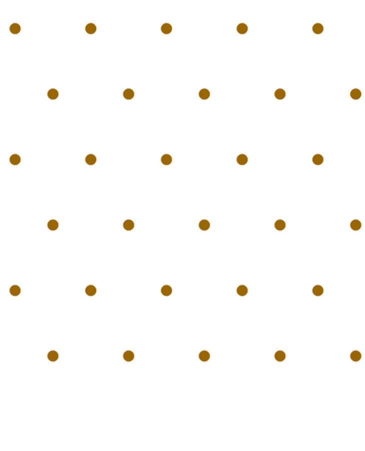
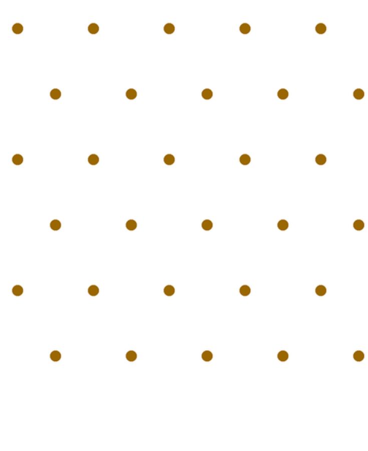
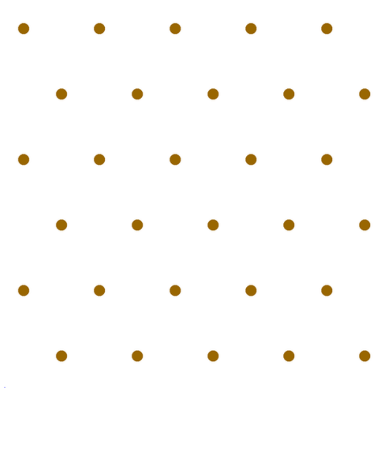
2. Is it possible to have a hcp insulator? Justify your answer, and, in case of positive answer, explain in details when it would be possible.

3. Consider now only the 2D in-plane structure which forms a triangular lattice. Make a picture of the corresponding reciprocal lattice, together with the first Brillouin zone, and give its area. *(You may use the pictures of the triangular lattices at the end of the page for your sketches)*

4. Make a picture also of the second and third Brillouin zones, indicating them clearly. Give also their areas.

5. Consider again the case of 2 valence electrons per site. Calculate the radius of the Fermi sphere (which is a "circle" in this 2D case).

6. Together with the sketch of the Brillouin zones, makes also the sketch of the Fermi sphere. In which Brillouin zones is the Fermi sphere lying? Translating the portions of the Fermi sphere lying in the n-th Brillouin zone through reciprocal lattice vectors, draw the branches of the Fermi sphere corresponding to the n-th band. *(as it is done e.g. in Fig. 9.15(b) of A&M)*

Exercise 2: *Semiclassical electron dynamics*

Consider a FCC crystal with lattice parameter a and a tight-binding single-orbital s-type energy band

$$E(\mathbf{k}) = -4\gamma\left(cos\frac{k_x a}{2}cos\frac{k_y a}{2} + cos\frac{k_y a}{2}cos\frac{k_z a}{2} + cos\frac{k_z a}{2}cos\frac{k_x a}{2}\right)$$

(the zero of the energy is properly chosen)

1. Write the expression of the Bloch velocity for this band.
2. Show explicitly that the velocity for $\mathbf{k}$ lying on the face of the Brillouin zone containing X=$\frac{2\pi}{a}(1,0,0)$ is parallel on such face *(the answer will be considered valid only if fully justified by calculations)*
3. The solid is in a static uniform magnetic field $\mathbf{H} = H\hat{z}$. Write the condition to be satisfied for the orbits in $\mathbf{k}$-space, for $k_z = 0$ and energy $E(\mathbf{k})$ = E.
4. For such orbits, write explicitly the expression of the velocity in the (x, y) plane for an electron in the given band.
5. In the particular case of $E = -4\gamma[3 - \delta^2/4]$ and $\delta \ll 1$, write the expression of the orbit, sketch it in the (k_x, k_y) plane, together with the section of the first Brillouin zone.
6. Estimate the period of that orbit (with respect to the free electron case) if $\gamma a^2 = \hbar^2/2m^*$, with $m^* = 0.067m_e$.

NOTE: Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.