Condensed Matter Physics II. – A.A. 2010-2011, July 12, 2011

(*time 3 hours)*

Solve the following two exercises, each has a maximum score of 18 for a total of 36. A score between 33 e 36 corresponds to 30 cum laude, between 30 e 32 is renormalized to 30 (the maximum official score, without laude).

NOTE:

- Give all details which help in understanding the proposed solution. Answers which only contain the final result or not enough detail will be judged insufficient and discarded;
- If you are requested to give evaluation/estimates, do so using 3 significant figures.

Exercise 1: *Model semiconductor in the degenerate and intrinsic regime*

Let's consider a model semiconductor in the degenerate, intrinsic regime: in other words, we consider a semiconductor for which it is **not** possibile to assume neither $\epsilon_c - \mu \gg K_B T$ nor $\mu - \epsilon_v \gg K_B T$. Moreover, we assume that the impurity concentration is negligible (intrinsic regime).The semiconductor density of states, however, satisfies: $g_v(\epsilon^* - \epsilon) = g_c(\epsilon^* + \epsilon)$, with $\epsilon^* = (\epsilon_c + \epsilon_v)/2$.

1. Assuming that the maximum of the conduction band is at $\epsilon_c + 2\Delta$, provide a qualitative sketch of $g_c(\epsilon)$, with the correct qualititative behavior at ϵ_c and $\epsilon_c + \Delta$: please indicate explicitely such qualitative behaviors. Here and in the following it is suggested to take ϵ^* as zero of energy.
2. Give a qualitative sketch (on the same graph) of $g_v(\epsilon)$ and $g_c(\epsilon)$.
3. Write down the condition that determines the chemical potential, keeping in mind that the Fermi distribution cannot be approximated in any way, due to the degenerate regime: in othe words, impose $n_c(T, \mu) = p_v(T, \mu)$, with obvious notation. It is suggested that you rearrange the two integrals providing the carrier concentrations in a form that allows the determination of μ by inspection. [*Note, you do not need to perfom any integral!*].
4. Consider now a density of states $g_c(\epsilon) = A\sqrt{(\epsilon - \epsilon_c)(2\Delta - \epsilon + \epsilon_c)}$. Determine A as function of ρ_L and Δ, knowing that the system is a Bravais with an atom/site, that the density of states results from just one band and that ρ_L is the density of lattice sites in space.
5. Express the effective mass at the bottom of the conduction band in terms of ρ_L and Δ.
6. Knowing that $\rho_L = 5.00 \times 10^{22} cm^{-3}$and $\Delta = 27.7eV$ evaluate m_c/m_e with 3 significant figures.

Exercise 2 *Excitation a linear Debye chain.*

Consider the harmonic vibrations (phonons) in an infinite linear chain of equispaced atoms, with lattice parameter a, and springs of constant G connecting each atom to its nearest neighbors.

1. Let $u_+(q,n,t) = \epsilon \exp[i(qna - \omega(q)t)]$ and $u_-(q,n,t) = u_+^*(q,n,t)$ be the 2N independent solutions (normal modes) of the dynamical problem, with q, n, t respectively a wavevector in the FBZ, a lattice position and time. We have in mind a chain of length $L = Na$, with PBC. Let's resort to Debye approximation and replace $\omega(q)$ with the linear behavior valid for $|q| \ll \pi/a$. Give explicitely $\omega(q)$ in such approximation for both positive and negative values of q.
2. Let's consider a superposition of the normal modes with coefficients
$$a_-(q) = a_+(q) = \frac{l\,a}{5L}\frac{1}{1+(ql)^2}$$
and (i) $l \gg a/\pi$. Calculate
$$u(n,t) = \sum_{\sigma=\pm,q} a_\sigma(q) u_\sigma(q,n,t).$$
We remark that due to the condition (i) above, the integral over q can be approximated extending it to all q-space, i.e., over the q-range $[-\infty, \infty]$.
3. Are there atoms displaced from the equilibrium positions at $t = 0$.
4. Calculate the speed of each atom at $t = 0$.
5. Which atoms are displaced from equilibrium at $t = ma/c$, with $ma \gg l$ and c the sound velocity: please, answer by giving also a qualitative sketch of the displacements along the chain!
6. Give a qualitative sketch of the velocity of the atoms along the chain, at $t = ma/c$, with $ma \gg l$ and c the sound velocity: please provide a detailed motivation of the sketch.

   Note:
$$\int_0^\infty dq \frac{\cos[qs]}{1+(ql)^2} = \frac{\pi}{2l} e^{-|s|/l}$$