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# **INDUSTRIAL PLANTS**

## **Chapter four:**

### **Study of market and product**

**DOUBLE DEGREE MASTER IN**

**“PRODUCTION ENGINEERING AND MANAGEMENT”**

**CAMPUS OF PORDENONE**

**UNIVERSITY OF TRIESTE**

## Generality

The **market research** or **marketing** permits to obtain the facts that:

- **analysis of market demand**

It allows to predict the entity of the overall demand for products and services, based on the evaluation of the phenomena of the market, the company intends to achieve and its variation over time;

- **formation process of the sale price**

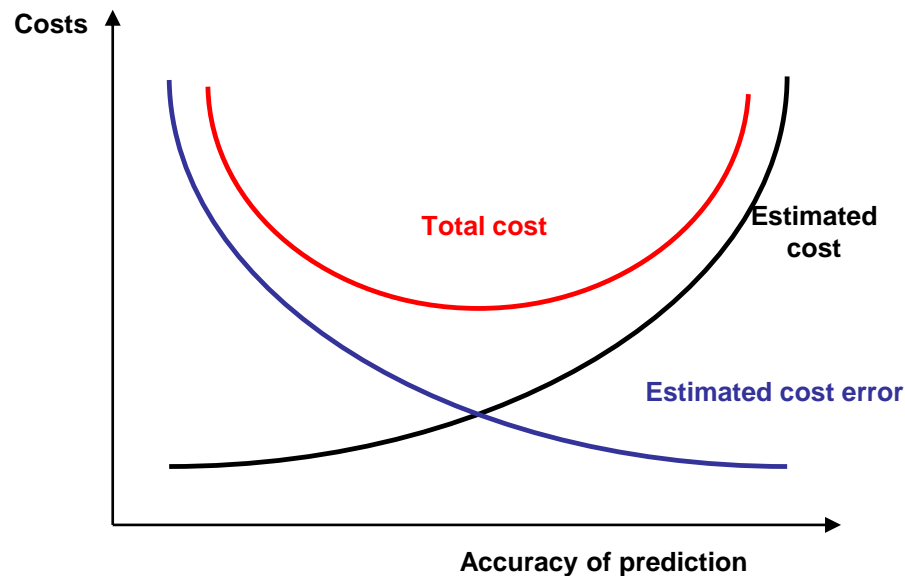
It determines the share of market demand or previously determined the volume of sales determined on the basis of the initiatives that the company can take yourself.



## Generality

The **objective of market research** is to determine the volume of business sales and the selling price for the new product.

The **purpose of the forecasts** is to reduce the risk of decision making and provide the company with accurate and reliable results from which to make their strategic choices. Miscalculations could lead companies to support the estimated costs useless.



## Generality

You can try to **reduce the risks** by using:

- acquisitions of specific commitments of the finished product by the distribution companies;
- clauses of the purchase price in order to modify the price on the basis of cost;
- market research on the factors of production and products to be placed on the market.



## Generality

The **sales volume** depends on:

- **phenomena of the market or passive levers**

Are the quantity and distribution of the income of families in the community, level of employment, preferences, geographic area of interest, inflation etc., and external phenomena are related to market behavior and the behavior independent of the company;

- **actions or decisions of the company**

Are changes in the price and conditions of sale, advertising etc. and are related to the behavior of the phenomena of company in the context in which it operates.



## Generality

The **sales volume** depends on:

- **demand analysis**

It considers the development of demand affected by the evaluation of market phenomena and is developed to study such effects generate passive levers on the volume of sales expected.

There are two types of question:

- a) **primary**, which the company received directly from the market;
- b) **secondary**, which is induced by the primary demand (parts);

- **analysis of market share and the price formation process**

The quantitative assessment of the demand may be obtained by varying the tools available to the company (advertising, price change etc.) and is developed to study what effects the active levers generate anticipated sales volume.

## Generality

The **predictions** made by a company are influenced by lead time (from receipt of the purchase order at the time when the vendor delivers the product to the customer). It is determined by:

- **internal inertia**

Is linked to internal parameters at company with technical and organizational components (duration of each operation of the production cycle);

- **external inertia**

Is linked to external parameters at company such as external suppliers (time of supply of materials).

The sum of the typical lead times, inside and outside, determines the **minimum delay prediction**.

## Generality

The **forecasts in market research** can be:

- **short-term or exercise**

Limited to production schedules with the system running of monthly period, quarterly or semi-annual and which corresponds to key aspects of analysis relating to economic or seasonal;

- **medium-term**

They are limited to the launch of new products to be implemented with existing systems and appropriately modified to match the key factors of analysis for the evaluation of trends and cyclical aspects;

- **long term or plant**

They are limited to the analysis of market demand regarding the construction of new facilities and meet the key factors in any analysis or assessment of technological trends.



# Generality

## The forecasts in market research

Oorganizational units	Verry short-term	Short-term	Medium-term	Long-term
Marketing	Sales by class of product or geographic area or customer	Total sales and the most important products, and price level	Total sales and the most important products and general economic situation	Total sales and the most important products, introduction of new products, sales of saturation of existing products and customer preferences and tastes
<b>Production</b>	<b>Demand for products of plant capacity and saturation</b>	<b>Total demand and by product category</b>	<b>Budget allocation, purchases of equipment and level of use of labor</b>	<b>Investment in the production, expansion of facilities and new technologies</b>
Finance	Sales revenues, production costs and cash flow and outflow	Total demand, inventory levels and price levels	Budget allocation and cash flow	Total sales, investment and allocation of financial resources
Purchases	Level of production and availability of capital	Demand for product and material, and delivery lead time	Demand for product and raw materials, and development of new products	Purchase contracts and preferences and tastes of customers

## Prediction models

The **criteria** used for determining the demand for a product are:

- **deterministic criteria**

It assesses the needs of the individual products on the basis of requests made to the company. Estimate the manufacturing needs of a product make to order and determines the demand of the secondary component. The criterion is not associated with the uncertainty in the nature and extent of primary and secondary demand is known and unchanging horizon programming;

- **statistic criteria**

Evaluate the needs of the individual product based on statistical principles and probability, and applies in the case of make to stock production where the demand is uncertain. The criterion refers to techniques used in conditions of uncertainty and is used for determining the primary demand.

## Prediction models

The **criteria for forecasting demand based on the analysis and extrapolation of historical data available** are:

**a) techniques to base multiperiodic or equitable-weights**

They are based on historical demand and are able to make short-term forecasts (product planning), medium (manpower planning) and long term (planning of production sites);

**b) techniques to base aperiodic or self-adaptive**

They do not consider the historical data, but only the most recent data. They are able to make short-term forecasts (produced programming);

**c) associative techniques**

They do not consider time series data, but different variables of time and are able to make predictions on the short (program product), medium (manpower planning) and long term (planning of production sites).

## Prediction models

**Autoregressive methods** are methods (projection to base multiperiodic, projection to base aperiodic) who consider the demand for a product function of time, while the other variables are constant.

Indication of  $D$  with the market demand for a product, with  $V$  the volume of sales of the company (always  $V \leq D$ ), with  $F$  the market phenomena, with  $I$  initiatives or decisions of the company and  $t$  the time, it is can therefore be noted that in the case of autoregressive methods:

$$D = f(t) \quad V = f(t)$$

Methods are simple and have satisfactory results.

## Prediction models

The **regressive methods** are methods that estimate the primary demand on the basis of the correlation between the demand and market phenomena and the actions and choices of the company, while the time is a constant:

$$D = f(F, I) \quad V = f(F, I)$$

In the analysis phase of the demand, assuming  $I = \text{constant}$ , we seek to know the function  $D = f(F)$ , while in the process of price formation, assuming  $F = \text{constant}$ , we seek to know the function  $V = f(I)$ .

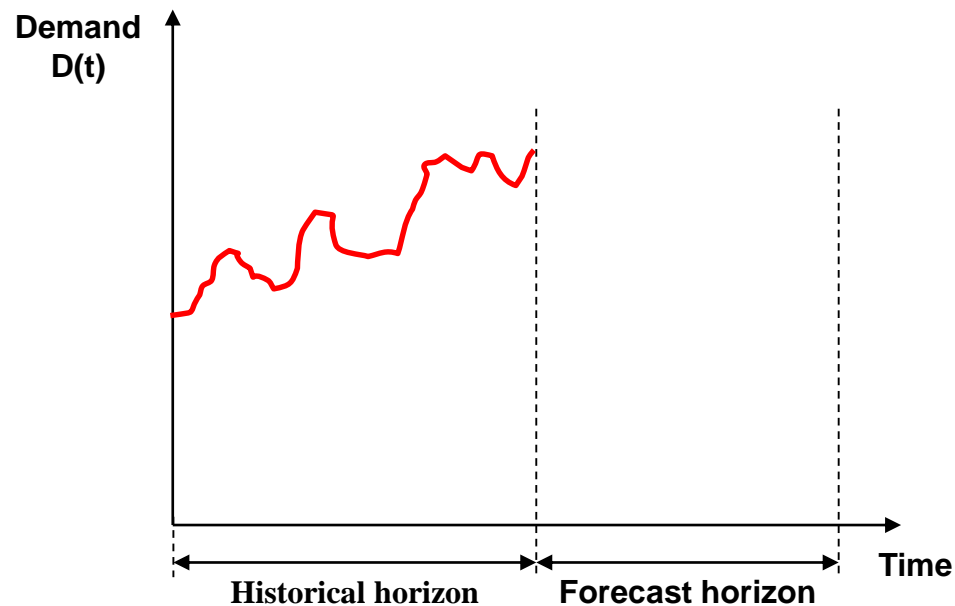
The phenomena associated with market demand are called **economic indicators** of the nation, region or city that has the market (gross national income or average per capita, building permits, production and registration of motor vehicles, consumer prices or wholesale, bank deposits etc.).

## Autoregressive method

**Autoregressive methods** can be divided into:

a) **technical data projection on to base multiperiodic or prediction equivalent-weight**

This technique considers historical data of the demand, assigning equal weight to all data series in a given time interval of the past



## Autoregressive method

**Autoregressive methods** can be divided into:

**a) technical data projection on to base multiperiodic or prediction equivalent-weight**

The development of demand  $D(t)$  is obtained from the sum of:

$$D(t) = T(t) + C(t) + S(t) + A(t)$$

- **systemic component**, due to individual action and measurable according to a deterministic nature. It breaks down into:
  - a) **trend component  $T(t)$**  in the long term;
  - b) **cyclical component  $C(t)$** , with cycle of period and amplitude varying in time;
  - c) **seasonal component  $S(t)$**  with a cycle of constant period in time in a period of one semester or one year;
- **random component**, set of non-specific actions in a stochastic ambient. It reads:
  - a) **random component  $A(t)$**  characteristic of random events that affect the development of demand

## Autoregressive method

**Autoregressive methods** can be divided into:

**a) technical data projection on to base multiperiodic or prediction equivalent-weight**

**a1) technical evaluation of the trend**

The positive or negative trend defines the trend of a time series of data sets. The trends of the dial trend are:

- constant trend:  $T(t) = \text{constant}$
- linear trend:  $T(t) = a + b \cdot t$
- parabolic trend:  $T(t) = a + b \cdot t + c \cdot t^2$
- exponential trend:  $T(t) = e \cdot (a + b \cdot t)$
- hyperbolic trend:  $T(t) = 1/(a + b \cdot t)$



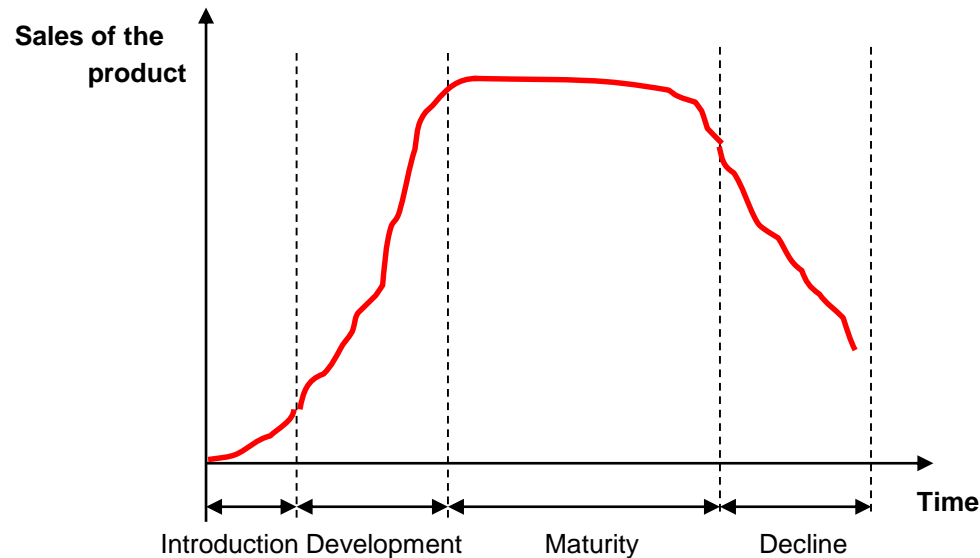
## Autoregressive method

**Autoregressive methods** can be divided into:

a) **technical data projection on to base multiperiodic or prediction equivalent-weight**

a1) **technical evaluation of the trend**

In the case of the life cycle of a product, the curve is:



## Autoregressive method

**Autoregressive methods** can be divided into:

a) **technical data projection on to base multiperiodic or prediction equivalent-weight**

a1) **technical evaluation of the trend**

If you consider the time horizon is not very large, can be approximated with a linear trend:

$$D(t) = a + b \cdot t$$

The values of the coefficients a and b using the method of least squares are:

$$a = \frac{[\sum D(t) \cdot \sum t^2] - [\sum t \cdot (\sum t \cdot D(t))]}{N \cdot \sum t^2 - (\sum t)^2} \quad b = \frac{N \cdot (\sum t \cdot D(t)) - [\sum t \cdot (\sum D(t))]}{N \cdot \sum t^2 - (\sum t)^2}$$

these expressions by translating the y-axis parallel to itself are:

$$a = \frac{\sum_{t=1}^N D(t)}{N} \quad b = \frac{\sum_{t=1}^N t \cdot D(t)}{\sum_{t=1}^N t^2}$$

## Autoregressive method

**Autoregressive methods** can be divided into:

- a) **technical data projection on to base multiperiodic or prediction equivalent-weight**
- a2) **technical evaluation of the seasonal**

The periodic fluctuations in demand compared to the average value represents the seasonal trend.

Identified period  $N_p$  that constitute the series, these periods are divided into cycles of equal length  $k$ , we define the component seasonal  $S_j$  (con  $j = 1, 2, \dots, k$ ) through the relation:

$$S_j = \frac{\text{value average of demand in the cycle}}{\text{value average of total demand}}$$

The value of demand in the months ahead can only be calculated on the assumption of trend and seasonality with the report:

$$\mathbf{D(t) = T(t) \cdot S_j(t)}$$

## Autoregressive method

Example: assuming you have a company for which the demand for product is characterized by the values of demand in different periods and for five years.

Period t	1	2	3	4	5	6	7	8	9	10	11	12	D <sub>total</sub>
D(t)– year 1	40	40	50	90	130	70	90	30	70	80	60	110	860
D(t)– year 2	50	50	70	100	120	90	80	30	60	80	90	120	940
D(t)– year 3	50	60	70	100	120	60	90	40	80	100	80	120	970
D(t)– year 4	40	60	50	130	130	110	80	50	100	70	110	130	1060
D(t)– year 5	60	80	70	100	120	120	110	80	50	100	100	120	1110
D <sub>p total</sub>	240	290	310	520	620	450	450	230	360	430	440	600	4940
D <sub>Mp</sub>	48	58	62	104	124	90	90	46	72	86	88	120	
$S_j = D_{Mp} / (D_{total}/60) = D_{Mp} / 82,33$	0,58	0,70	0,75	1,26	1,51	1,09	1,09	0,56	0,87	1,04	1,07	1,46	
$T(t) = 82,33 + 0,52 \cdot t$	82,85	83,37	83,89	84,41	84,92	85,45	85,97	86,49	87,01	87,53	88,05	88,57	
$D(t) = T(t) \cdot S_j(t)$	48,30	58,73	63,17	106,62	127,91	93,41	93,98	48,32	76,09	91,43	94,11	129,09	

They then determined the values of the coefficients a and b of the valuation technique of linear trend, pointing out that the shifting center of gravity making the y-axis is equal to 30.5 times:

$$a = \frac{\sum_{t=1}^N D(t)}{N} = \frac{4940}{60} = 82,33$$

$$b = \frac{\sum_{t=1}^N t \cdot D(t)}{\sum_{t=1}^N t^2} = \frac{9530}{17995} = 0,52$$

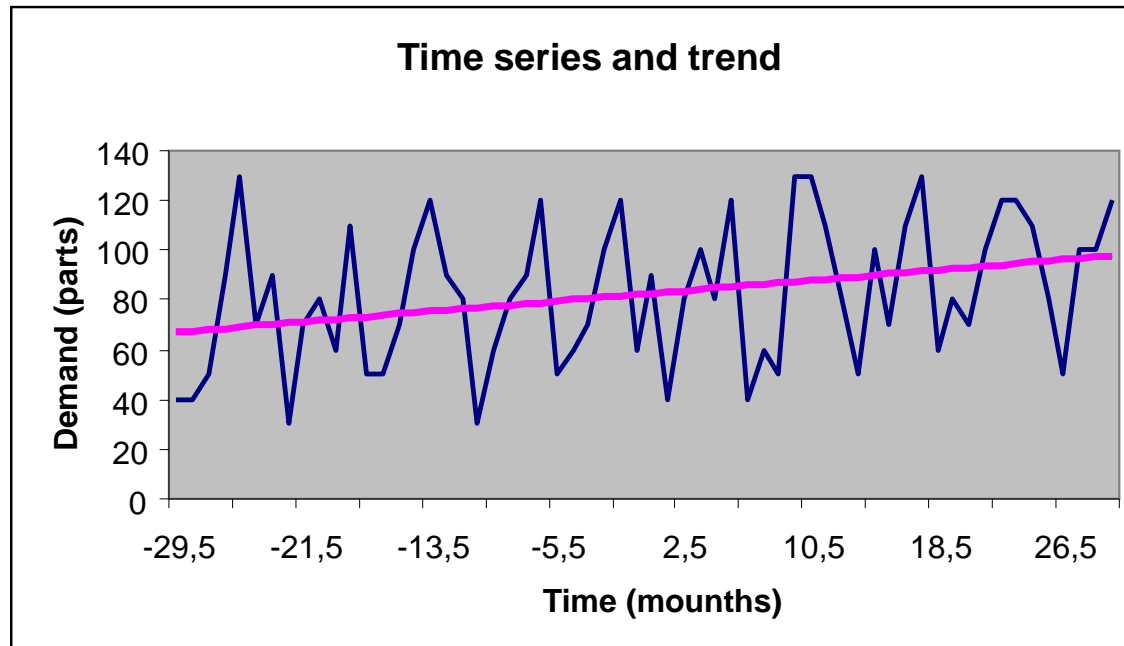
## Autoregressive method

Example:

The expression of the trend is equal to:

$$T(t) = 82,33 + 0,52 \cdot t$$

Recalling that the average value of the total demand is equal to 82.33, we calculated the seasonal coefficients  $S_j(t)$ , the trend  $T(t)$  and the value of demand in the twelve months following  $D(t)$ .



## Autoregressive method

**Autoregressive methods** can be divided into:

- a) **technical data projection on to base multiperiodic or prediction equivalent-weight**
- a3) **technical evaluation of uncertainty**

The component of uncertainty  $A(t)$  considers the deviations from the basic trend, but do not present a cyclical but random. In order to obtain the series of random component must de-trend and de-seasonally adjust the value of demand in the horizon:

$$A(t) = D(t) - T(t) \cdot S_j(t)$$

To characterize then the law of statistical distribution, the average value of  $A_m$  are defined, the standard deviation  $A_s$  and variance  $A_v$ :

$$A_m = \frac{\sum_{t=1}^N A(t)}{N} \quad A_s = \sqrt{\frac{\sum_{t=1}^N (A(t) - A_m)^2}{N - 1}} \quad A_v = \frac{\sum_{t=1}^N (A(t) - A_m)^2}{N - 1}$$

even if they fail to provide the entity of the deviations from the mean value of the variable.

## Autoregressive method

**Autoregressive methods** can be divided into:

- a) **technical data projection on to base multiperiodic or prediction equivalent-weight**
- a3) **technical evaluation of uncertainty**

You can also define the coefficient of variation  $C_v$ :

$$C_v = \frac{A_s}{A_m}$$

which is a parameter of opinion on the effect of the random component on demand.

The value of demand forecasting horizon in equi-weight method is a function of:

$$D = f(t, a, b, S_j, A_m, A_s)$$

## Autoregressive method

**Autoregressive methods** can be divided into:

### b) **technical data projection on to base aperiodic**

It gives equal weight to all data of the historical series, even if you have to consider the most recent data, it determines a single value, which represents the single period forecast horizon

#### b1) **moving average method**

Considering the historical values of the demand for a product defined by  $D_1, D_2, \dots, D_n$ , the moving average is considered the average of a number of specific data of the most recent questions:

$$D_{n+1} = \frac{\sum_{t=1}^n D_t}{n}$$

The oldest value is discarded whenever they become available new value of the application and the second average is:

$$D_{n+2} = \frac{\sum_{t=2}^{n+1} D_t}{n}$$



## Autoregressive method

**Autoregressive methods** can be divided into:

**b) technical data projection on to base aperiodic**

**b1) moving average method**

The method seeks to eliminate some of the forecast errors induced by the projection of historical data of demand. Is an important value to be assigned in the higher this parameter the greater the damping effect is obtained with moving averages.

In the case where  $n=1$ , this approach leads to the question of the last period and the value of demand forecast for the next instant coincides with the actual value of the last period of demand. The demand forecast for period  $t+1$   $PD_{t+1}$  amounted to actual demand at the time  $D_t$ :

$$PD_{t+1} = D_t$$

The optimal value of  $n$  is one that minimizes the error  $E_t$ :

$$E_t = D_t - PD_t$$

# Autoregressive method

**Autoregressive methods** can be divided into:

**b) technical data projection on to base aperiodic**

**b1) moving average method**

The optimal value of  $n$  is the one that minimizes the standard absolute deviation of errors (mean absolute deviation - MAD):

$$MAD = \frac{\sum_{t=1}^n |E_t|}{n}$$

Example:

You know the daily demand of a certain product:

Day	1	2	3	4	5	6	7	8	9	10	11
Demand	5	8	12	7	9	6	11	8	6	9	10

You must determine the values of the projected demand using the moving average of 3 days and 6 days and you have to compare the MAD of the forecast errors.

# Autoregressive method

**Autoregressive methods** can be divided into:

**b) technical data projection on to base aperiodic**

**b1) moving average method**

In the case of moving average of the three periods we have:

Day	Effective demand $D_t$	Forecast demand $PD_t$	Forecast error $E_t$	Sum of absolute values $E_t$
1	5			
2	8			
3	12			
4	7	8,3	-1,3	1,3
5	9			
6	6			
7	11			
8	8			
9	6			
10	9			
11	10			

## Autoregressive method

**Autoregressive methods** can be divided into:

**b) technical data projection on to base aperiodic**

**b1) moving average method**

In the case of moving average of the three periods we have:

Day	Effective demand $D_t$	Forecast demand $PD_t$	Forecast error $E_t$	Sum of absolute values $E_t$
1	5			
2	8			
3	12			
4	7	8,3	-1,3	1,3
5	9	9,0	0,0	1,3
6	6	9,3	-3,3	4,7
7	11	7,3	3,7	8,3
8	8	8,7	-0,7	9,0
9	6	8,3	-2,3	11,3
10	9	8,3	0,7	12,0
11	10	7,7	2,3	14,3

mean absolute deviation

$$MAD = \frac{\sum_{t=1}^n |E_t|}{n} = \frac{14,3}{11} = 1,3 \text{ demands / day}$$

# Autoregressive method

**Autoregressive methods** can be divided into:

**b) technical data projection on to base aperiodic**

**b1) moving average method**

In the case of moving average of the six periods we have:

Day	Effective demand $D_t$	Forecast demand $PD_t$	Forecast error $E_t$	Sum of absolute values $E_t$
1	5			
2	8			
3	12			
4	7			
5	9			
6	6			
7	11	7,8	3,2	3,2
8	8			
9	6			
10	9			
11	10			

## Autoregressive method

**Autoregressive methods** can be divided into:

**b) technical data projection on to base aperiodic**

**b1) moving average method**

In the case of moving average of the six periods we have:

Day	Effective demand $D_t$	Forecast demand $PD_t$	Forecast error $E_t$	Sum of absolute values $E_t$
1	5			
2	8			
3	12			
4	7			
5	9			
6	6			
7	11	7,8	3,2	3,2
8	8	8,8	-0,8	4,0
9	6	8,8	-2,8	6,8
10	9	7,8	1,2	8,0
11	10	8,2	1,8	9,8

mean absolute deviation  $MAD = \frac{\sum_{t=1}^n |E_t|}{n} = \frac{9,8}{11} = 0,9 \text{ demands / day}$

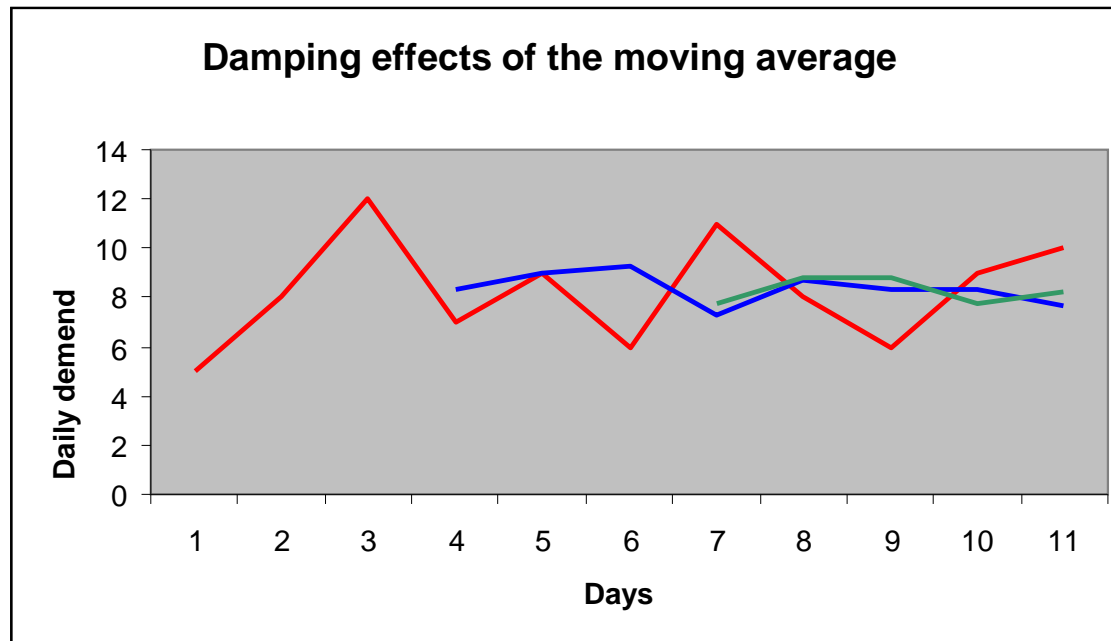
## Autoregressive method

**Autoregressive methods** can be divided into:

**b) technical data projection on to base aperiodic**

**b1) moving average method**

The results are visible in the figure, from which you can see that is convenient to use the method of moving average that provides the lowest mean absolute deviation MAD and then in the case with  $n = 6$



## Autoregressive method

**Autoregressive methods** can be divided into:

**b) technical data projection on to base aperiodic**

**b2) weighted moving average method**

The prediction of the demand is made by assigning the same number of times a different weight  $W$ , whose attribution is made to allow the application of more recent periods may affect more than forecast and remote, this allows for a faster response to recent changes the demand forecast.

The advantage is the ability to judge the relative importance of the periods referred to in the media, allowing the auditor to consider the trend or seasonal effects. The disadvantage is that the choice of weights is subjective.



## Autoregressive method

**Autoregressive methods** can be divided into:

**b) technical data projection on to base aperiodic**

**b2) weighted moving average method**

The weighted moving average to define the forecast demand for period  $n+1$   $D_{n+1}$ , notice the weight assigned to period  $t$   $W_t$ , which is expressed by the relation:

$$D_{n+1} = \frac{\sum_{t=1}^n (D_t \cdot W_t)}{\sum_{t=1}^n W_t}$$

When you assign a weight 1 for each of the periods, returns to the simple moving average forecast.

## Autoregressive method

**Autoregressive methods** can be divided into:

**b) technical data projection on to base aperiodic**

**b2) weighted moving average method**

Example:

If you consider the actual demand during the five days of the week, consider the prediction on the first day of the next question with the technique of weighted moving average where the weights have been attributed to effective demand.

Day	1	2	3	4	5	6
Effective demand $D_t$	50	60	55	60	50	55
Weight $W_t$	0,1	0,13	0,16	0,26	0,35	

$$D_{n+1} = \frac{\sum_{t=1}^n (D_t \cdot W_t)}{\sum_{t=1}^n W_t} = \frac{50 \cdot 0,1 + 60 \cdot 0,13 + 55 \cdot 0,16 + 60 \cdot 0,26 + 50 \cdot 0,35}{1,0} = 54,7 \cong 55$$

## Autoregressive method

**Autoregressive methods** can be divided into:

b) **technical data projection on to base aperiodic**

b3) **exponential smoothing method**

The method level values of the time series of historical demand, assigning a different weight to the data series and decreasing depending on seniority. The most recent data are those that contribute more to the value of forecasting demand.

The weights are determined by choosing a damping factor  $\alpha$  ( $0 \leq \alpha \leq 1$ ) and the demand forecast for period  $t+1$   $PD_{t+1}$ , note the actual demand at time  $t$   $D_t$ , is:

$$PD_{t+1} = PD_t + \alpha \cdot (D_t - PD_t)$$

where  $(D_t - PD_t)$  represents the forecast error in period  $t$ .

It is observed that when  $\alpha = 0$  then  $PD_{t+1} = PD_t$ , and then when  $\alpha=1$  then  $PD_{t+1} = D_t$ .

## Autoregressive method

**Autoregressive methods** can be divided into:

**b) technical data projection on to base aperiodic**

**b3) exponential smoothing method**

The damping coefficient  $\alpha$  is related to the number of periods considered, so that the limit of sensitivity of exponential smoothing is expressed by the relation:

$$N_p = \frac{(2 - \alpha)}{\alpha}$$

where NP is the number of periods.

It is noted that when  $\alpha=1$  the number of periods is  $N_p=1$  and then  $\alpha$  decreases when  $N_p$  increases.

## Autoregressive method

**Autoregressive methods** can be divided into:

**b) technical data projection on to base aperiodic**

**b3) exponential smoothing method**

The determination of the damping coefficient can be made:

- method attempts

The value of the damping coefficient  $\alpha$  is such as to minimize the difference between the forecast value and actual value, and therefore it must clear the error:

$$E_t = D_t - PD_t$$

- method of the coefficient of interference CD

The coefficient CD provides information on the distribution data respect the mean. If data are missing respect the average value, the CD is very high:

$$CD = \frac{MAD}{D_m} = \frac{\sum_{t=1}^n |E_t|}{\sum_{t=1}^n D_t}$$

## Autoregressive method

**Autoregressive methods** can be divided into:

b) **technical data projection on to base aperiodic**

b3) **exponential smoothing method**

- method of the coefficient of interference CD

The value to assign to the damping coefficient is the lower the higher the value of the CD.

In the case of companies that produce durable goods, which are characterized by stable demand, the value of the damping coefficient is around 0.2, while in the case of companies that produce luxury goods, which are characterized by an unstable demand, the value of the damping coefficient is around 0.7.

## Autoregressive method

**Autoregressive methods** can be divided into:

**b) technical data projection on to base aperiodic**

**b3) exponential smoothing method**

Example:

In case you have a question in the days and a forecast real, suppose you take as a first attempt value of damping coefficient  $\alpha = 0,2$ :

Day	Effective demend $D_t$	Forecast demend $PD_t$	Forecast error $(D_t - PD_t)$	Adjustment $\alpha \cdot (D_t - PD_t)$	Forecast damped	$\sum_{t=1}^n  E_t $
1	9	9,0	0	0	9,0	
2	6	9,0	-3,0	-0,6	8,4	3,0
3	11	8,4	2,6	0,5	7,9	5,6
4	8	7,9	-0,1	0	7,9	5,7
5	6	7,9	-1,9	-0,4	7,5	7,6

$$MAD = \frac{\sum_{t=1}^n |E_t|}{n} = \frac{7,6}{4} = 1,9 \text{ demands / day}$$

## Autoregressive method

**Autoregressive methods** can be divided into:

b) **technical data projection on to base aperiodic**

b3) **exponential smoothing method**

Example:

If we consider instead a damping coefficient  $\alpha = 0,7$  we have:

Day	Effective demend $D_t$	Forecast demend $PD_t$	Forecast error $(D_t - PD_t)$	Adjustment $\alpha \cdot (D_t - PD_t)$	Forecast damped	$\sum_{t=1}^n  E_t $
1	9	9,0	0	0	9,0	
2	6	9,0	-3,0	-2,1	6,9	3,0
3	11	6,9	4,1	2,9	9,8	7,1
4	8	9,8	-1,8	-1,3	8,5	8,9
5	6	8,5	-2,5	-1,8	6,7	11,4

$$MAD = \frac{\sum_{t=1}^n |E_t|}{n} = \frac{11,4}{4} = 2,9 \text{ demands / day}$$



## Autoregressive method

**Autoregressive methods** can be divided into:

**b) technical data projection on to base aperiodic**

**b4) associative projection method**

It does not determine the question with respect to the time variable, but in terms of other variables. Through the correlation is established the existence of a relationship between the demand and another variable (gross national income for abitant or wholesale consumer prices etc.). You can verify the existence of this link through the use of the **correlation diagram**.

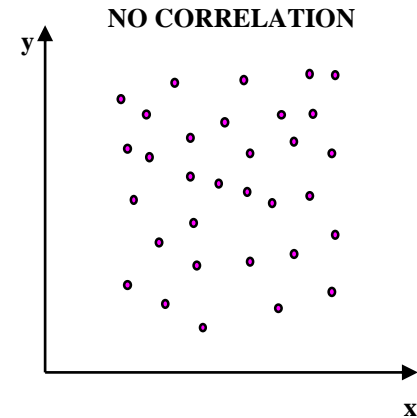
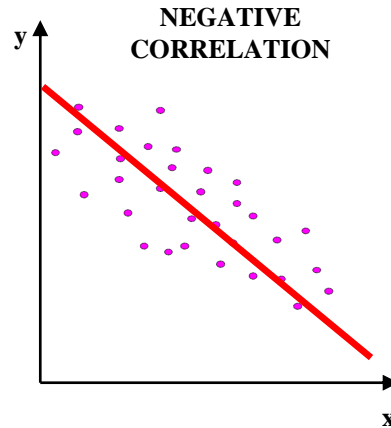
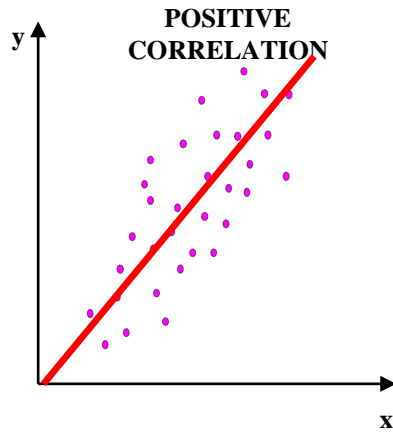
An analysis of the diagram can be found a positive correlation in which they are accumulating points on a straight line with positive slope, but if the slope is negative then it has a negative correlation, while if there is no correlation that is void and the points are distributed in a completely random in the Cartesian plane.

# Autoregressive method

**Autoregressive methods** can be divided into:

b) **technical data projection on to base aperiodic**

b4) **associative projection method**



## Autoregressive method

**Autoregressive methods** can be divided into:

b) **technical data projection on to base aperiodic**

**b4) associative projection method**

You can verify the existence of this link through the use of the **correlation coefficient**.

Qualitative examination of the correlation diagram, the correlation coefficient quantitatively evaluates the report:

$$r = \frac{\sum_{t=1}^n (D_t - \bar{D})(x_t - \bar{x})}{\sqrt{\sum_{t=1}^n (D_t - \bar{D})^2 \cdot \sum_{t=1}^n (x_t - \bar{x})^2}}$$

with a value of the correlation coefficient  $-1 \leq r \leq 1$

## Autoregressive method

**Autoregressive methods** can be divided into:

**b) technical data projection on to base aperiodic**

**b4) associative projection method**

Example:

Consider a machine which completes its cycle of production in a time-dependent by temperature level at which the same is maintained during the production process. Suppose you have found a set of values of time and temperature for a sample of 10 production cycles (Table). It is noted that the average values are respectively 2.825 minutes and 23.645°C, while the median values are respectively 2.90 minutes and 23.61°C.

Time (minutes)	2,43	2,61	2,64	2,68	2,71	2,94	2,96	3,32	3,32	3,36
Temperature (°C)	23,73	23,74	23,68	23,64	23,66	23,63	23,57	23,48	23,48	23,46

## Autoregressive method

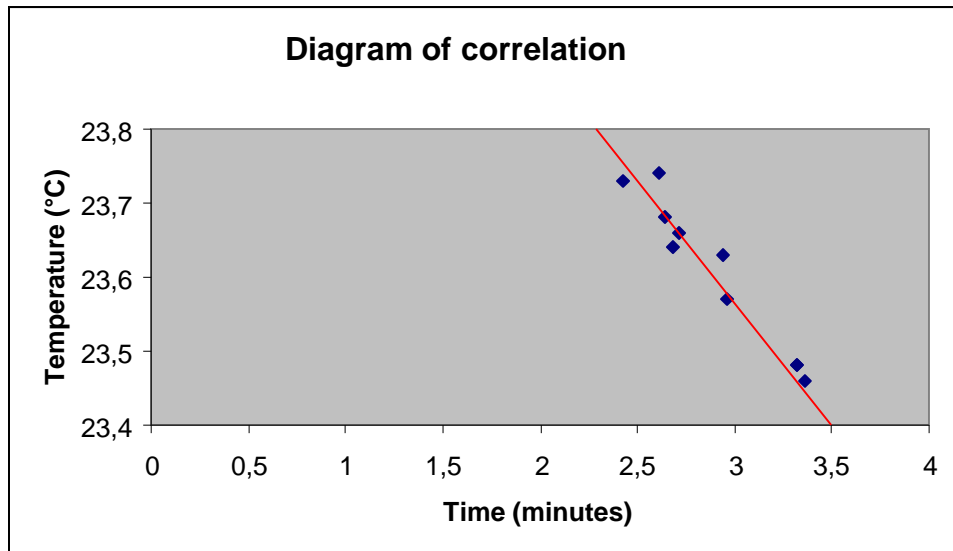
**Autoregressive methods** can be divided into:

**b) technical data projection on to base aperiodic**

**b4) associative projection method**

Example:

For a diagram of correlation of figure, is associated with a correlation coefficient which is equal to:



$$r = \frac{\sum_{t=1}^n (D_t - \bar{D})(x_t - \bar{x})}{\sqrt{\sum_{t=1}^n (D_t - \bar{D})^2 \cdot \sum_{t=1}^n (x_t - \bar{x})^2}} = \frac{-0,31}{\sqrt{1,03 \cdot 0,10}} = -0,97$$