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INDUSTRIAL PLANTS

Chapter nine:

Use of economic variables in the decisions in industrial plants

DOUBLE DEGREE MASTER IN

“PRODUCTION ENGINEERING AND MANAGEMENT”

SEAT OF PORDENONE

UNIVERSITY OF TRIESTE

Generality

A **decision is a choice between several alternatives**. In most cases, the decision concerning the management of a company can be translated into quantitative terms the different alternatives that are faced. The phases that characterize it are:

- **defining of the problem;**
- **identification of possible alternatives;**
- **identification of the criterion of preference among alternatives;**
- **evaluation and measurement of the consequences of the alternatives, expressed in quantitative terms, consistent with the evaluation criteria chosen;**
- **evaluation of the consequences that can not be expressed in terms of quantity and relevance of such consequences in relation to measurable variables;**
- **decision through the choice of a line of action**.

Generality

We analyze in this area:

- methods for assessment of economic investment of industrial plants;
- method for assessment of economic of the replacement of industrial plants;
- raising of capital;
- equilibrium of the company in the short term, through the analysis of business costs.

Methods of assessing the economic convenience of investments of the industrial plants

The concept of cash flow

After the investment, the company presents a succession, in different time, outputs (or output costs) and revenue (revenue or inputs), which represent cash flows with opposite sign (cash flow).

To evaluate an investment project, it is necessary to know the estimates of the cash flow that year j-th F_j is:

$$F_j = (R_j - C_j - A_j) - Im_j + A_j$$

R_j = sum of revenues at j-th year;

C_j = sum of costs at j-th year;

A_j = amortization at j-th year;

Im_j = taxes on the j-th year related to the difference ($R_j - C_j - A_j$).

In the case where there is a bank loan:

$$F_j = (R_j - C_j - A_j - Qi_j) - Im_j + A_j - Qc_j$$

Qi_j = interest portion of the loan to the j-th year;

Qc_j = principal amount of the loan to the j-th year.

Methods of assessing the economic convenience of investments of the industrial plants

<u>The concept of cash flow</u>

Industrial investments typically have cash flows of type C.I.P.O. (Continuous Input Point Output).

The **technical-economic calculations of convenience** require the following information:

a) the estimate of duration of the investment;
b) the estimate of production volume and of selling prices;
c) the estimate of total costs future of production and operation;
d) the temporal sequence of investment required to purchase, construction, commissioning, and operation plant (labor, raw materials, energy, maintenance etc.), salvage value and return on assets (at the end of economic life);
e) the estimate of time required for completion and commissioning of the plant.

Methods of assessing the economic convenience of investments of the industrial plants

The concept of cash flow

After determining the investment alternatives, the next step is to calculate the cash flows associated with each of them, which may be absolute or relative.

They say **cash flows absolute** are compared with those cash flow null or zero corresponding to the alternative investment "don'ts" while said **cash flows differentiated** or related those derived from the comparison of the flows of two alternatives A and B.

Methods of assessing the economic convenience of investments of the industrial plants

Criteria for evaluating investments

The evaluation criteria can be divided into two main categories:

- **simple or arithmetic approximate methods** (method of the recovery period simple);
- **precise methods based on the concept of discounting** (net present value method and the discounted rate of return method).

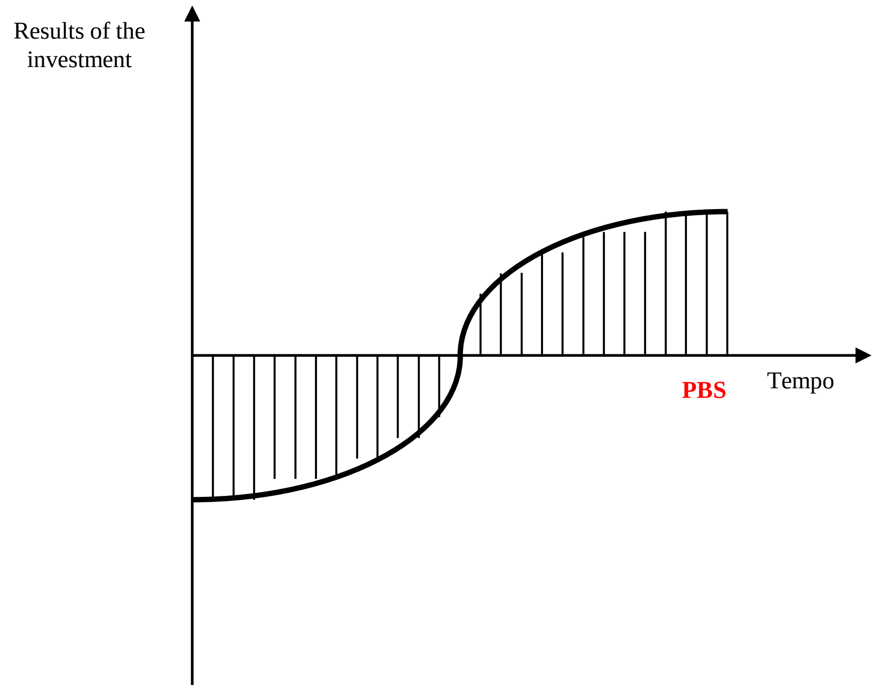
Methods of assessing the economic convenience of investments of the industrial plants

<u>Criteria for evaluating investments</u>

Simple approximate methods

a) simple payback period

The index measures the number of years within which the cash flow from investment is can "recover" the investment.



Methods of assessing the economic convenience of investments of the industrial plants

Criteria for evaluating investments

Simple approximate methods

a) **simple payback period**

Exist the relationship:

$$-\sum_{j=0}^{m} I_j + \sum_{k=1}^{PBS} F_k = 0$$

I_j = investment in the j-th;

F_k = cash flow for the year k-th;

PBS = simple payback period;

$$m \leq PBS.$$

To provide a positive opinion should be:

$$\mathbf{PBS \leq PBS_{limit}}$$

The usual value is 3-4 years.

Between two or more investment alternatives will choose one that has the smallest PBS provided that it is ≤ PBS_{limit}.

Methods of assessing the economic convenience of investments of the industrial plants

Criteria for evaluating investments

Simple approximate methods

a) simple payback period

Example: between three investment alternatives (table), from which shows that the best alternative is A as it retrieves the first investment made (1.000 k€)

Year	Alternative A		Alternative B		Alternative C	
	Investment	Cash flow	Investment	Cash flow	Investment	Cash flow
0	1.000	-	1000	-	1000	-
1	-	350	-	200	-	50
2	-	650	-	300	-	150
3	-	500	-	500	-	300
4	-	650	-	400	-	500
Simple payback period	2 years		3 years		4 years	

Methods of assessing the economic convenience of investments of the industrial plants

Criteria for evaluating investments

Simple approximate methods

a) simple payback period

The method has two basic flaws:

- does not measure the return on investment since it ignores the existence of each cash flow (positive or negative) after the recovery period simple;
- does not take into account the value of money in the time, that is from the same monetary value on cash flows the same, but obtained at different times.

This criterion is used when:

- among several alternatives to compare it is preferred that which presents a lower recovery time;
- two or more options having the same recovery period are considered indifferent for the purposes of choice.

Methods of assessing the economic convenience of investments of the industrial plants

<u>Criteria for evaluating investments</u>

Precise methods based on the concept of discounting

a) **Net present value (NPV) or economic result discounted (REA)**

The method of the net present value is used for the choice as:

- between a number of investments, will be preferred that to REA highest;
- in the choice between two alternatives, if these have the same REA, they will say then indifferent, but not necessarily identical;
- when you have to evaluate whether or not to make an investment, be biased to the first hypothesis (do) if the REA is greater than zero.

An investment is convenient if:

$$REA = -I_o + \sum_{k=1}^{n} \frac{F_k}{(1+i_A)^k} > 0$$

and also:

$$\mathbf{REA \geq REA_{limit}}$$

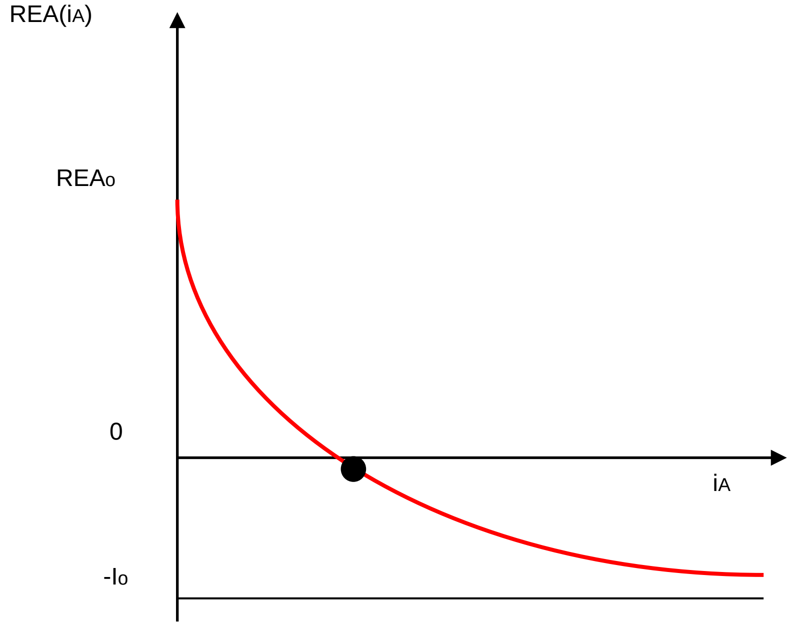
Methods of assessing the economic convenience of investments of the industrial plants

<u>Criteria for evaluating investments</u>

Precise methods based on the concept of discounting

a) Net present value (NPV) or economic result discounted (REA)

Graphically:



The rate "r" is called the internal rate of return on investment

Criteria for evaluating investments

Precise methods based on the concept of discounting

a) Net present value (NPV) or economic result discounted (REA)

Example:

Year	Present value of output (M€)	Revenue (M€)	Discount rate and actual values					
			i_A = 5%	VA (5%)	i_A = 7,5%	VA (7,5%)	i_A = 10%	VA (10%)
0	100.000	-	-	-	-	-	-	-
1	-	26.000	0,952	24.752	0,930	24.180	0,909	23.634
2	-	26.000	0,907	23.582	0,865	22.490	0,826	21.476
3	-	26.000	0,864	22.464	0,805	20.930	0,751	19.526
4	-	26.000	0,823	21.398	0,749	19.474	0,683	17.758
5	-	26.000	0,783	20.358	0,696	18.096	0,612	16.146
Total		130.000	112.552		104.936		98.540	

Assuming a discount rate of 7.5% and applying it to the cash flow, it has a discounted value greater at investment, so the project could be approved. The difference (from 104,936 to 100,000) = 4,936 M € indicates that the wealth of the company will be increased by this amount and that, therefore, will contribute to satisfy the hope of shareholder value as 4,936 M € are a useful

Methods of assessing the economic convenience of investments of the industrial plants

Criteria for evaluating investments

Precise methods based on the concept of discounting

b) internal rate of return (IRR) or discounted cash flow rate of return (DCF RR)

The purpose of this criterion is to determine the value of "r" or IRR of i_A that cancels the REA distribution of costs and returns on the investments:

$$-I_o + \sum_{k=1}^{n} \frac{F_k}{(1+TRI)^k} = 0$$

An investment project A is acceptable if the IRR is:

$$\mathbf{IRR \geq IRR_{limit}}$$

while among the most investments you choose the one with the highest IRR provided that:

$$\mathbf{IRR_A > IRR_B} \qquad \mathbf{IRR_A \geq IRR_{limit}}$$

Methods of assessing the economic convenience of investments of the industrial plants

Criteria for evaluating investments

Precise methods based on the concept of discounting

b) Internal rate of return (IRR) or discounted cash flow rate of return (DCF RR)

Example:

Year	Investment or recovery	Revenue	Costs	Profits + gros tax depreciation	Taxable income	Taxes	Net revenue	Investment[1] and cash flow
0	- 150	-	-	-	-	-	-	-150
1	-150	-	-	-	-	-	-	-150
2	-400	-	-	-	-	-	-	-400
3	-	1.200	750	450	350	140	210	+310
4	-	1.200	750	450	350	140	210	+310
5	-	1.200	750	450	350	140	210	+310
6	-	1.200	750	450	350	140	210	+310
7	-	1.200	750	450	350	140	210	+310
Fine 7	+200	[1] $F_j = R_j – C_j – A_j – Im_j + A_j$						+200

The amortization is (700 – 200) / 5 = 100 k€. For taxes asssume a rate of 40%.

Methods of assessing the economic convenience of investments of the industrial plants

Criteria for evaluating investments

Precise methods based on the concept of discounting

b) Internal rate of return (IRR) or discounted cash flow rate of return (DCF RR)

May be set as the calculation of the IRR (values in €):

Year	Cash flow	$i_A = 20\%$		$i_A = 25\%$		$i_A = 27\%$		$i_A = 28\%$	
		Discount factor	Accouint discounted	Discount factor	Accouint discounted	Discount factor	Accouint discounted	Discount factor	Accouint discounted
0	-150	1	-150,0	1	-150,0	1	-150,0	1	-150,0
1	-150	0,833	-125,0	0,800	-120,0	0,787	-118,1	0,781	-117,2
2	-150	0,694	-277,6	0,640	-256,0	0,620	-248,0	0,610	-244,1
3	+310	0,579	+179,4	0,512	+158,7	0,488	+151,3	0,477	+147,8
4	+310	0,482	+149.4	0,410	+127,0	0,384	+119,2	0,373	+115,5
5	+310	0,402	+124,6	0,328	+101,6	0,303	+93,8	0,291	+90,2
6	+310	0,335	+103,9	0,262	+81,3	0,238	+73,9	0,227	+70,5
7	+310	0,279	+85,3	0,210	+65,0	0,188	+58,2	0,178	+55,1
End 7	+200	0,233	+72,2	0,168	+33,6	0,148	+29,6	0,139	+27,8
REA		+162,3		+41,2		+9,9		-4,4	

<u>Criteria for evaluating investments</u>

Precise methods based on the concept of discounting

b) Internal rate of return (IRR) or discounted cash flow rate of return (DCF RR)

The value of the IRR can be found with a linear interpolation:

$$x = \frac{REA_{27}}{REA_{27} - REA_{28}} = \frac{9{,}9}{9{,}9 - (-4{,}4)} = \frac{9{,}9}{14{,}3} = 0{,}69$$

IRR = (27 + x)% = (27 + 0,69)% = 27.69%

Methods of assessing the economic convenience of investments of the industrial plants

The Ruggeri has developed a rational process that expects to apply a characteristic model of the exponential variation of the management costs, wear, and obsolescence as a function of the value attributed to technical progress, which for simplicity is considered a variable between 0 and 1.

The determination of the optimal replacement policy requires knowledge of some fundamental values:

a) **value of the plant new to the beginning of the year in question**, defined by the notation: $\mathbf{C_{n,t}}$ with **$t = 0, 1, \ldots, n$**

for t =0 it has: $C_{n,0}$ represents the cost of purchase of the new (the best on the market) at time zero;

for t=1 it has: $C_{n,1}$ represents the cost of the new plant (still the best) that will be available early in the second year;

etc.

The cost of the new plant decreases according to the report:

$$C_{n,t} = C_{n,0} \cdot e^{-g \cdot t}$$

g represents the technical progress.

Methods of assessing the economic convenience of investments of the industrial plants

b) **realizable value of the new plant at the end of its useful life**, defined by: $\mathbf{C_{nr,t}}$ with $\mathbf{t = t_n}$ duration of useful life of plant.

It depends on three key factors:

- by age of the plant;
- by depreciation or obsolescence of the plant;
- by real wear physical.

With increasing of age of the plant there is an increase of wear, which entails higher costs maintenance, reduction of the efficiency of the plant and increase in the waste product; at time t_n the realization value depends on from both technical progress g from the effect of wear and tear u and is equal to:

$$C_{nr,tn} = C_{n,0} \cdot e^{-(g+u)\cdot t}$$

If you work in conditions of stationary economy (technical progress g equal zero), having the same plants on the market, and u > 0, the residual value of the plant decreases again as a result of the wear.

c) **annual cost of managing the new plant at the beginning of the year in question**, defined by the notation: $\mathbf{E_{n,t}}$ with $\mathbf{t = 0, 1, \ldots, n}$

It comprises the direct labor costs, maintenance, energy and raw materials.

Technological progress make the cost of management of the plants, which will be presented later on the market, is decreasing over time as it reduces the labor cost per unit of product.

The relation of the annual cost of operating the new plant is:

$$E_{n,t} = E_{n,0} \cdot e^{-g \cdot t}$$

where:

$E_{n,0}$ = cost of management of the new plant at zero time (just installed).

d) technical progress g

The accuracy that one must set at the prevention the value to be attributed at g is closely linked to the knowledge of the evolution of the market.

Surely you must consider that the phenomenon evolves linearly over time is entirely theoretical and never perfectly found in practice.

Methods of assessing the economic convenience of investments of the industrial plants

e) **value of the old plant to subject to assessment in the year in question**, defined by the notation: $\mathbf{C_{v,t}}$ with **$t = 0, 1, \ldots, m$**

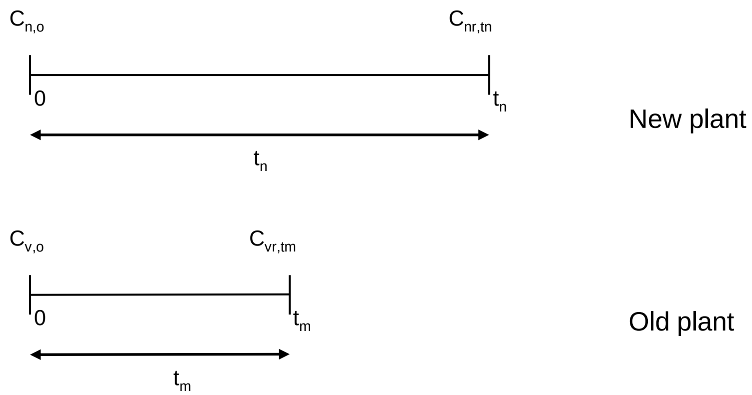
This value is a function of age and remaining expected life of the plant, and can be determined by the profitability in the company or at a market value of the latter depends on the expected future income and thus varies with the changing of rate of obsolescence.

The value $C_{v,t}$ is therefore not obtained through the use of mathematical expressions more or less sophisticated, but from practical assessments.

Methods of assessing the economic convenience of investments of the industrial plants

Methodology

1st year



The equivalent annual cost of the new facility at the time t = 0 is:

$$\left[C_{n,o} - \frac{C_{nr,tn}}{(1+i)^{tn}} \right] \cdot \alpha_{tn\neg i} + E_{n,o} = EAC(n_o)$$

$\alpha_{tn\neg i}$ = unitary term of depreciation, which is the annual installment postponed necessary to pay off in t_n years the debt of € 1 and expressible by:

$$\alpha_{tn\neg i} = \frac{(1+i)^{tn} \cdot i}{(1+i)^{tn} - 1}$$

Methods of assessing the economic convenience of investments of the industrial plants

<u>Methodology</u>

The equivalent annual cost of the new plant at time t = 0 is expressible:

$$\left[C_{n,o} - C_{nr,tn}\right] \cdot \alpha_{tn\neg i} + C_{nr,tn} \cdot i + E_{n,o} = EAC(n_o)$$

For the old plant into operating, the annual cost equivalent to the time t = 0 is:

$$\left[C_{v,o} - C_{vr,tm}\right] \cdot \alpha_{tv\neg i} + C_{vr,tm} \cdot i + E_{v,o} = EAC(v_o)$$

The convenience to replace is:

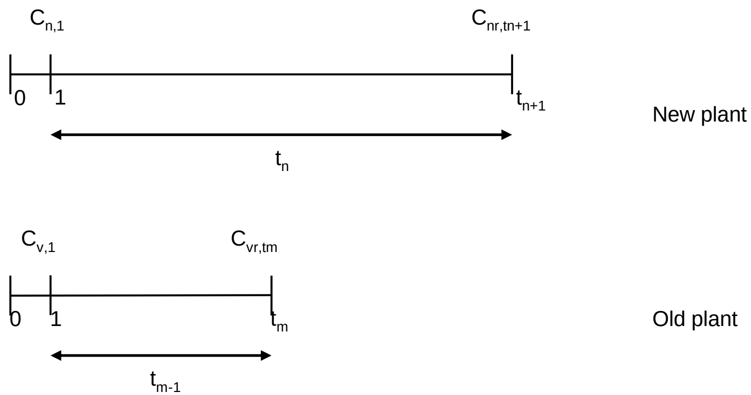
$$EAC(n_o) < EAC(v_o)$$

Otherwise we consider what happens next year.

Methods of assessing the economic convenience of investments of the industrial plants

Methodology

2nd year



The equivalent annual cost of the new and old system at time t = 1 is:

$$\left[C_{n,o} \cdot e^{-g \cdot 1} - C_{nr,tn+1}\right] \cdot \alpha_{tn\neg i} + C_{nr,tn+1} \cdot i + E_{n,o} \cdot e^{-g \cdot 1} = EAC(n_1)$$

$$\left[C_{v,1} - C_{vr,tm}\right] \cdot \alpha_{tv-1\neg i} + C_{vr,tm-1} \cdot i + E_{v,1} = EAC(v_1)$$

The convenience to replace is:

$$EAC(n_1) < EAC(v_1)$$

Otherwise we consider what happens next year.

Methods of assessing the economic convenience of investments of the industrial plants

Methodology

If you do not have this inequality, we analyze what happens in the third year and so on until the inequality is:

$$EAC(n_t) < EAC(v_t)$$

where will agree to replace the old system with a new one on the market, although preferable not to make forecasts beyond five years.

The method is used to determine the economic convenience to the replacement of the plant in operation in the presence of demand expansion, that can be qualitative (production of new products) and/or quantitative (increase in the current production).

Methods of assessing the economic convenience of investments of the industrial plants

Methodology

In the presence of demand growth that can be qualitative (production of new products), using the **method of equivalent annual cost** compared to the actual annual production and cost to replace the existing system at the beginning of the s-th year with a new plant is:

$$C_{un,s} < C_{uv,s}$$

having expressed the unit cost of the new and old system at time t = s as:

$$\frac{\left[C_{n,o} \cdot e^{-g \cdot s} - C_{nr,tn+s}\right] \cdot \alpha_{tn \neg i} + C_{nr,tn+s} \cdot i + E_{n,o} \cdot e^{-g \cdot s}}{P_{n,s}} = C_{un,s}$$

$$\frac{\left[C_{v,s} - C_{vr,tm-s}\right] \cdot \alpha_{tv-s \neg i} + C_{vr,tm-s} \cdot i + E_{v,s}}{P_{v,s}} = C_{uv,s}$$

where:

$P_{n,s}$ = actual annual production of the new plant at the time t = s

$P_{v,s}$ = actual annual production of the old plant at the time t = s

Methods of assessing the economic convenience of investments of the industrial plants

Methodology

In the presence of demand growth that can be qualitative (production of new products) and/or quantitative (increase in the current), we consider the **method of maximum useful**. Convenience to replace the existing system at the beginning of the s-th year with a new system is:

$$Z_{n,s} > Z_{v,s}$$

having expressed the annual profit of the new and old equipment at the time t = s:

$$R_{n,s\cdot} \cdot P_{n,s} - \left\{ \left[C_{n,o} \cdot e^{-g \cdot s} - C_{nr,tn+s} \right] \cdot \alpha_{tn \neg i} + C_{nr,tn+s} \cdot i + E_{n,o} \cdot e^{-g \cdot s} \right\} = Z(n_s)$$

$$R_{v,s} \cdot P_{v,s} - \left[C_{v,s} - C_{vr,tm} \right] \cdot \alpha_{tv-s \neg i} + C_{vr,tm-s} \cdot i + E_{v,s} = Z(v_s)$$

with:

$R_{n,s}$ = revenue per unit of output of the new plant at the time t =s

$P_{n,s}$ = actual annual production of the new plant at the time t = s

$R_{v,s}$ = revenue per unit of output of the old plant at the time t =s

$P_{v,s}$ = actual annual production of the old plant at the time t = s

Methods of assessing the economic convenience of investments of the industrial plants

Methodology

Example

New plant	Old plant
R_n = 1000,00 €/piece ……….. time s	R_v = 1000,00 €/piece ……….. time s
$P_{n,0}$ = 10.000 piece	P_v = 9.000 pieces …………… ime s
$P_{n,1}$ = 12.000 piece	$E_{v,0} = 2 . 10^6$ €
$P_{n,2}$ = 15.000 piece	$E_{v,1} = 3 . 10^6$ €
$E_{n,0} = 2 . 10^6$ €	$E_{v,2} = 4{,}5 . 10^6$ €
$C_{n,0} = 2 . 10^7$ €	$C_{v,0} = 5 . 10^6$ €
t_n = 10 years	t_m = 4 anni
$C_{nr,tn} = C_{n,t} . e^{-(g+u).t}$	$C_{vr,tm} = 10^7$ €
i = 10%	i = 10%
G = 01,4 valued	$C_{v,t} = C_{v,0} – t.(5 . 10^5)$
U = 0,06 valued	

Methodology

Example

a) equivalent annual cost method

- at the beginning of the first year

$$\left\{\left[20-20\cdot e^{-(0,14+0,06)\cdot 10}\right]\cdot \alpha_{10\neg 0,10}+20\cdot e^{-(0,14+0,06)\cdot 10}\cdot 0,10+2\right\}\cdot 10^{6}=5,8\cdot 10^{6}\ €$$

$$\left\{\left[5-1\right]\cdot \alpha_{4\neg 0,10}+1+2\right\}\cdot 10^{6}=3,36\cdot 10^{6}\ €$$

- at the beginning of the second year

$$\left\{\left[20\cdot e^{-0,14\cdot 1}-20\cdot e^{-0,14\cdot 1}\cdot e^{-(0,14+0,06)\cdot 10}\right]\cdot \alpha_{10\neg 0,10}+20\cdot e^{-0,14\cdot 1}\cdot e^{-(0,14+0,06)\cdot 10}\cdot 0,10+2\cdot e^{-0,14\cdot 1}\right\}\cdot 10^{6}=$$
$$=4,43\cdot 10^{6}\ €$$

$$\left\{\left[4,5-1\right]\cdot \alpha_{4-1\neg 0,10}+1\cdot 0,10+3\right\}\cdot 10^{6}=4,51\cdot 10^{6}\ €$$

from which it follows: the replacement cost at the beginning of 2nd year

<u>Methodology</u>

Example

b) method of the minimum unit cost

- at the beginning of the first year

$$\frac{1}{10000}\cdot\left\{\left[20-20\cdot e^{-(0,14+0,06)\cdot 10}\right]\cdot\alpha_{10\neg 0,10}+20\cdot e^{-(0,14+0,06)\cdot 10}\cdot 0,10+2\right\}\cdot 10^{6}=508\ €/\,pezzo$$

$$\frac{1}{9000}\cdot\left\{\left[5-1\right]\cdot\alpha_{4\neg 0,10}+1+2\right\}\cdot 10^{6}=373\ €/\,pezzo$$

- at the beginning of the second year

$$\frac{1}{12000}\cdot\left\{\begin{matrix}\left[20\cdot e^{-0,14\cdot 1}-20\cdot e^{-0,14\cdot 1}\cdot e^{-(0,14+0,06)\cdot 10}\right]\cdot\alpha_{10\neg 0,10}+\\ 20\cdot e^{-0,14\cdot 1}\cdot e^{-(0,14+0,06)\cdot 10}\cdot 0,10+2\cdot e^{-0,14\cdot 1}\end{matrix}\right\}\cdot 10^{6}=369\ €/\,pezzo$$

$$\frac{1}{9000}\cdot\left\{\left[4,5-1\right]\cdot\alpha_{4-1\neg 0,10}+1\cdot 0,10+3\right\}\cdot 10^{6}=501\ €/\,pezzo$$

from which it follows: the replacement cost at the beginning of 2nd year

Methodology

Example

c) method of maximum profit

- at the beginning of the first year

$$10^3 \cdot 10^4 \cdot \left\{ \left[20 - 20 \cdot e^{-(0,14+0,06)\cdot 10} \right] \cdot \alpha_{10\neg 0,10} + 20 \cdot e^{-(0,14+0,06)\cdot 10} \cdot 0,10 + 2 \right\} \cdot 10^6 = 4,92 \cdot 10^6 \text{ €}$$

$$10^3 \cdot 9 \cdot 10^3 \cdot \left\{ \left[5 - 1 \right] \cdot \alpha_{4\neg 0,10} + 1 + 2 \right\} \cdot 10^6 = 5,64 \cdot 10^6 \text{ €}$$

- at the beginning of the second year

$$10^3 \cdot 1,2 \cdot 10^4 \cdot \left\{ \begin{array}{l} \left[20 \cdot e^{-0,14\cdot 1} - 20 \cdot e^{-0,14\cdot 1} \cdot e^{-(0,14+0,06)\cdot 10} \right] \cdot \alpha_{10\neg 0,10} + \\ 20 \cdot e^{-0,14\cdot 1} \cdot e^{-(0,14+0,06)\cdot 10} \cdot 0,10 + 2 \cdot e^{-0,14\cdot 1} \end{array} \right\} \cdot 10^6 = 7,57 \cdot 10^6 \text{ €}$$

$$10^3 \cdot 9 \cdot 10^3 \cdot \left\{ \left[4,5 - 1 \right] \cdot \alpha_{4-1\neg 0,10} + 1 \cdot 0,10 + 3 \right\} \cdot 10^6 = 4,49 \cdot 10^6 \text{ €}$$

from which it follows: the replacement cost at the beginning of 2nd year

Retrieval of capital

The **capital** refers to the heritage, in the form of money or goods that can be used to generate further wealth. In the financial market, the supply of capital is given mainly by investors who want to invest their money for a period longer or shorter is acquired.

- **use of social capital, which consists of the contributions of members and is formalized**

  In the Limited Company (S.p.A.) will be the shareholders' meeting to decide whether to increase the share capital, issue new shares, which are securities representing a share capital of a company and give the holder rights and powers administrative and capital, have a right to share in profits and therefore constitute a risky investment. The issued shares will initially be offered to the shareholders of the company and only later was not available in the stock market to buy them.

  The Limited Liability Company (S.r.l.) is excluded the right to issue bonds.

Retrieval of capital

The **capital** is acquired:

- *use of capital acquisition through the issuance of bonds*

  Are securities arising from a liability which the offered company has towards investors who have signed up and pay a fixed income (interest)
- ***acquisition of a bank loan***

  It is a capital loan constitutes a financing medium and long-term through credit institutions. These grant loans to medium and long term especially for industrial investment.

  The company in this case has to deliver in time both the percentage interest that the share capital, according to the method agreed with the bank

Retrieval of capital

The **capital** is acquired:

- **acquisition of a loan in project financing**

  It is an operation of long-term funding, which consists in the use of a newly formed company (SPC, Special Purpose Company) which serves to maintain separate assets of the project by those proponents of the initiative investment ("promoters").

  The SPC is financed both by equity capital (shares), provided by the promoters and generally should not exceed the amount of 15-20%, the remaining 80-85% of debt financing (bonds) normally obtained from a syndicate of banks. In this way, through the allocation of assets and liabilities at SPC you can keep a tighter control on the project

Interest and mast

The **interest** is the fee that the one who lends a sum of money (or capital) C.

The interest can be paid:

- globally (for short-term operations);
- periodically;
- in advance;
- in arrears.

The **mast** M is the sum of the principal plus the interest:

$$M = C + In$$

and the amount that is paid on the date specified, the debtor when the interest is due largely in arrears.

Main models of financial mathematics

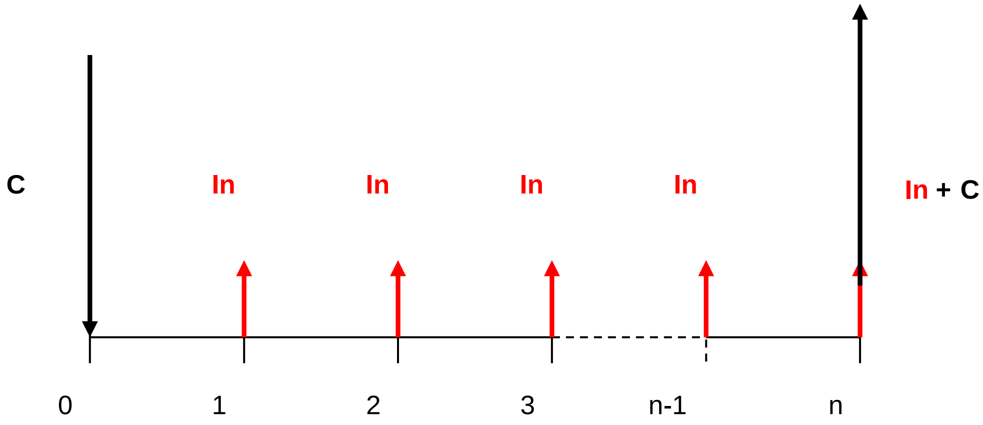
Simple interest

The simple interest is the interest on a capital C taken for a time n and is expressed by the relation:

$$\mathbf{In = C \cdot n \cdot i}$$

The mast the sum total capital plus interest paid after n time units:

$$\mathbf{M = C + In = C + C \cdot n \cdot i = C \cdot (1 + n \cdot i)}$$



Counpound interest

The debtor retains, rather than pour, the interest each period, competing with this form to increase the share paid, then you have to pay interest on this new capital

The mast at the end of the first year is:

$$M_1 = C \cdot (1 + i)$$

at the end of the second year is:

$$M_2 = M_1 \cdot (1 + i) = C \cdot (1 + i)^2$$

at the end of n years is:

$$M_n = M_{n-1} \cdot (1 + i) = C \cdot (1 + i)^n$$

Constant rate and the present value

Since the mast of a capital C due or payable to a specific future date:

$$M = C \cdot (1+i)^n$$

the current value of this sum is given by:

$$C = M \cdot (1+i)^{-n}$$

If the equation of compound interest arises:

$$v^n = \frac{1}{(1+i)^n}$$

and the discount factor is defined compound (corresponding to the present value of 1.00 € payable in n periods), we obtain:

$$C = M \cdot v^n$$

It's called an annual series of equal payments made at the end of the same periods of time.

An annuity is called ordinary when payments are made at the end of each interest period.

Constant rate and the present value

One can determine the value at the initial "0" of an annuity ordinary having the payment of 1.00 € at the end of each period. At the time of "0", the payment of 1.00 € made a later period, has the value of v^1

Similarly, the payment of 1.00 € made after two periods of interest from the time of "0" has a value of v^2

Generalizing, the present value of interest of 1.00 € paid in n periods is of interest v^n

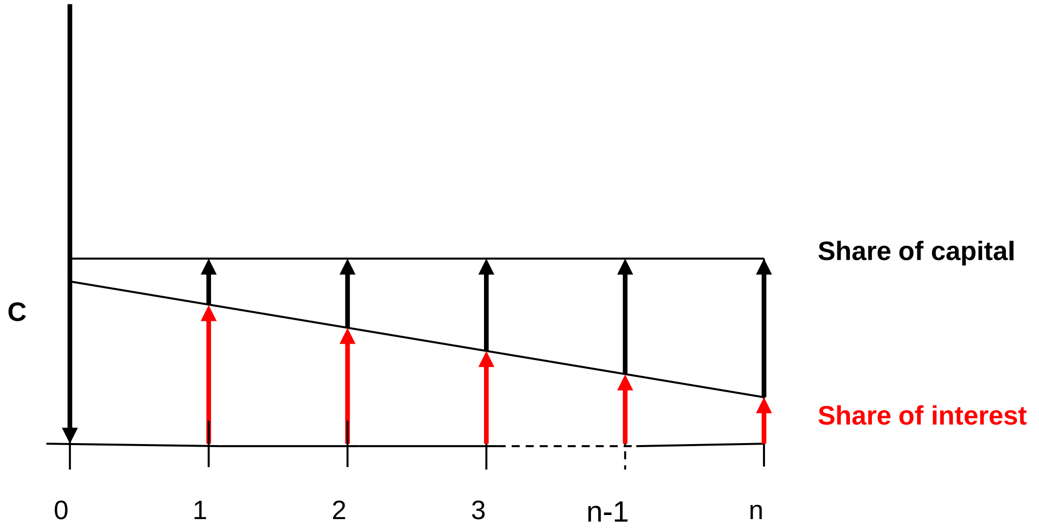
$$A = R \cdot a_{\overline{n}|i} \qquad R = \frac{A}{a_{\overline{n}|i}}$$

with:

$$a_{\overline{n}|i} = \frac{(1+i)^n - 1}{(1+i)^n \cdot i}$$

Constant rate and the present value

The payment plan of equal annual installments of a loan is:



Is determined:

- the share of capital of the k-th installment:

$$C_k = \frac{A \cdot i \cdot (1+i)^{k-1}}{[(1+i)^n - 1]}$$

- the share of interest of the k-th installment:

$$I_k = A \cdot i \cdot (1+i)^{k-1} \cdot \frac{[(1+i)^{n-(k-1)} - 1]}{[(1+i)^n - 1]}$$

Share of costant capital

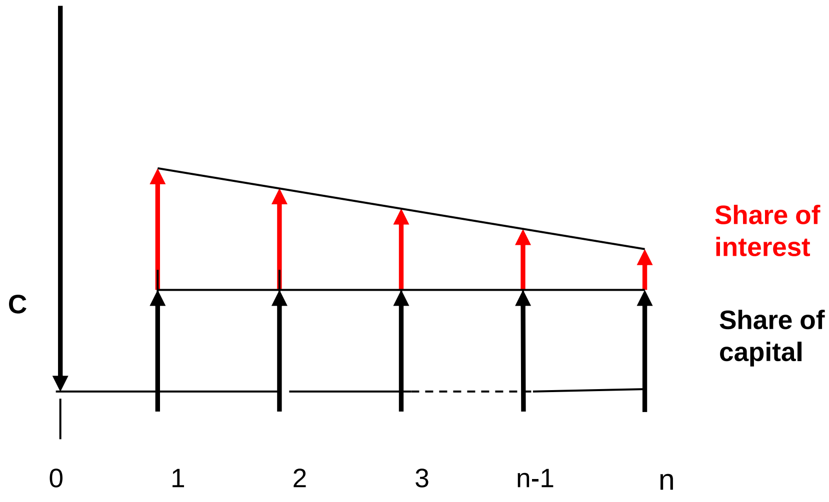
The share of capital is returned to those who have lent capital is constant for each year. To this value will be paid the value of the accrued interest for the remaining capital that you must return.

The value of the installment of the k-th year is:

$$R_k = C + I_k = \frac{A \cdot [1 + i \cdot (n - k + 1)]}{n}$$

with:

$$C = \frac{A}{n}$$



Main models of financial mathematics

Share of costant capital

The share of interest of the k-th installment is:

$$I_k = \frac{A \cdot i \cdot (n-k+1)}{n}$$

The debt paid off by the k-th payment is:

$$E_k = \frac{A \cdot k}{n}$$

while the remaining debt to the k-th payment is:

$$D_k = \frac{A \cdot (n-k)}{n}$$

Equilibrium of the company in the short term through the analysis of business costs

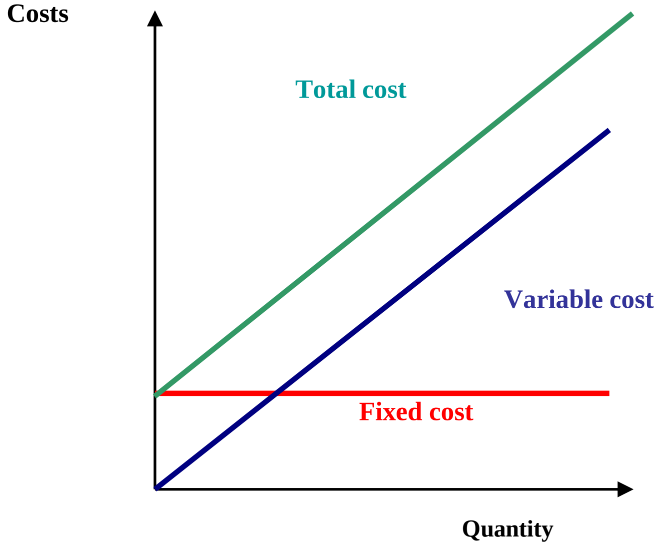
Costs

It is the sacrifice and effort that must be addressed in order to achieve the supply of a good or service.

Corporate costs are classified:

- **variables**, directly proportional to the quantity produced
- **fixed**, independent of the volume of production

whose sum gives the **total cost**.



Equilibrium of the company in the short term through the analysis of business costs

Costs

A particular case is represented by the **cost semi-variables**, which have a fixed part and a variable (indirect personnel commensurate with bands of part production etc.) or are stepped in correspondence with the variation of certain quantities.

The **corporate costs** are built through the aggregation of significant cost elements in relation to the purposes for which the cost should respond.

At **costs directly attributable** (raw materials, labor remuneration, depreciation specific etc.) must be aggregated **costs to general criteria** related to the (costs of commercial facilities, office and administrative management of the materials etc.).

Equilibrium of the company in the short term through the analysis of business costs

Costs

The general scheme of the general costs is:

raw material direct labor specific cost of production	costs of processing	total cost
general production costs selling expenses general administrative expenses financial charges	costs of period	

Equilibrium of the company in the short term through the analysis of business costs

Costs

The **average unit costs** are given by the ratio between the total cost of the period and the corresponding level of production, expressed in hours, tons, pieces, sizes, etc.:

$$CM = \frac{C(q)}{q}$$

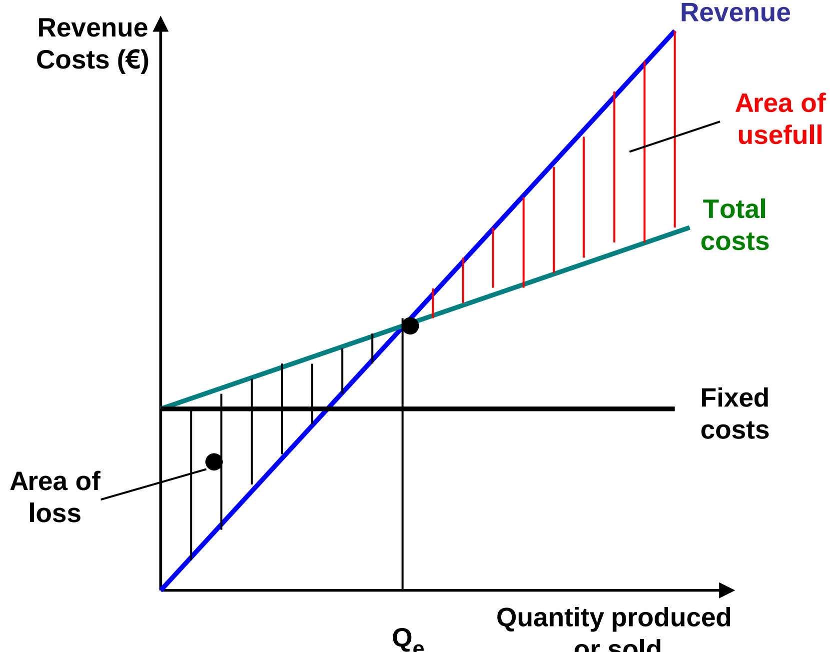
The **marginal cost** C'(q) is equal to:

$$C^{'}(q) = \frac{dC(q)}{dq}$$

Equilibrium of the company in the short term through the analysis of business costs

Break-even point

The **diagram of profitability** allows you to have a global vision of business management, as highlights the relationship between variable costs, fixed costs and sales, in order to determine the point of equilibrium or balance point of the company. The breakeven point is the quantity of production which is the balance between revenue and costs, at the intersection of the respective lines



Equilibrium of the company in the short term through the analysis of business costs

Break-even point

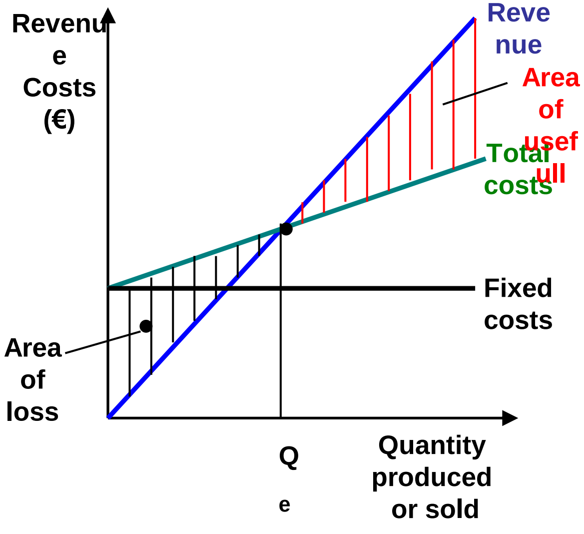
The area of the profit is bounded by the rays R and CT which radiate from the point of balance and is represented by the expression:

$$P = R - CT = Q \cdot p - (CF + Q \cdot c_V) = Q \cdot (p - c_V) - CF > 0$$

while the area of loss enclosed between the half-lines that converge towards the breakeven point is defined by the expression P < 0.

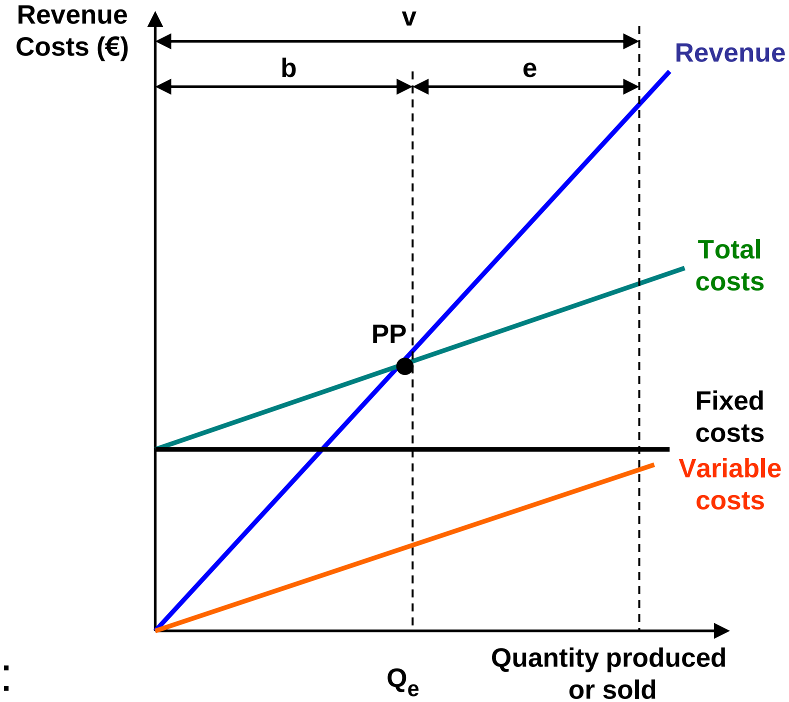
The amount that corresponds to the break-even point, under the condition P = 0 is therefore given by:

$$Q_e = \frac{C_F}{(p - c_V)}$$



Equilibrium of the company in the short term through the analysis of business costs

Break-even point



Volumes and ratios:

v = total sales volume

b = sales volume to the break-even point

e = volume of sales in addition to the break-even point

b/v = percentage of recovery of fixed costs on the average sales

e/v = safety margin

Equilibrium of the company in the short term through the analysis of business costs

Break-even point

The examination of the diagram of profitability allows identification of the possible changes in the corporate structure to change the breakeven point, with the same unit prices. The four cases are considered respectively:

1. a reduction in fixed costs;
2. a reduction in variable costs;
3. an increase in the volume of sales;
4. an additional share of sales under price.

A fifth case of an increase in profit was due to the increase in the unit price, which leads to an increase in revenues equal costs according to the report:

$$R = CF + CV + P$$

Equilibrium of the company in the short term through the analysis of business costs

Contribution margin

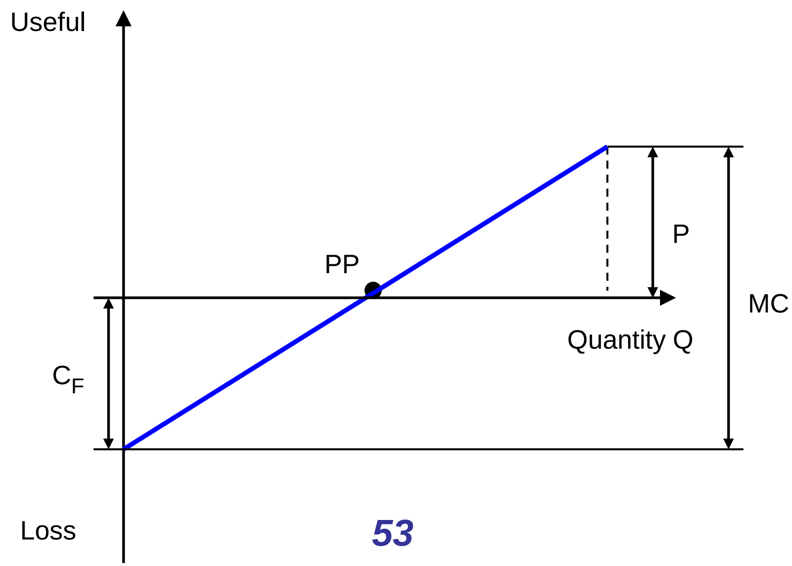
Given the relation:

$$R = CF + CV + P$$

where: R = revenue, CF = fixed costs, CV = variable costs, P = usefull or profit

$$R - CV = CF + P = MC$$

The **diagram contribution/volume** (P/V) contrasts the contribution to fixed costs in relation to the volume of sales



Equilibrium of the company in the short term through the analysis of business costs

Contribution margin

The analysis of the diagram shows the following significant values:

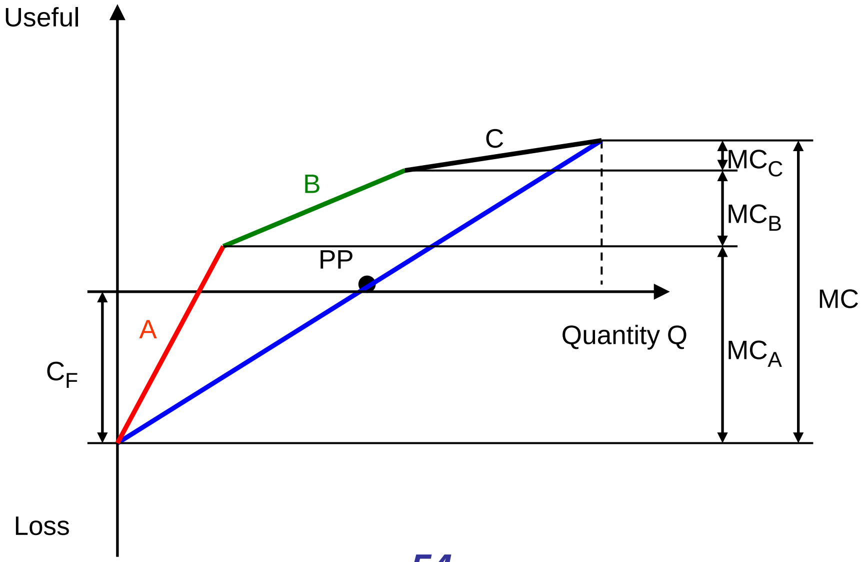
MC/CF= percentage of covering fixed costs

MC/CF= 1 equilibrium condition to the point of sale with balance PP

< 1….. loss situation

> 1….. situation of profit or useful

For most products we have the following situation:



Equilibrium of the company in the short term through the analysis of business costs

Contribution margin

Example

Costs and revenue (€)	Prodocts			Total
	A	B	C	
R	100	200	300	600
C_v	55	65	170	290
$M_c = R - C_v$	45	135	130	310
C_f				
$P = M_c - C_f$				
Assumption of proportional				
allocation of C_f	40	80	120	240
$C_{fi} = f\ (R)$	5	55	10	70
$P_i = M_{ci} - C_{fi}$	5 %	27,5 %	3.3 %	240
P_i / R_i	1,12	1,68	1,08	70
M_c / C_f				
Quantity produced	400	200	600	1200
P / R=11.6%	$Q_e = C_F /(p - c_v) = 240 / (600 – 290):1200 = 929$