

## **A**GENDA

- Measures
- Real and nominal IR
- IR and returns
- Demand/supply and liquidity preference
- Models of asset pricing
- Risk and IR
- Term structure

### **MEASURES OF IR**

Types of credit market instruments:

- Simple/balloon loan: principal is repaid entirely at maturity with interests
- <u>Fixed-payment (fully amortised) loan</u>: repayment occurs periodically and represents interests and a portion of principal
- <u>Coupon bond</u>: repayments of interests occur periodically, whereas principal (face/par/nominal value) is repaid entirely at maturity
- <u>Discount (zero-coupon) bond</u>: no coupons are paid, therefore the present (purchase) value is under its face value, that is repaid entirely at maturity

Variations exist (f.i. variable IR, adjustable maturities, different amortisation plans, etc.)



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#### MEASURES OF IR

How to compare different bond instruments quickly and easily?

Yield to maturity (or internal rate of return, or effective interest rate):

- the IR that balances the PV of future cash-flows with its current value
- For simple loans, YTM equals the simple interest rate
- For ZC bonds:  $i_{\it YTM} = \sqrt[n]{\frac{NV}{CV}} 1$
- For fixed-payment loans and coupon bonds, calculation is more complex (usually solved through *goal-seek* and similar spreadsheet functions):

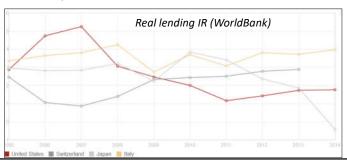
$$\begin{aligned} CV_{FP} &= \sum_{t=1}^{n} \frac{FP}{(1+i_{YTM})^{t}} \\ CV_{CB} &= \sum_{t=1}^{n} \frac{C}{(1+i_{YTM})^{t}} + \frac{NV}{(1+i_{YTM})^{n}} \end{aligned}$$

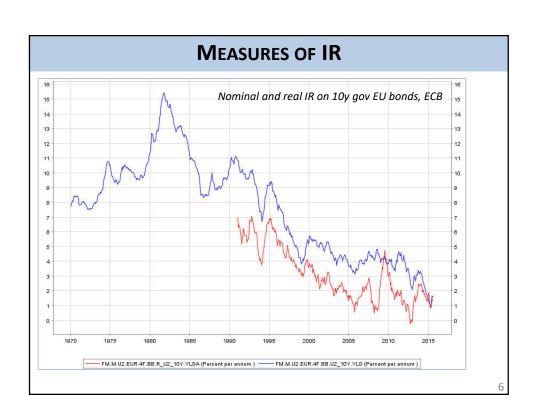
• Note that the greater the YTM, the smaller the current value, meaning that increases in IR reduce the value of a debt instrument

# **MEASURES OF IR**

But as any simple tool, the YTM has its limitations

- You need to keep the instrument until maturity
- Coupons are reinvested at the same IR
- It is a nominal measure... but inflation is known when it's too late
- Ex-ante real IR are adjusted by expected inflation  $i = i_r + \pi^e + i_r \cdot \pi^e$
- Ex-post real IR can actually measure performance... when it's over
- But also taxes should be considered to know the net performance (effective real IR)





# **NEGATIVE IR???**

- Pay to lend?
  - Central banks: ECB -0.2% on deposits in 9/2014 (but also DEN, SWE, CH)
  - Governments: DE from -0.4% to 0 for 1m-8y bonds (but also NED, SWE, DEN, CH, AUT), with FIN and DE issuing bonds with negative IR from inception on 2/2015
  - Corporations: Nestlé for its 4y € bonds in 2/2015, f.i.
- Should be good if you are a borrower?
  - Maybe, unless people keep money at home
  - Maybe, unless this shrinks profitability of commercial banks
  - · Maybe, until this triggers a currency war
- · Does it make any sense?
  - Real IR mostly do, considering deflation
  - Storing money, building wealth reserves, accessing settlement services: all cost
  - A number of bonds give access to CB lending, increasing their demand
  - Taxation applies on nominal interest rates



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### **IR** AND RETURNS

- Rate of return: payments to the owner of a security plus the change in its value as a fraction of its purchase price
- IR and RoR are related but usually differ because of capital gains:

$$RoR = \frac{C + P_{t+1} - P_{t}}{P_{t}} = \frac{C}{P_{t}} + \frac{P_{t+1} - P_{t}}{P_{t}} = i_{c} + g$$

- If holding period equals time to maturity, return equals yield to maturity only for ZCs: reinvestment risk
- The bigger the time to maturity, the bigger the effect on capital gains due to changes in IR: longer term bonds are more volatile (<u>interest-rate</u> <u>risk</u>)
- <u>Increasing IR produces capital losses</u>, decreasing IR produces gains
- Despite capital gains and losses are unrealised, they represent missed opportunities to earn greater rates of return (opportunity cost)
- If holding period is longer than time to maturity, this is another source for reinvestment risk (uncertainty over future IR)



### **IR** AND RETURNS

So, how can we compare bonds with different maturities, coupons and prices?

A simple way is to use the <u>duration</u> (effective maturity)

- It's the weighhed average lifetime of a debt instruments' cashflows
- · For ZCs equal to the time to maturity
- Other instruments are seen as a portfolio of ZCs, weighted by their proportion over the portfolio (a useful additive property)

$$DUR = \frac{\sum_{t=1}^{n} \frac{CF_{t}}{(1+i)^{t}} \cdot t}{\sum_{t=1}^{n} \frac{CF_{t}}{(1+i)^{t}}}$$

- Longer terms and smaller coupons mean bigger duration
- · Increases in interest rates decrease duration
- For small changes in IR, duration is a good proxy of interest rate risk

$$\%\Delta P = \frac{(P_{t+1} - P_t)}{P_t} = -DUR \cdot \frac{\Delta i}{(1+i)}$$

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### **DEMAND AND SUPPLY FRAMEWORK**

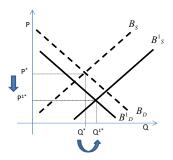
Reasons for changes in interest rates:

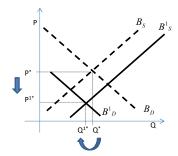
- Bonds' demand:
  - (+) Wealth owned by an individual
  - (+) Expected return relative to other assets
  - (-) Expected future interest rates
  - (–) Expected future inflation
  - (–) Risk (uncertainty in return) relative to other assets
  - (+) Liquidity (how quickly and cheaply turned into cash) relative to other assets
- Bonds' supply:
  - (+) Profitability of investments made with loan proceedings
  - (+) Expected inflation, leading to cheaper borrowing
  - (+) Government deficits, leading to greater issues of public debt

# **DEMAND AND SUPPLY FRAMEWORK**

#### Changes in IR due to inflation:

- An increase in expected inflation affects simultaneously demand (decrease of expected return) and supply (cheaper borrowing)
- IR will increase (prices fall)
- Effect on quantity is not readily predictable



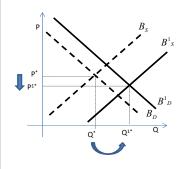


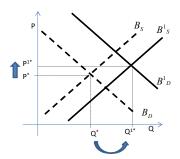
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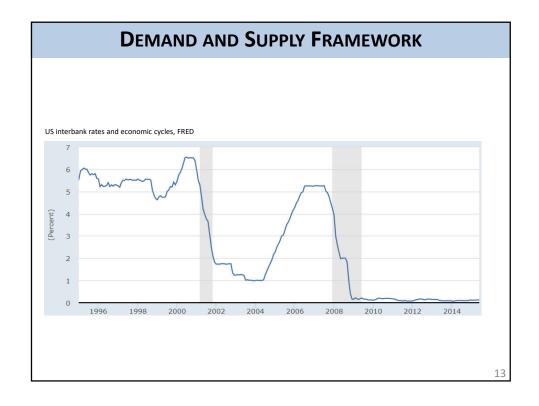
# **DEMAND AND SUPPLY FRAMEWORK**

#### Changes in IR due to business cycles:

- An economic expansion affects simultaneously demand (increase of wealth) and supply (greater expected returns on investments)
- · Quantity will increase
- IR can increase or decrease (usually, increase and decrease during recessions)







# LIQUIDITY PREFERENCE FRAMEWORK

- · Adds to the general model by focusing on bonds and money
- Assumptions:
  - Only risky bonds with return i and safe but costly money (opportunity cost) exist to store wealth
  - Money supply is fixed by the CB
- The bigger *i*, the smaller the quantity of money demanded because of its opportunity cost
- Demand for money changes:
  - (+) because of changes in income, meaning more wealth and more frequent uses of money
  - (+) because of changes in inflation, since people care of wealth in real terms
- Supply of money changes (+) when CB changes its quantity
- When income or inflation rise, IR rise



# LIQUIDITY PREFERENCE FRAMEWORK

When CBs increase the money supply, IR should decline, but evidence is mixed:

- Immediate liquidity effect reducing IR
- Economic stimulus: more income (**income effect**) and IR, but it takes time to have effects (wages, investments, ...)
- More inflation (price-level effect) and IR, but it takes time to adjust prices
  of goods and services
- More expected inflation (expected-inflation effect) and IR, with speed of effects depending on people's speed of adjusting expectations
- Result
  - If the liquidity effect is dominant, sharp reduction in IR, then recovery up to a smaller final value
  - If the liquidity effect is insufficient, sharp reduction in IR, then recovery up to a higher final value
  - If the liquidity effect is marginal, people adapt their expectations on inflation and the reduction in IR does not take place, and final IR are higher immediately

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### **RISK AND IR**

IR differ also for bonds with equal duration because of default risk:

- government bonds were considered risk-free, yet only few of them now are really like that
- the higher the risk the bigger the risk premium (spread)
- specialised firms (rating agencies) provide judgment over borrowers' default-risk (investment grade VS junk/high yield bonds)
- IR differ also for <u>liquidity risk</u> (adding to the risk premium)

## FU long-term gov. IR

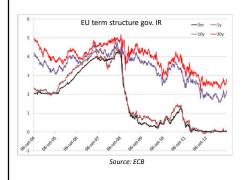
### FU long-term go

• Finally, some bonds have tax incentives (municipal bonds, Italy's gov., ...)

## **TERM STRUCTURE OF IR**

IR differ also based on bonds' maturity:

- Differences in IR can be plotted at different maturities to derive the term structure of IR (vield curve)
- <u>Usually yield curves are upward-sloping</u>, meaning that longer maturities are charged with higher IR
- Flat or even downward-sloping or inverted yield curves are rare



- · Different maturities move similarly
- When short-term IR are high, inversion is more likely
- Inverted yield curves seem to anticipate recessions ('81, '91, 2000, '07), steep upward curves are associated with economic booms

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# **TERM STRUCTURE OF IR**

Three theories for explaining the term structure of IR:

#### **Expectations theory**

- If bonds at different maturities are perfect substitutes, their expected return must be equal
- $(1+i_{n,0})^n = (1+i_{1,0})(1+i^e_{1,1}) \cdot \ldots \cdot (1+i^e_{1,n-1}) \rightarrow i_{n,0} \approx \frac{i_{1,0}+i^e_{1,1}+\ldots+i^e_{1,n-1}}{n}$
- Predicts flat curves, whereas instead are usually upward-sloping

#### Market segmentation theory

- Bonds at different maturities are not substitutes and each has a specific market, as well as each investor has a preferred maturity
- Together with interest-rate risk aversion, <u>explains why longer investments require a risk premium</u>
- Does not explain why IR move together along time
- Does not explain why with high short-term IR inversion is more likely

# **TERM STRUCTURE OF IR**

#### Liquidity premium theory

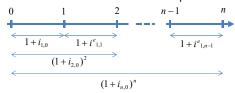
- · Combines the other two in a comprehensive way
- Adds to expectations theory a liquidity premium for longer term bonds that is subject to market (demand, supply) conditions for that segment
- Bonds are substitutes as long as investors' preferences are compensated with a term (liquidity) premium that is always positive and grows as maturity gets longer
- $i_{n,0} \approx \frac{i_{1,0} + i^e_{1,1} + \dots + i^e_{1,n-1}}{n} + l_{n,n}$
- <u>Explains inverted term structures</u>: when future expectations on short-term IR are of a wide fall, so that their average is not balanced even by a positive liquidity premium (more likely when short-term rates are high)
- Support empirical evidence that:
  - Term structure is a predictor of business cycles and inflation
  - Term structure is less reliable for intermediate movements

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## **TERM STRUCTURE OF IR**

#### Forward and spot rates:

• Term structures allow to measure expected IR



- Expected future IR are forward rates, in contrast to spot rates
- Knowing spot IR we can derive market expectations

F.i.: 
$$i_{1,1}^e = \frac{(1+i_{2,0})^2}{1+i_{1,0}} - 1$$
 or, generalising:  $i_{1,k}^e = \frac{(1+i_{k+1,0})^{k+1}}{(1+i_{k,0})^k} - 1$ 

• Including liquidity premiums:  $i_{1,k}^{\varepsilon} = \frac{(1+i_{k+1,0}-l_{k+1,0})^{k+1}}{(1+i_{k,0}-l_{k,0})^k} - 1$ 

## **EXAMPLES**

- 1. A selling agent needs a car for his/her job. It is worth 20,000 € today and will allow to earn 15,000 € every year for three years. A three-year loan to buy the car is available at 50% annual interest rate, paid in fixed installments:
- -Is it worth it?
- -Is the charged IR 'ethical'?
- a) Loan's installment:  $R = 20,000 \cdot \alpha_{3-50\%} = 14,210.53$
- b) Financial and economic plan:

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	0	1	2	3
Inflow		15,000.00	15,000.00	15,000.00
Outflow		-14,210.53	-14,210.53	-14,210.53
Net flow		789.47	789.47	789.47
Loan	20,000.00	15,789.47	9,473.68	0.00
Earnings		15,000.00	15,000.00	15,000.00
Interests		-6,000.00	-4,736.84	-2,842.11
Profit/loss		9,000.00	10,263.16	12,157.89

# **EXAMPLES**

- 2. What is the present value of:
- a) zero-coupon bond due in 3y for 2,000 with a YTM of 5%
- b) bond due in 5y for 3.000 with an annual coupon of 3% and a YTM of 6%
- c) perpetuity of 100 with YTM of 8%

a) 
$$PV = \frac{2,000}{(1+5\%)^3} = 1,727.68$$

a) 
$$PV = \frac{2,000}{(1+5\%)^3} = 1,727.68$$
  
b)  $PV = \sum_{t=1}^{5} \frac{90}{(1.06)^t} + \frac{3,000}{(1.06)^5} = 2,620.89$ 

c) 
$$PV = \frac{100}{8\%} = 1,250$$

### **EXAMPLES**

- 3. What is the price effect on the following bonds of market IR increasing from 4% to 4.25%?
- a) zero-coupon bond due in 3y for 2,000 with a YTM of 5%
- b) bond due in 5y for 3.000 with an annual coupon of 3% and a YTM of 6%
- c) a portfolio made of 40% of the bond sub-a) and 60% of the bond sub-b)
- d) what if IR drop from 4% to 3% on all three alternatives?

a) 
$$DUR = 3$$
 % $\Delta P \approx -3 \cdot \frac{0.25\%}{1 + 4\%} = -0.72\%$ 

**b)** 
$$DUR = \left(\sum_{t=1}^{5} t \cdot \frac{90}{1.04^t} + 5 \cdot \frac{3,000}{1.04^5}\right) / \left(\sum_{t=1}^{5} \frac{90}{1.04^t} + \frac{3,000}{1.04^5}\right) = 4.71$$
  $\%\Delta P \approx -4.71 \cdot \frac{0.25\%}{1+4\%} = -1.13\%$ 

c) 
$$DUR = 3 \cdot 40\% + 4.71 \cdot 60\% = 4.03$$
  $\% \Delta P \approx -4.03 \cdot \frac{0.25\%}{1 + 4\%} = -0.97\%$ 

d) 
$$\%\Delta P_1 \approx -3 \cdot \frac{-1\%}{1+4\%} = 2.88\%$$
  $\%\Delta P_2 \approx -4.71 \cdot \frac{-1\%}{1+4\%} = 4.53\%$   $\%\Delta P_3 \approx -4.03 \cdot \frac{-1\%}{1+4\%} = 3.87\%$ 

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#### **EXAMPLES**

4. Extract from The Economist, 29th June 2013

"[...] Bankers in the rich world have moaned incessantly [...] about how low interest rates are squeezing [their profits]. Now [...] long-term interest rates have risen [...] and changes in short-term rates seem closer than they once did [...]. Rising rates may restore banks' profitability but too sudden an increase may damage their health."

Whv?

"[...] The immediate threat to banks is a fall in the market value of assets that banks hold. [...] A hypothetical three-percentage-point increase in yields across all bond maturities could result in losses to all holders of government bonds equivalent to 15-35% of GDP in countries such as France, Italy, Japan and Britain."

Is that all?

"But simply looking at holdings of government bonds probably understates the risk [...] since they hold many other fixed-income assets that would also fall in value."

Is there anything else?

"[...] A third risk to banks from higher rates is that more of their customers will struggle to repay their loans."

So?

"[...] keeping rates low for long is dangerous. So is letting them rise too quickly."

#### **EXAMPLES** 5. On $2^{nd}$ October 2013, the following spot interest rates on AAA Euro-area government bonds were recorded (by maturity): 0.06% 0.22% 0.44% 0.69% 0.96% 1.22% 1.47% 1.70% 1.90% 2.09% IR spot a) What are the expected one-year IRs? b) What would be the expected IR on a three and five years bond issued in 1, 2 and 3 years from 0.06% 0.22% 0.96% 2.09% 0.44% 0.69% 1.22% 1.47% 1.70% 1.90% $IRspot\_t$ IRfwd\_1 0.38% 0.88% 1.44% 3.32% 2.05% 2.53% 2.98% 3.51% 3.82% 3y bond 0.90% 1.46% 2.01% 5y bond 1.46% 1.98% 2.47%

