LEZIONE 5-6

VORTEX ELEMENT METHODS FOR FLUID DYNAMIC ANALYSIS OF ENGINEERING SYSTEMS

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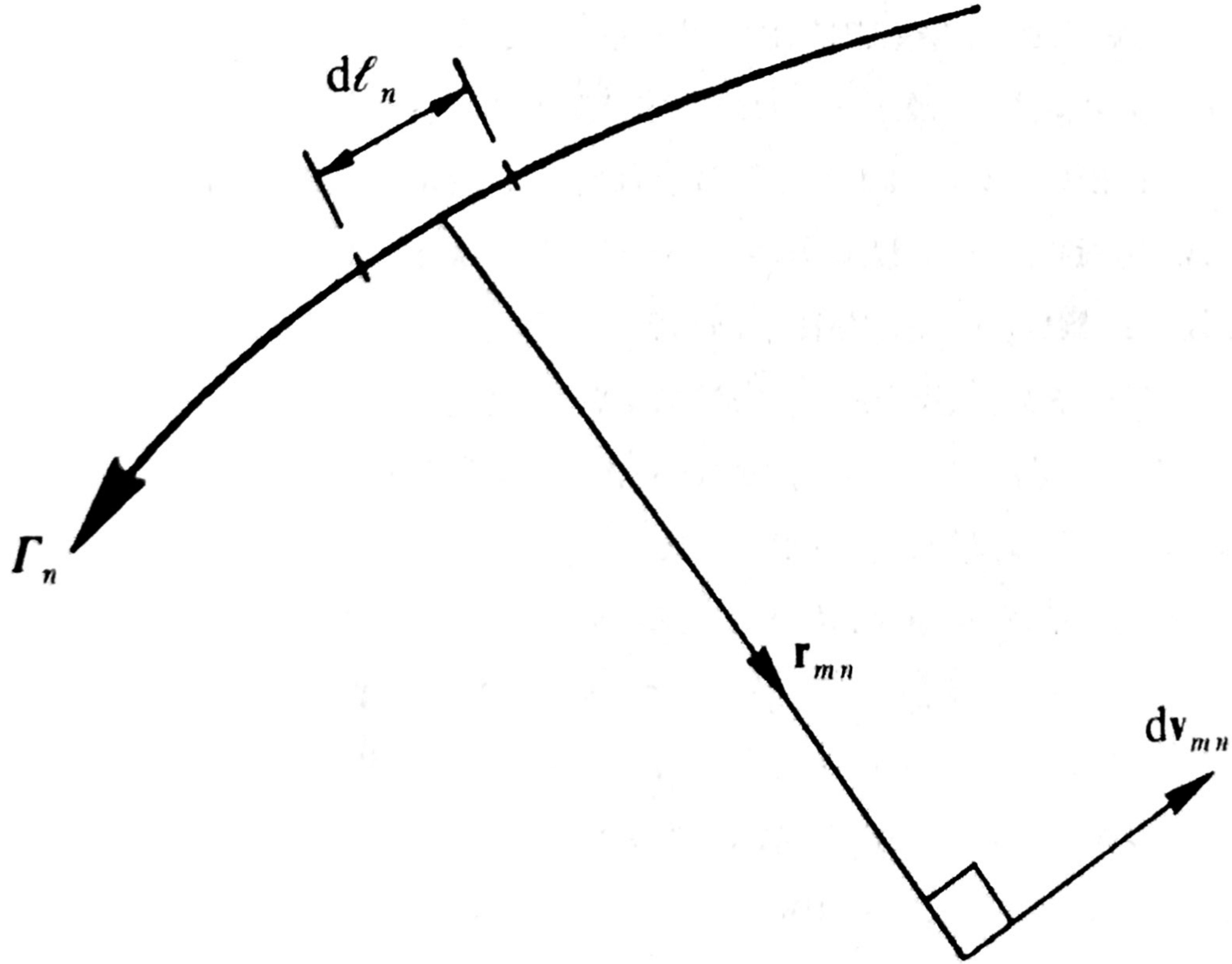


Fig. 1.2. Velocity induced by a line vortex element.

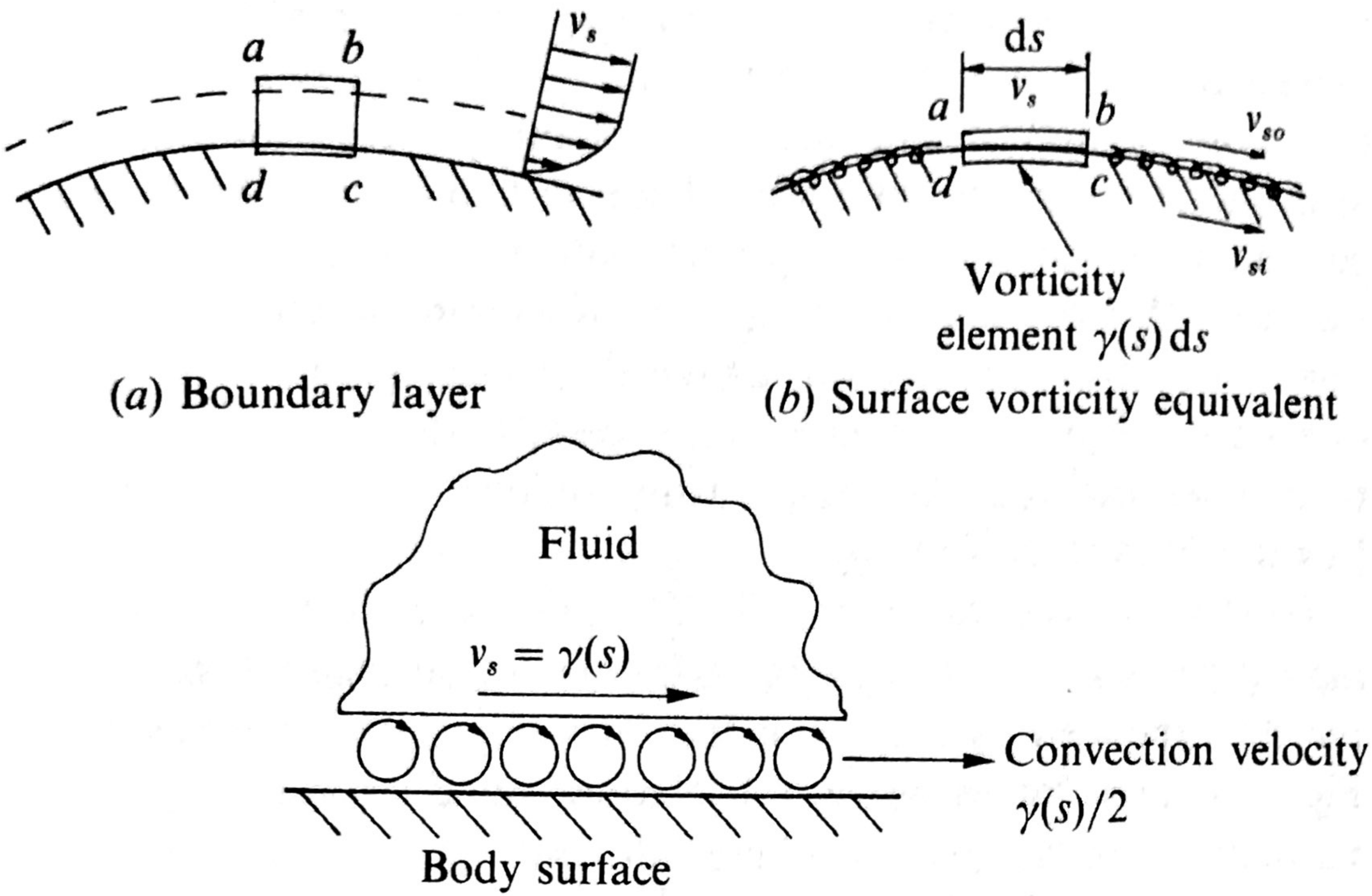
velocity induced at m by a small line vortex element at n of strength $\boldsymbol{\Gamma}_n$ per unit length* and of length $\mathrm{d}l_n$ is given by the Biot–Savart law, namely, with reference to Fig. 1.2,

$$\mathrm{d}\mathbf{v}_{mn} = \frac{\boldsymbol{\Gamma}_n \, \mathrm{d}l_n X \mathbf{r}_{mn}}{4\pi r_{mn}^{\;3}} \tag{1.8}$$

By taking the cross product of $\mathrm{d}\mathbf{v}_{mn}$ with the unit vector $\mathbf{i}_m$ normal to the surface at m twice, we obtain the velocity parallel to the surface at m induced by the line vortex element. Thus

$$\mathrm{d}\mathbf{v}_{smn} = \mathbf{i}_m X (\mathrm{d}\mathbf{v}_{mn} X \mathbf{i}_m)$$

$$= \frac{\mathbf{i}_m X ((\boldsymbol{\Gamma}_n X \mathbf{r}_{mn}) X \mathbf{i}_m) \, \mathrm{d}l_n}{4\pi r_{mn}^{\;3}} \tag{1.9}$$



(*a*) Boundary layer

(*b*) Surface vorticity equivalent

(*c*) Self convection of a surface vorticity sheet

Fig. 1.3. Boundary layer and surface vorticity equivalent in potential flow.

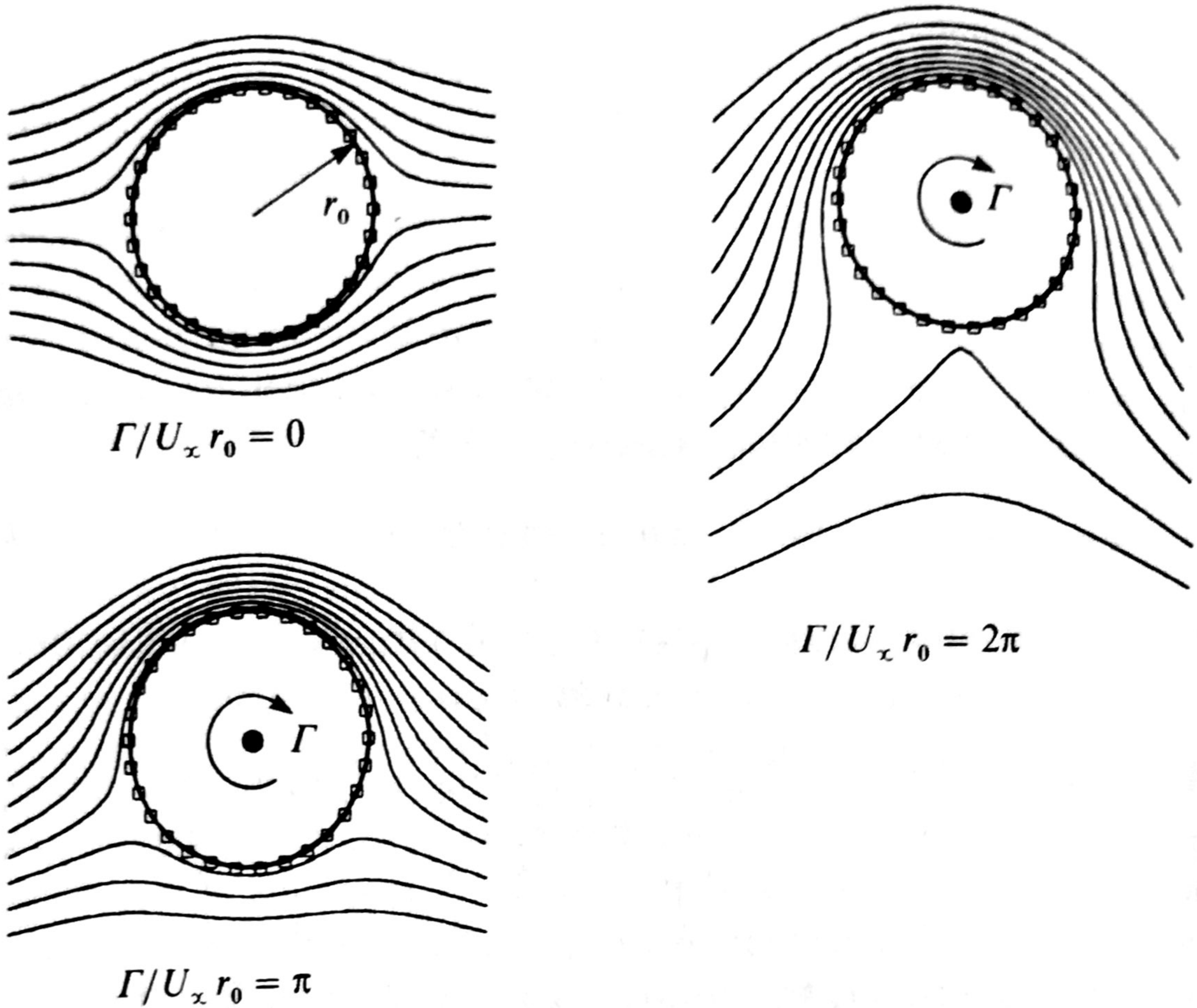


Fig. 2.1. Flow induced by cylinder with circulation in a uniform derived by the surface vorticity method.

through α_∞ that this expression transforms to

$$v_s = 2W_\infty \sin(\theta - \alpha_\infty) + \frac{\Gamma}{2\pi r_0} \tag{2.2a}$$

As shown by Glauert by integrating the surface pressure on the cylinder, a lift force L is generated in the direction normal to W_∞ given by the Magnus law.

$$L = \rho W_\infty \Gamma \tag{2.3}$$

Introducing the usual definition of lift coefficient

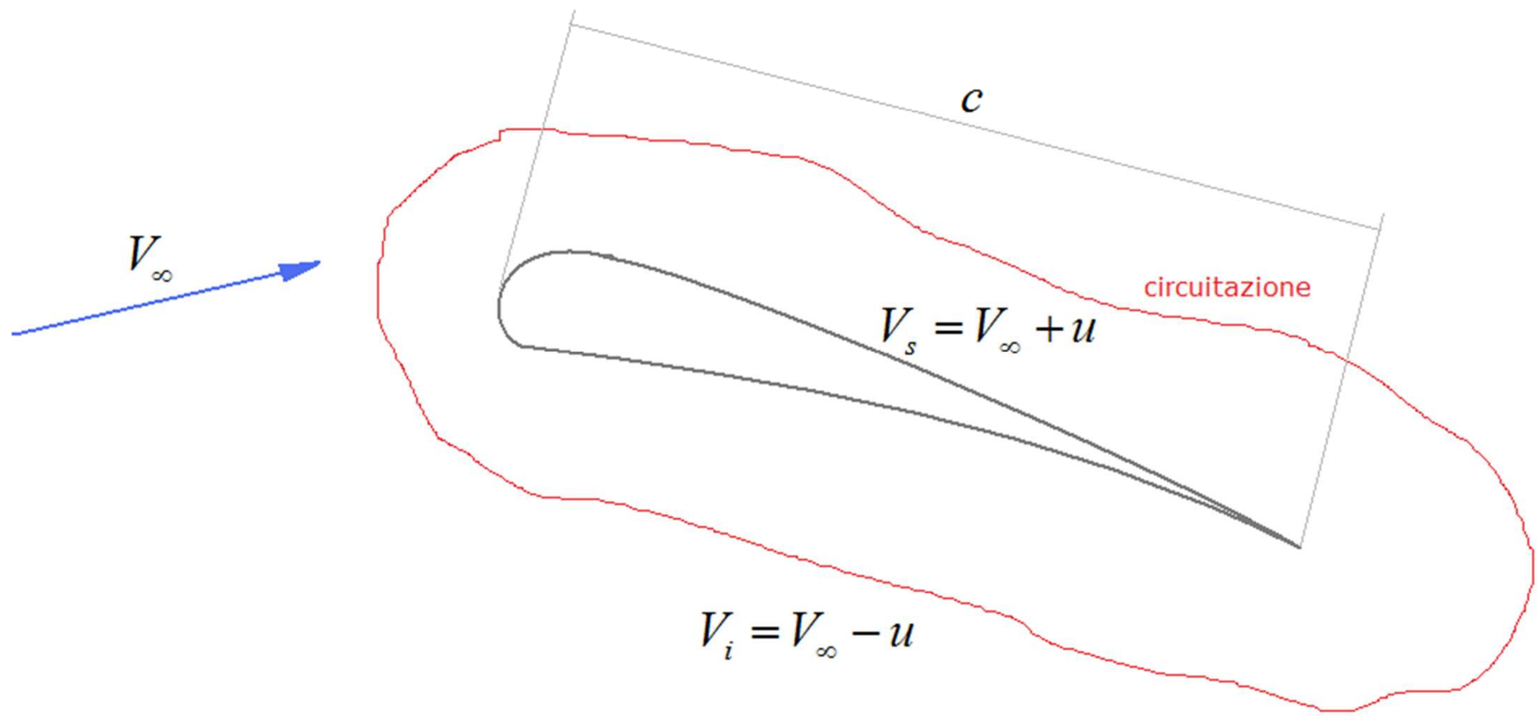
$$C_L = \frac{L}{\frac{1}{2}\rho W_\infty{}^2 \ell} \tag{2.4}$$

where ℓ is a typical dimension of the body, in this case its diameter $2r_0$, we have

$$C_L = \Gamma / W_\infty r_0 \tag{2.5}$$

The dimensionless parameter $\Gamma/U_\infty r_0$ previously referred to is in fact the lift coefficient with uniform stream U_∞. To estimate the

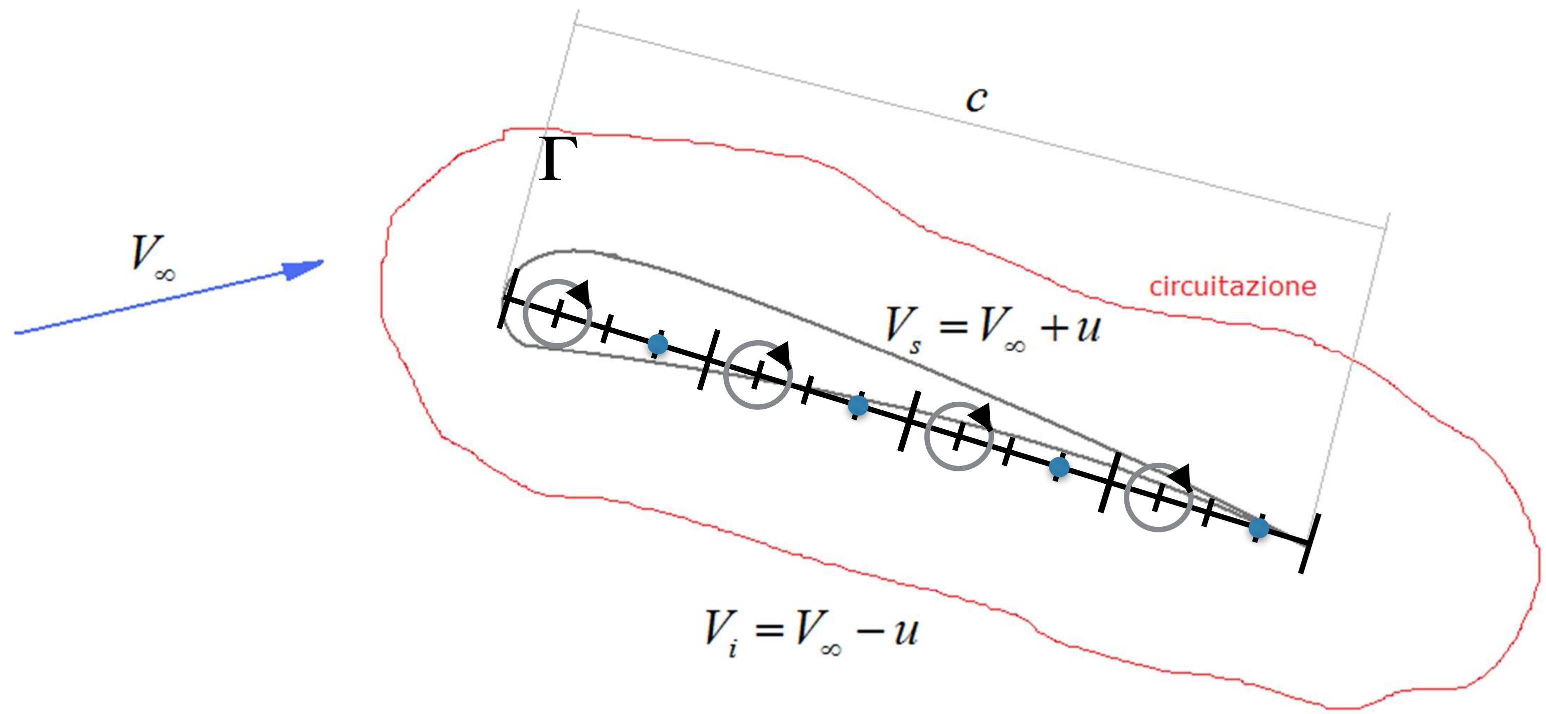
teorema di Kutta-Jukowsky



$$\Gamma = \oint_\ell \overline{v} \cdot d\,\overline{\ell} \qquad L = \rho \Gamma V_\infty$$

$$\Delta p = p_i - p_s = \frac{1}{2}\rho\left(V_s^2 - V_i^2\right) = 2\rho u V_\infty \qquad \Gamma = 2cu$$

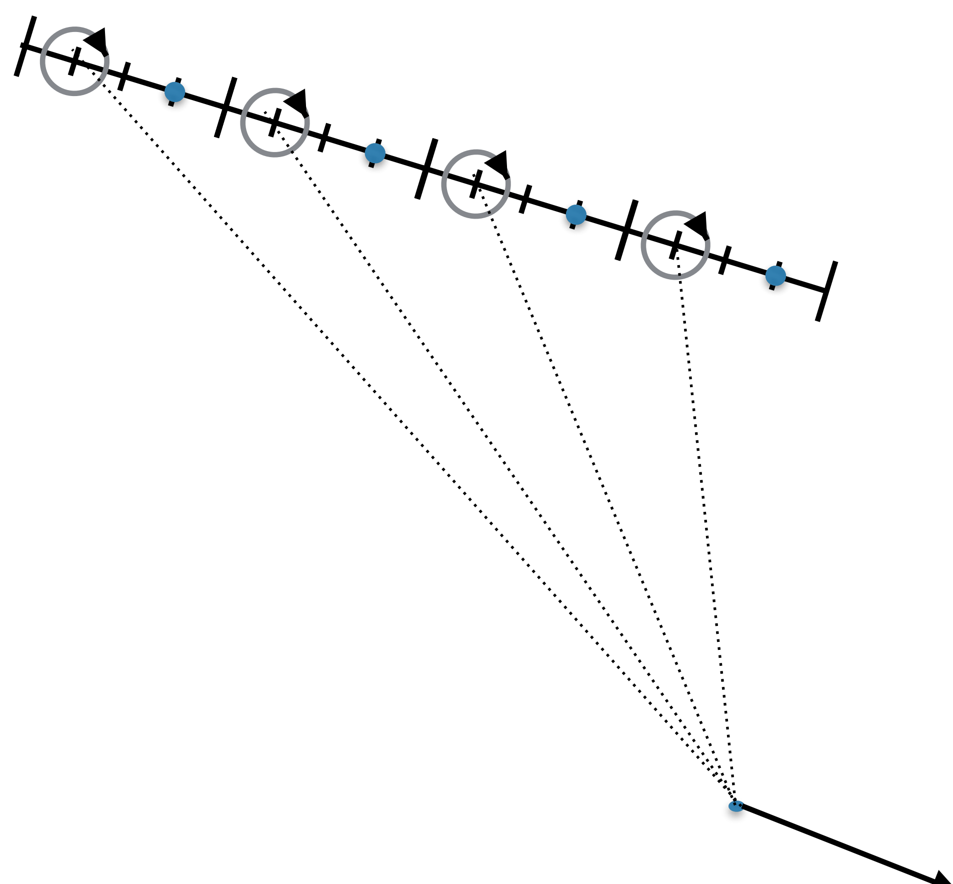
Metodo delle singolarità vorticose



$$\Gamma = \oint_{\ell} \overline{v} \cdot d\,\overline{\ell} \qquad L = \rho \Gamma V_\infty$$

$$\Delta p = p_i - p_s = \frac{1}{2}\rho\left(V_s^2 - V_i^2\right) = 2\rho u V_\infty \qquad \Gamma = 2cu$$

Metodo delle singolarità vorticose



$$V\infty+\sum \frac{\Gamma_i}{2\Pi\ r_i}$$

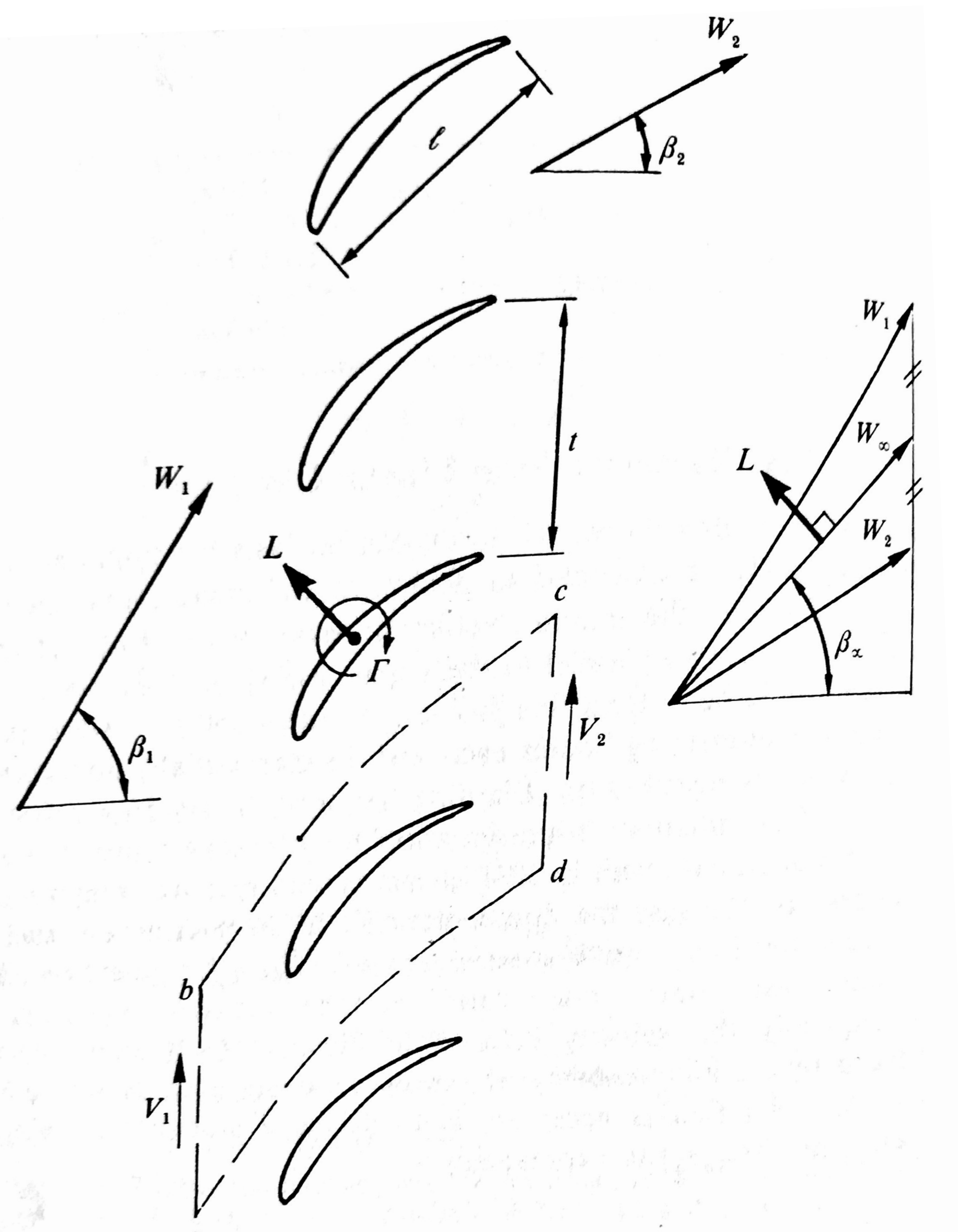
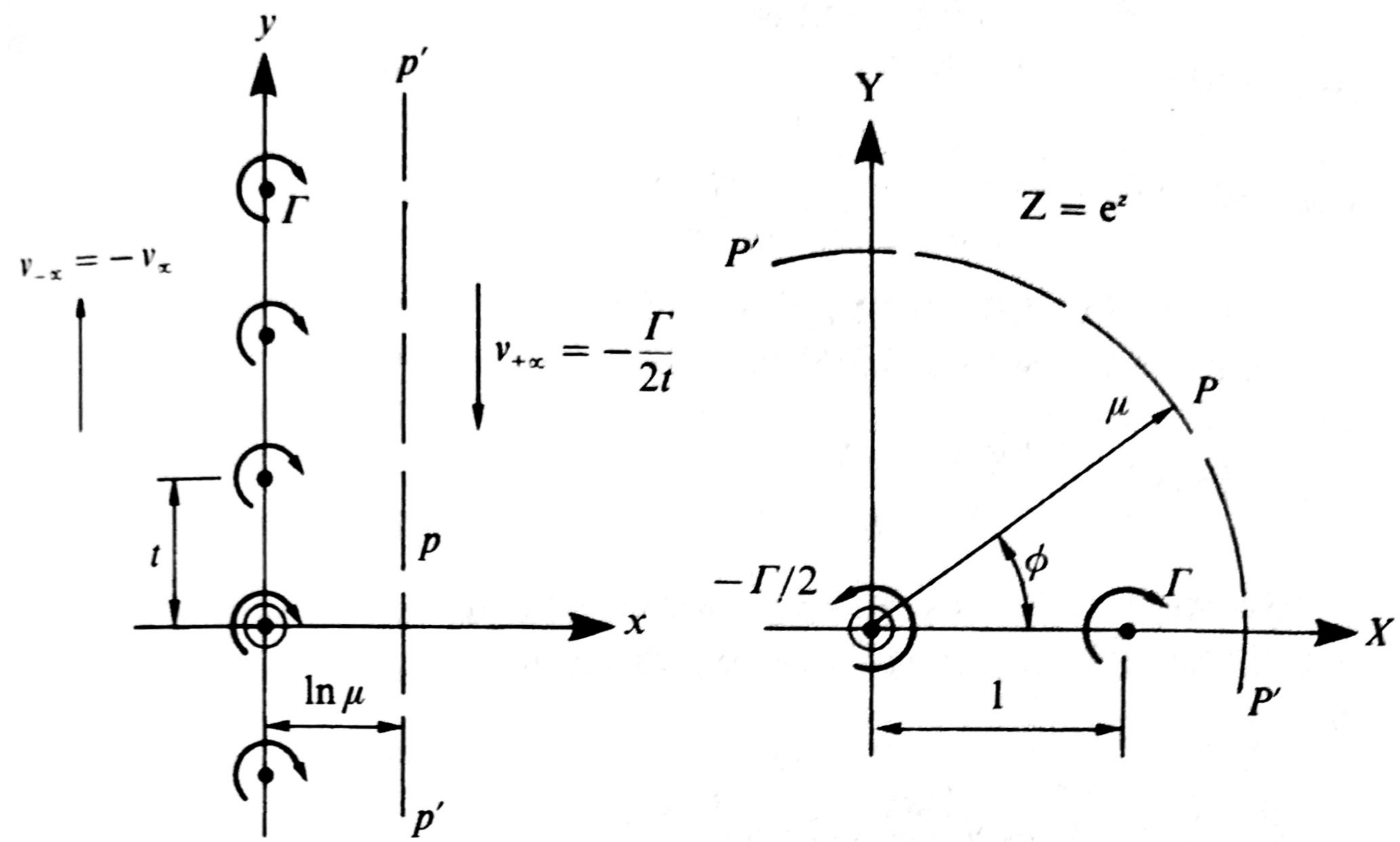


Fig. 2.12. Cascade geometry and velocity triangles.



(a) z plane (b) Z plane

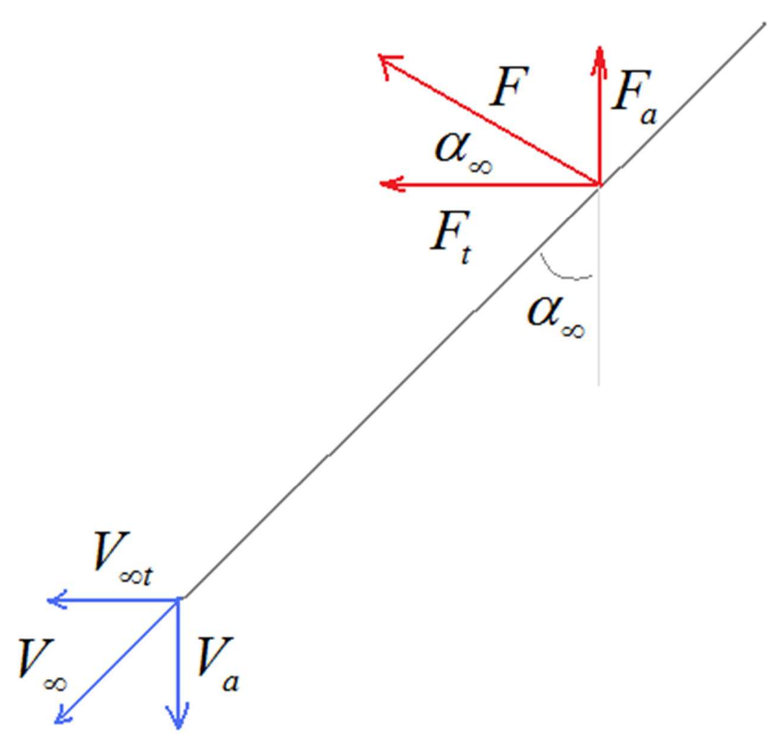
Fig. 2.13. Transformation of vortex array in z plane to vortex pair in Z plane.

Schiere di pale

$\Delta p_0 = 0$ ipotesi perdite nulle

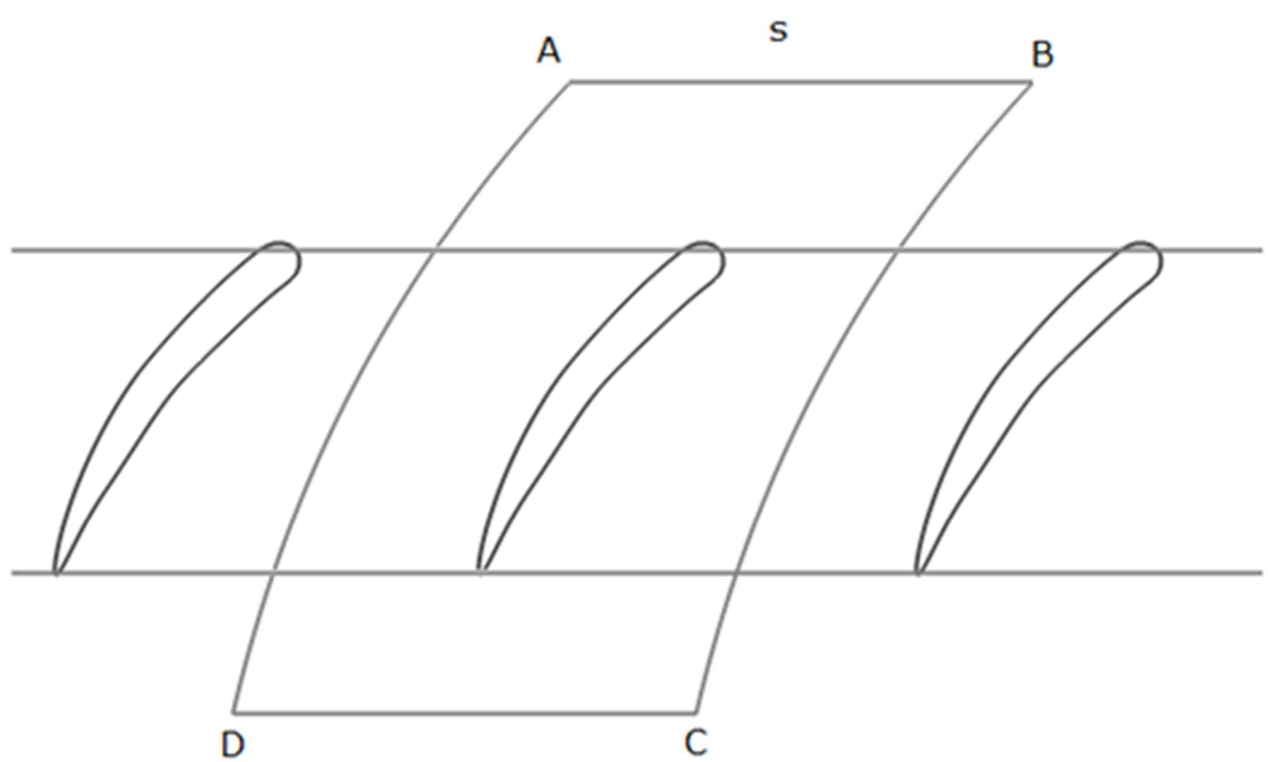
$$F_a = s\rho V_{\infty t}\left(V_{2t} - V_{1t}\right)$$
$$F_t = s\rho V_a\left(V_{1t} - V_{2t}\right)$$

$$\frac{V_{\infty t}}{V_a} = -\frac{F_a}{F_t} = \tan\alpha_\infty$$



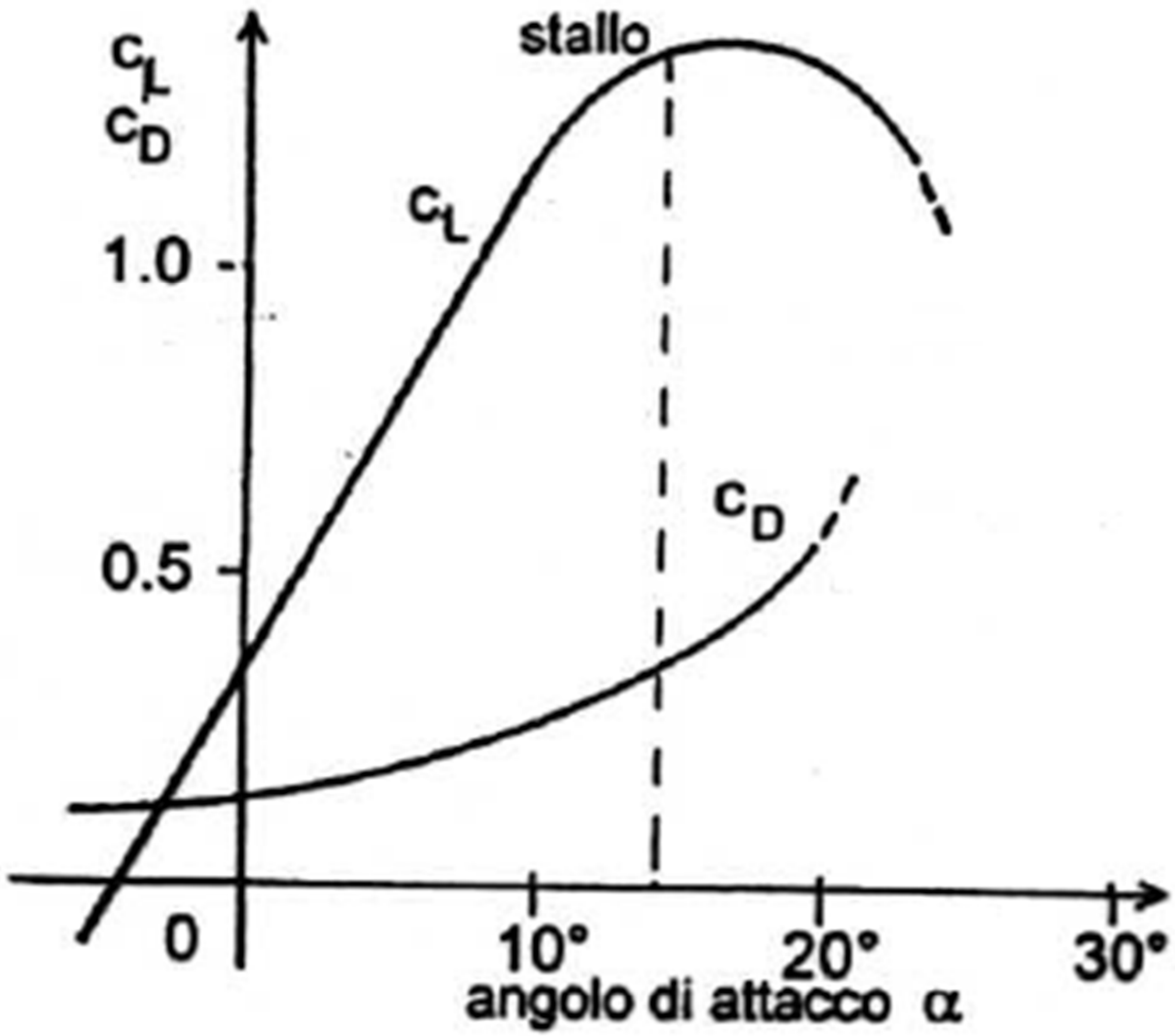
$$F = \frac{F_t}{\cos\alpha_\infty} = \rho\frac{V_a}{\cos\alpha_\infty}s\left(V_{1t} - V_{2t}\right) = \rho V_\infty s\left(V_{1t} - V_{2t}\right) = L$$

Schiere di pale

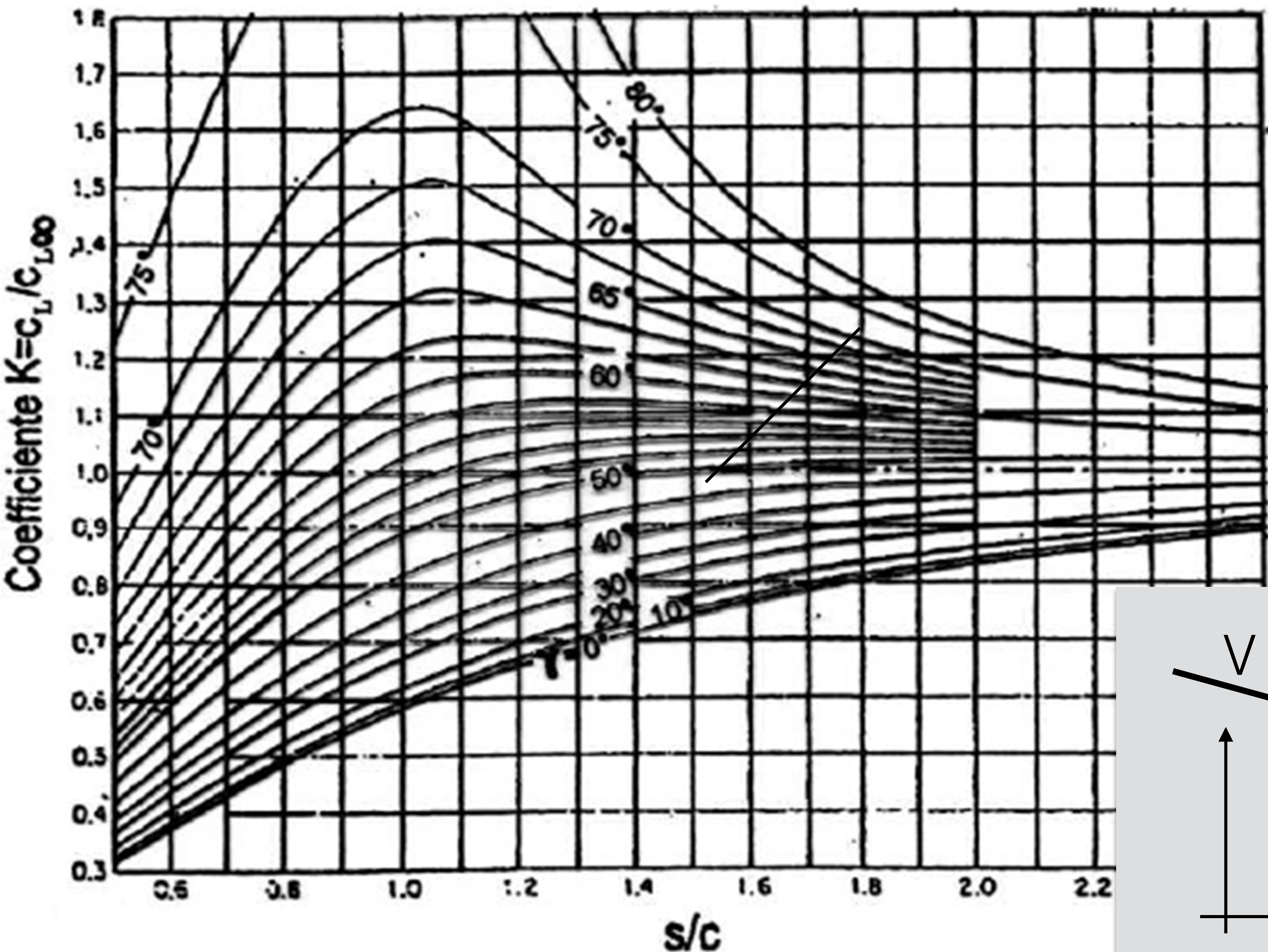


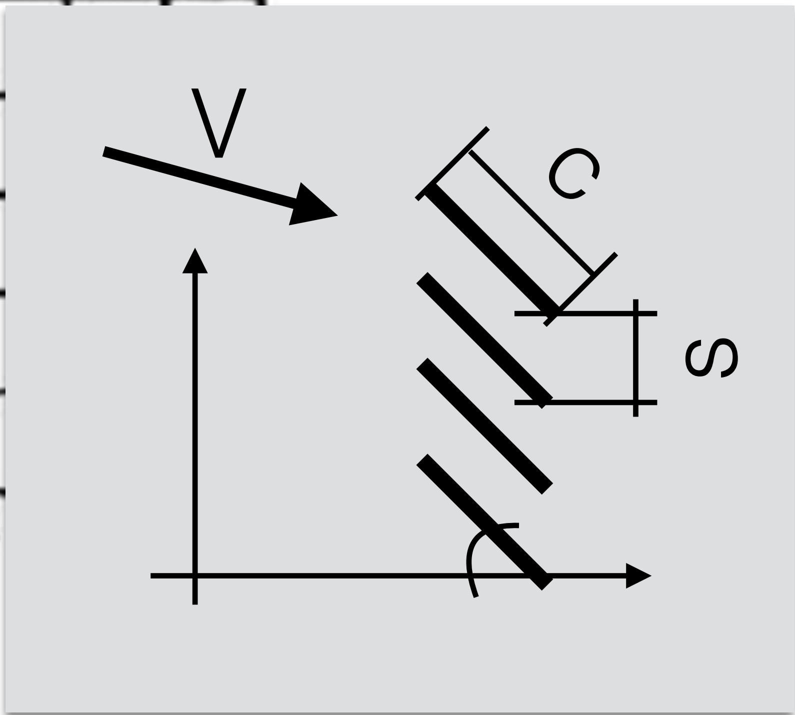
$$\Gamma = s\left(V_{1t} - V_{2t}\right) \quad \rightarrow \quad L = \rho V_{\infty}\Gamma$$

Effetto schiera sulle prestazioni del profilo



Effetto schiera sulle prestazioni del profilo





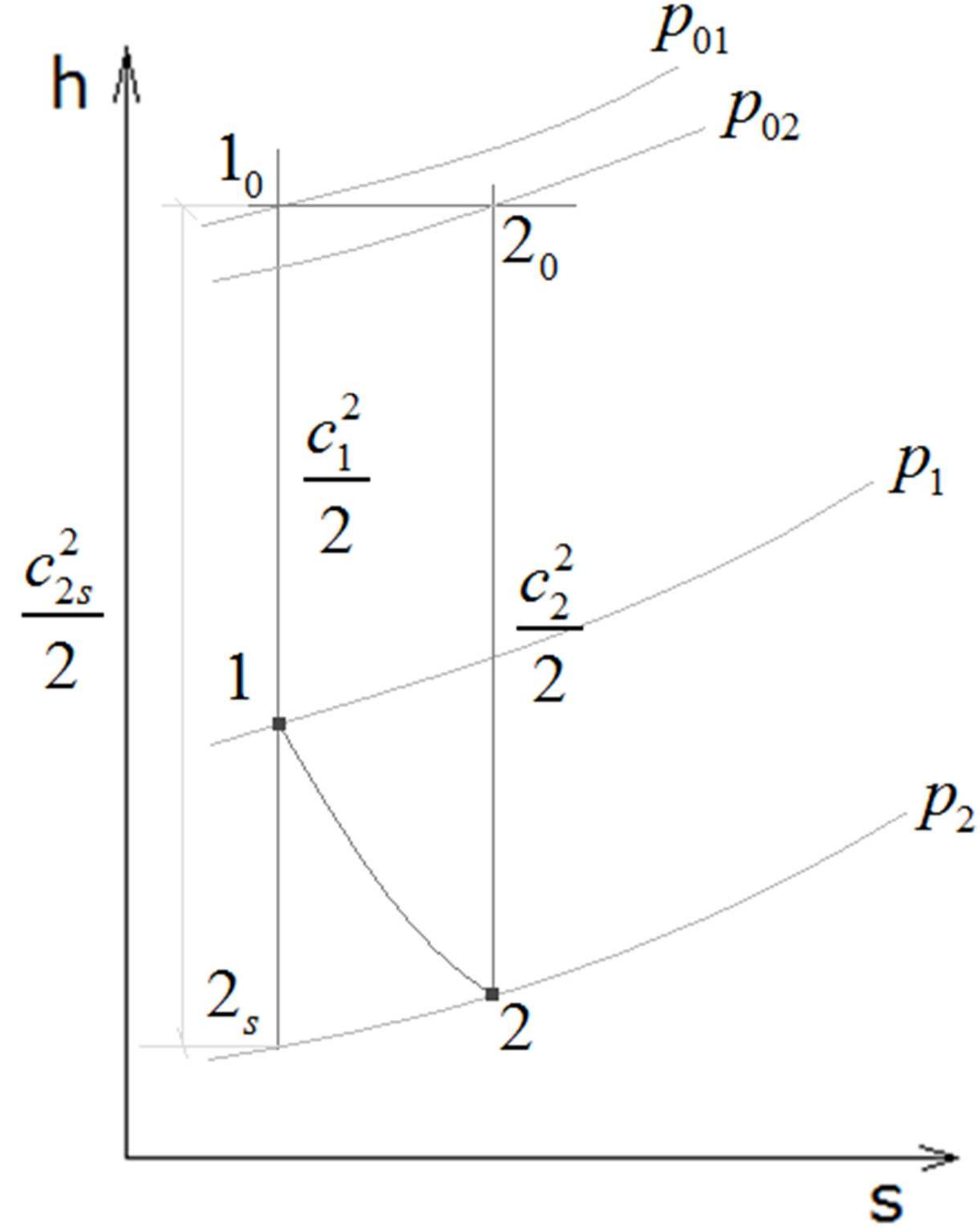
ugelli e diffusori

Distinguiamo due casi:

1) Nell’elemento abbiamo un incremento di velocità a spese di una riduzione di pressione. Questi saranno gli *ugelli*.
2) Nell’elemento l’energia cinetica diminuisce ed aumenta la pressione. Questi saranno i *diffusori*.

ugelli

$$\eta_{is} = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{\frac{c_2^2}{2} - \frac{c_1^2}{2}}{\frac{c_{2s}^2}{2} - \frac{c_1^2}{2}} = \frac{c_2^2 - c_1^2}{c_{2s}^2 - c_1^2}$$



ugelli

(Ma<0,3)

$$p_{01} = p_1 + \frac{1}{2}\rho c_1^2 \quad \rightarrow \quad c_1^2 = \frac{2}{\rho}\left(p_{01} - p_1\right)$$

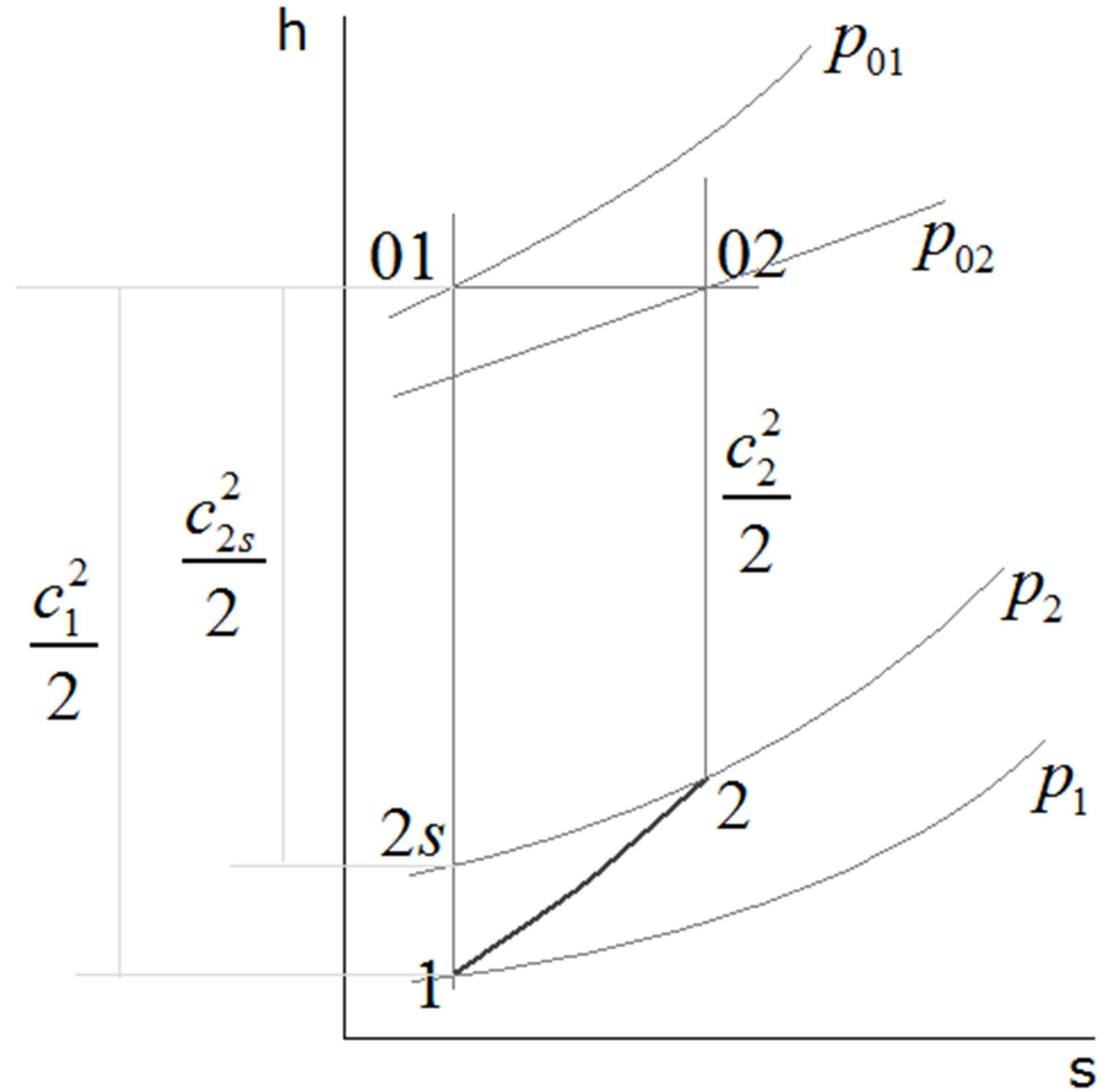
$$p_{01} = p_2 + \frac{1}{2}\rho c_{2s}^2 \quad \rightarrow \quad c_{2s}^2 = \frac{2}{\rho}\left(p_{01} - p_2\right)$$

$$p_{02} = p_2 + \frac{1}{2}\rho c_2^2 \quad \rightarrow \quad c_2^2 = \frac{2}{\rho}\left(p_{02} - p_2\right)$$

$$\eta_{is} = \frac{p_{02} - p_2 - \left(p_{01} - p_1\right)}{\cancel{p_{01}} - p_2 - \left(\cancel{p_{01}} - p_1\right)} = \frac{p_{02} - p_2 - \left(p_{01} - p_1\right)}{p_1 - p_2} = 1 - \frac{\Delta p_0}{p_1 - p_2}$$

Diffusori

$$\eta_{is} = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{c_1^2 - c_{2s}^2}{c_1^2 - c_2^2}$$



Diffusori

(Ma<0,3)

$$\eta_{is} = \frac{p_{01} - p_1 - \left(p_{01} - p_2\right)}{p_{01} - p_1 - \left(p_{02} - p_2\right)} = \frac{p_2 - p_1}{p_2 - p_1 + \left(p_{01} - p_{02}\right)} = \frac{1}{1 - \frac{\Delta p_0}{p_2 - p_1}}$$

coeff. recupero di pressione: $c_p = \dfrac{p_2 - p_1}{p_{01} - p_1}$

Diffusori

(Ma<0,3)

Legame tra c_p e η_{is}

$$\eta_{is} = \frac{p_2 - p_1}{p_2 - p_1 + \left(p_{01} - p_{02}\right)}$$

$$\frac{1}{\eta_{is}} = \frac{p_2 - p_1 + \left(p_{01} - p_{02}\right)}{p_2 - p_1} = \frac{p_{01} - p_1 - \left(p_{02} - p_2\right)}{p_2 - p_1} = \frac{1}{c_p} - \frac{p_{02} - p_2}{p_2 - p_1}$$

$$c_{pi} = \frac{p_2 - p_1 + \left(p_{01} - p_{02}\right)}{p_{01} - p_1}$$

Diffusori

(Ma<0,3)

$$p_2 = p_{02} - \frac{1}{2}\rho c_2^2$$

$$p_1 = p_{01} - \frac{1}{2}\rho c_1^2$$

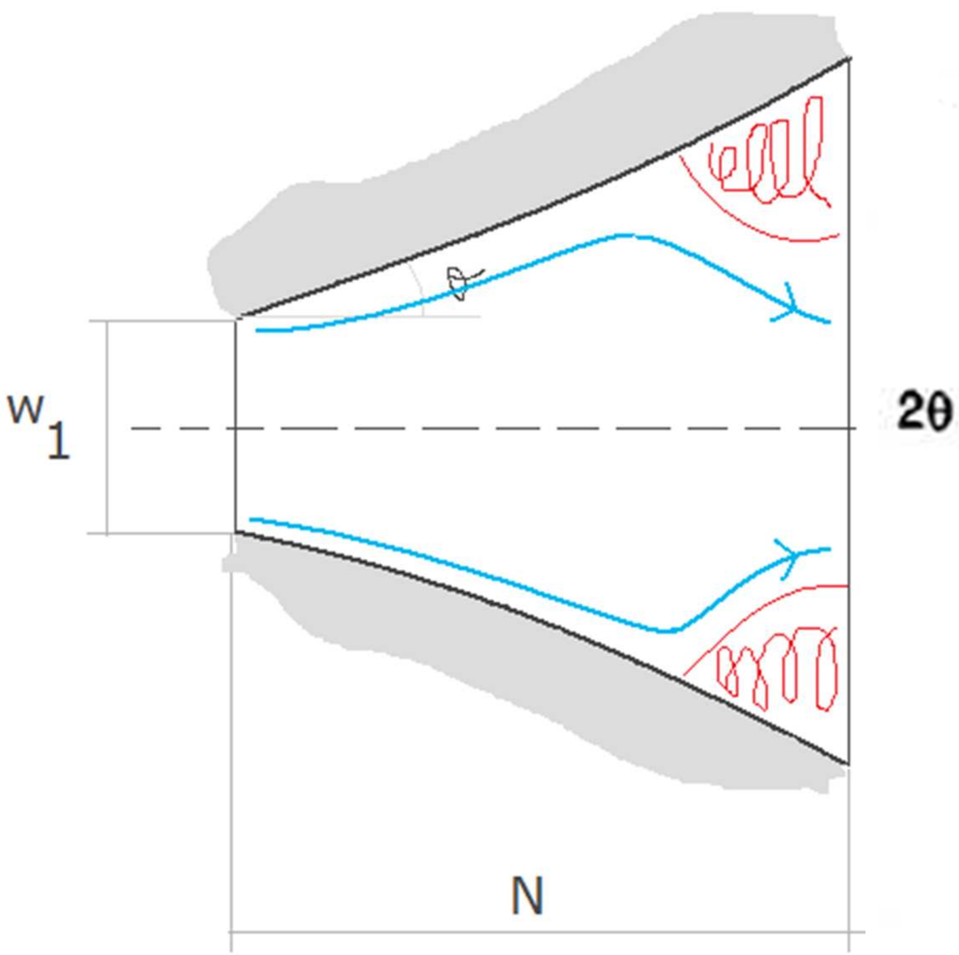
$$c_{pi} = \frac{c_1^2 - c_2^2}{c_1^2} = 1 - \left(\frac{c_2}{c_1}\right)^2 = 1 - \left(\frac{A_1}{A_2}\right)^2 = 1 - \frac{1}{A_R^2}$$

Diffusori

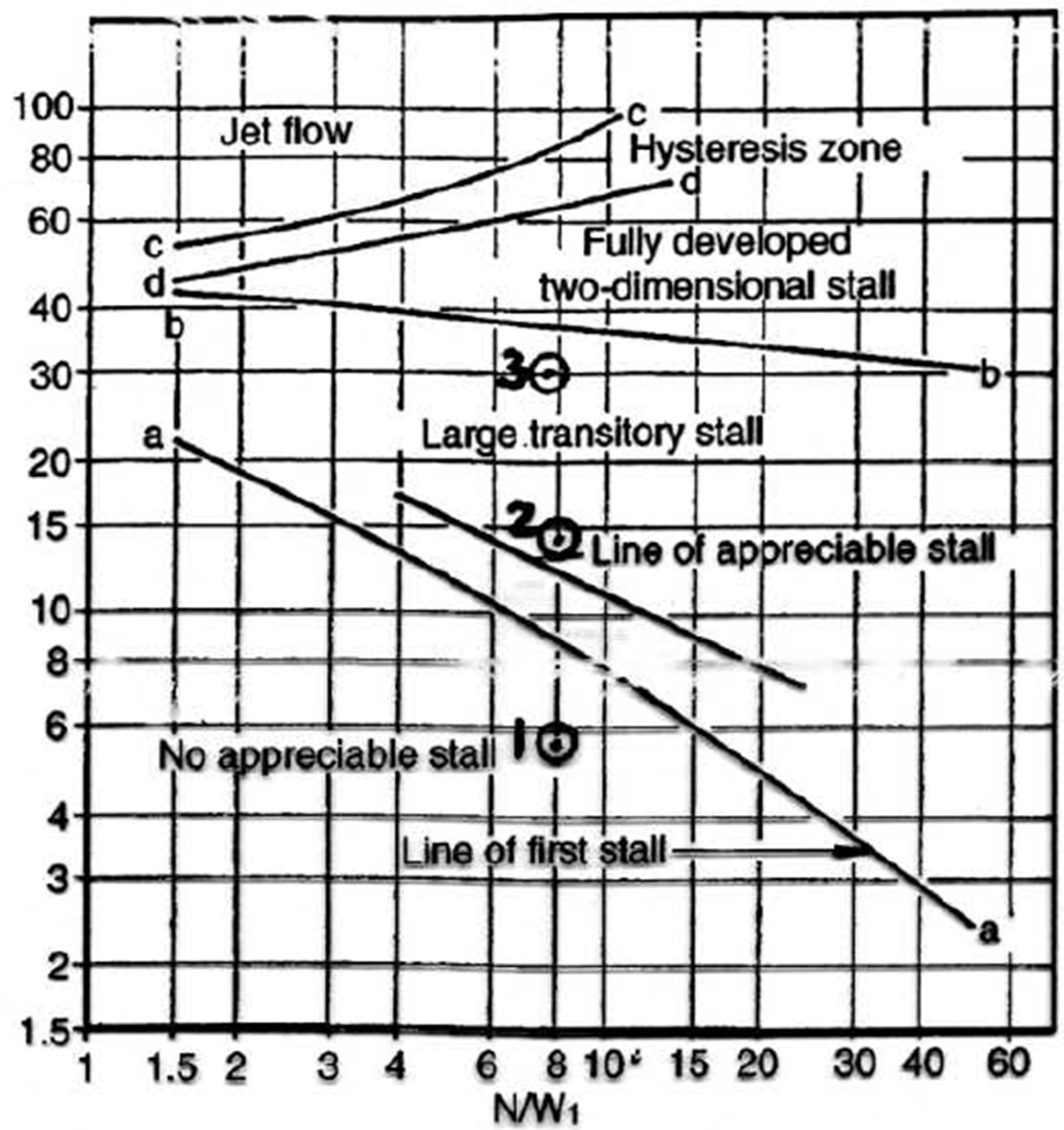
Legame tra c_p , η_{is} *e* c_{pi}

$$\frac{c_p}{c_{pi}} = \frac{p_2 - p_1}{p_{01} - p_1} \cdot \frac{p_{01} - p_1}{\left(p_2 - p_1\right) + \left(p_{01} - p_{02}\right)} = \eta_{is}$$

Diffusori



w1
2θ
N



Jet flow
Hysteresis zone
Fully developed two-dimensional stall
Large transitory stall
Line of appreciable stall
No appreciable stall
Line of first stall
N/W1
100
80
60
40
30
20
15
10
8
6
4
3
2
1.5
1
1.5
2
3
4
6
8
10
15
20
30
40
60
a
b
c
d
1
2
3

Diffusori

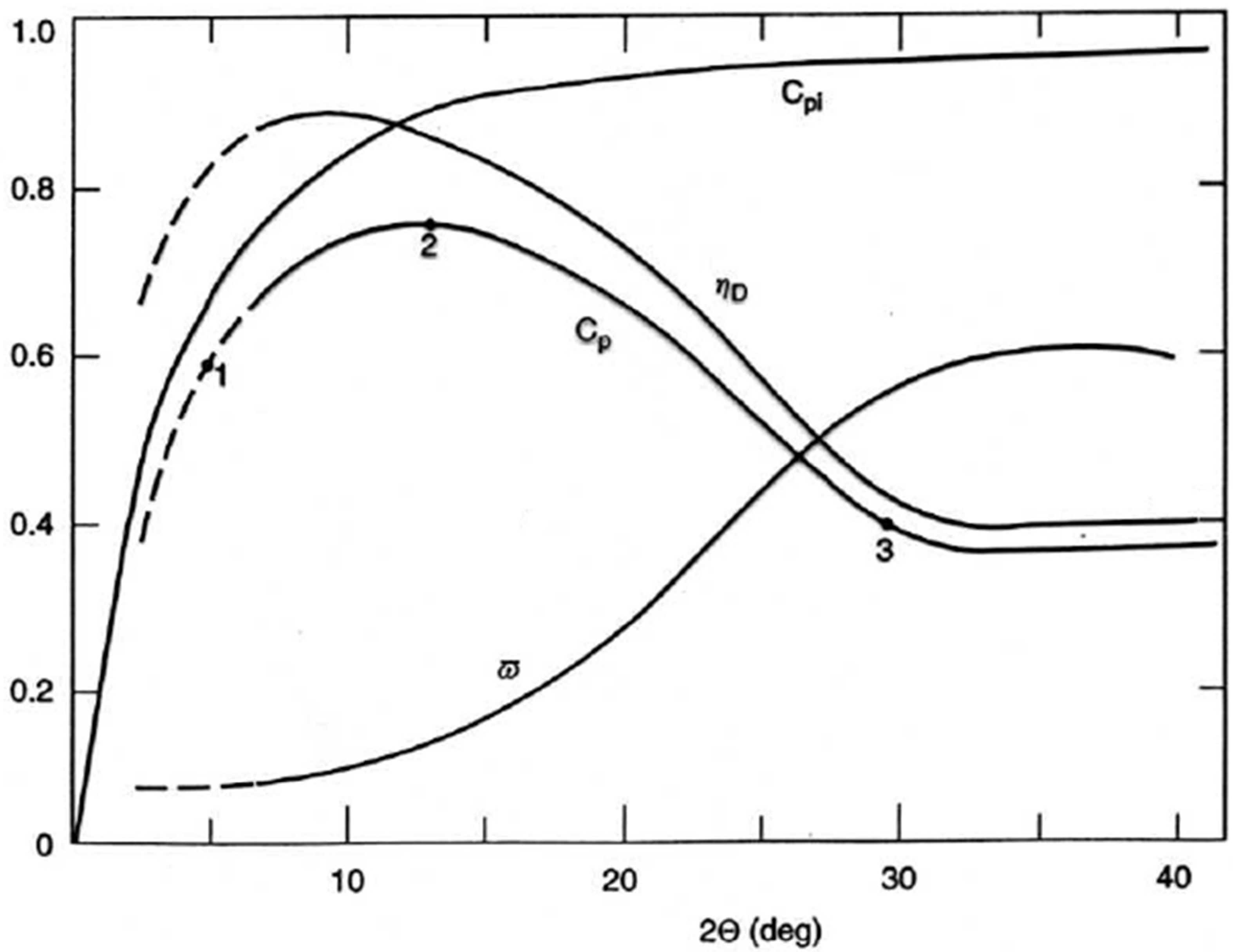


FIG. 2.16. Typical diffuser performance curves for a two-dimensional diffuser, with $L/W_1 = 8.0$ (adapted from Kline *et al.* 1959).

https://www.youtube.com/watch?v=JhlEkEk7igs&list=PL0EC6527BE871ABA3&index=8