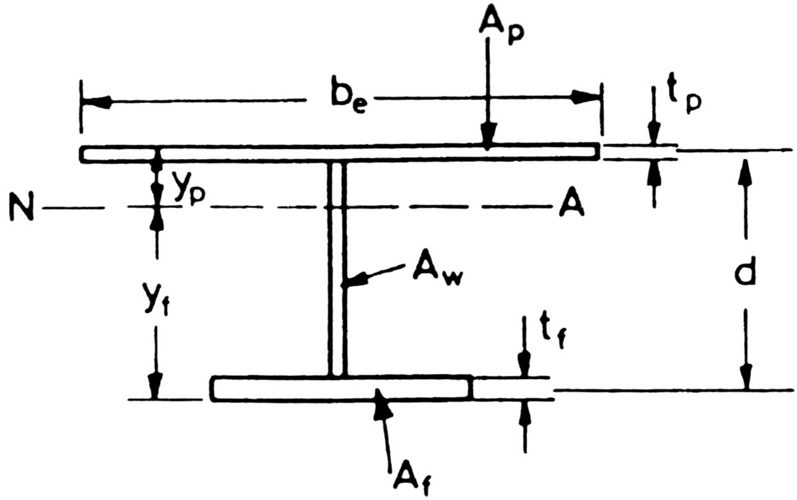



Geometric properties of beams attached to plating

$$y_f = \frac{1}{2} t_f + d\,C_2$$

$$y_p = \frac{1}{2} t_p + d(1 - C_2)$$

$$I_e = A_T\, d^2\, C_1$$

$$C_1 = \frac{A_w\left(\frac{A_T}{3} - \frac{A_w}{4}\right) + A_f A_p}{A_T^2}$$

$$C_2 = \frac{\frac{A_w}{2} + A_p}{A_T}$$

$A_p$ = effective plate area = $b_e t_p$

$A_w$ = web area

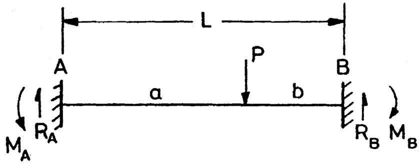
$A_f$ = flange area

$A_T$ = total area = $A_p + A_f + A_w$

$t_p$ = plate thickness

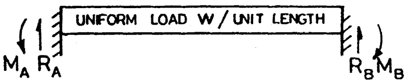
$t_f$ = flange thickness

$d$ = distance from center of plate to center of flange



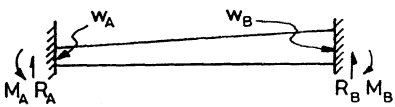

$$R_A = \frac{Pb^2(3a+b)}{L^3} \qquad R_B = \frac{Pa^2(a+3b)}{L^3}$$

$$M_A = \frac{Pab^2}{L^2} \qquad M_B = -\frac{Pa^2b}{L^2}$$



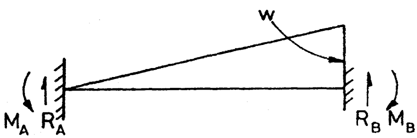

$$R_A = \frac{wL}{2} \qquad R_B = \frac{wL}{2}$$

$$M_A = \frac{wL^2}{12} \qquad M_B = -\frac{wL^2}{12}$$



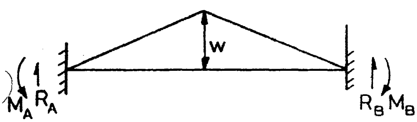

$$R_A = \frac{L}{20}(7w_A + 3w_B) \qquad R_B = \frac{L}{20}(3w_A + 7w_B)$$

$$M_A = \frac{L^2}{60}(3w_A + 2w_B) \qquad M_B = \frac{-L^2}{60}(2w_A + 3w_B)$$



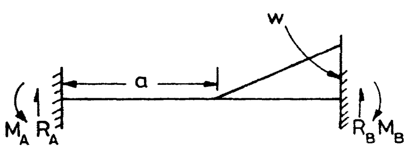

$$R_A = \frac{3}{20}wL \qquad R_B = \frac{7}{20}wL$$

$$M_A = \frac{wL^2}{30} \qquad M_B = \frac{-wL^2}{20}$$




$$R_A = \frac{wL}{4} \qquad R_B = \frac{wL}{4}$$

$$M_A = \frac{5wL^2}{96} \qquad M_B = \frac{-5wL^2}{96}$$




$$R_A = \frac{w(L-a)^3(3L+2a)}{20L^3}$$

$$R_B = \frac{w(L-a)}{20L^3}\left[10L^3 - (L-a)^2(3L+2a)\right]$$

$$M_A = \frac{w}{60L^2}(L-a)^3(2L+3a)$$

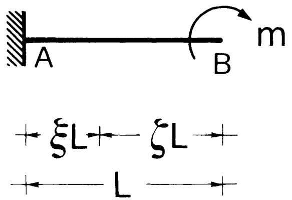
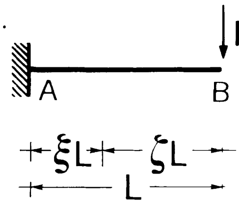
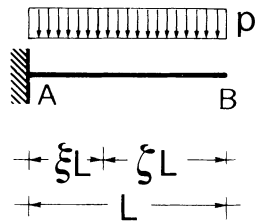
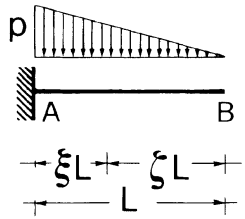
$$M_B = \frac{-w}{60L^2}(L-a)^2(3L^2+4aL+3a^2)$$


$$R_A = -R_B = \frac{12\ EI\ v_o}{L^3}$$

$$M_A = M_B = \frac{6\ EI\ v_o}{L^2}$$

Fixed-end reaction forces and moments

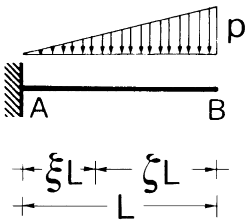
**Tabella 6. Travi ad asse rettilineo, dati per varie situazioni di carico e vincolo**

| | | | |
|---|---|---|---|
| 1.   | 2.   | 3.   | 4.   |
| $Y_A = 0; \quad M_A = -m$ | $Y_A = P; \quad M_A = -PL$ | $Y_A = PL; \quad M_A = -\frac{1}{2} PL^2$ | $Y_A = \frac{1}{2} PL; \quad M_A = -\frac{1}{6} PL^2$ |
| $T = cost = 0$ | $T = cost = P$ | $T = pL\,\zeta$ | $T = \frac{1}{2} pL\,\zeta^2$ |
| $M = cost = -m$ | $M = -PL\,\zeta$ | $M = -\frac{1}{2} pL^2\,\zeta^2$ | $M = -\frac{1}{6} pL^2\,\zeta^3$ |
| $\varphi = \frac{mL}{EI}\,\xi$ | $\varphi = \frac{1}{2}\frac{PL^2}{EI}\,\xi\,(1+\zeta)$ | $\varphi = \frac{1}{6}\frac{pL^3}{EI}\,(1-\zeta^3)$ | $\varphi = \frac{1}{24}\frac{pL^3}{EI}\,(1-\zeta^4)$ |
| $\varphi_B = \frac{mL}{EI}$ | $\varphi_B = \frac{1}{2}\frac{PL^2}{EI}$ | $\varphi_B = \frac{1}{6}\frac{pL^3}{EI}$ | $\varphi_B = \frac{1}{24}\frac{pL^3}{EI}$ |
| $v = \frac{1}{2}\frac{mL^2}{EI}\,\xi^2$ | $v = \frac{1}{6}\frac{PL^3}{EI}\,\xi^2\,(3-\xi)$ | $v = \frac{1}{24}\frac{pL^4}{EI}\,[3-\zeta\,(4-\zeta^3)]$ | $v = \frac{1}{120}\frac{pL^4}{EI}\,[4-\zeta\,(5-\zeta^4)]$ |
| $v_B = \frac{1}{2}\frac{mL^2}{EI}$ | $v_B = \frac{1}{3}\frac{PL^3}{EI}$ | $v_B = \frac{1}{8}\frac{pL^4}{EI}$ | $v_B = \frac{1}{30}\frac{pL^4}{EI}$ |

*(segue)*

*(seguito tabella 6)*

5.




$Y_A = \frac{1}{2}\, pL; \quad M_A = -\frac{1}{3}\, pL^2$

$T = \frac{1}{2}\, pL\, \zeta\, (2-\zeta)$

$M = -\frac{1}{6}\, pL^2\, (3-\zeta)\, \zeta^2$

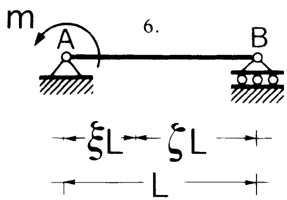
$\varphi = \frac{1}{24}\, \frac{pL^3}{EI}\, [3-\zeta^3\, (4-\zeta)]$

$\varphi_B = \frac{1}{8}\, \frac{pL^3}{EI}$

$v = \frac{1}{120}\, \frac{pL^4}{EI}\, [11-15\, \zeta+\zeta^4\, (5-\zeta)]$

$v_B = \frac{11}{120}\, \frac{pL^4}{EI}$

6.




$Y_A = -Y_B = \frac{m}{L}$

$T = cost = \frac{m}{L}$

$M = -m\, \zeta$

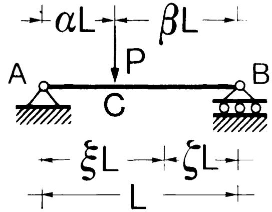
$\varphi = \frac{1}{6}\, \frac{mL}{EI}\, (1-3\, \zeta^2)$

$\varphi_A = -\frac{1}{3}\, \frac{mL}{EI}$

$\varphi_B = \frac{1}{6}\, \frac{mL}{EI}$

$v = -\frac{1}{6}\, \frac{mL^2}{EI}\, \zeta\, (1-\zeta^2)$

7.




– tronco AC:

$Y_A = P\, \beta$

$T = cost = P\beta$

$M = PL\, \beta\, \xi$

$\varphi = \frac{1}{6}\, \frac{PL^2}{EI}\, \beta\, (1-\beta^2-3\, \xi^2)$

$\varphi_A = \frac{1}{6}\, \frac{PL^2}{EI}\, \beta\, (1-\beta^2)$

$\varphi_C = \frac{1}{3}\, \frac{PL^2}{EI}\, \alpha\, \beta\, (\beta-\alpha)$

$v_A = \frac{1}{6}\, \frac{PL^3}{EI}\, \beta\, \xi\, (1-\beta^2-\xi^2)$

$v_C = \frac{1}{3}\, \frac{PL^3}{EI}\, \alpha^2\, \beta^2$

– tronco CB:

$Y_B = P\, \alpha$

$T = cost = -P\, \alpha$

$M = PL\, \alpha\, \zeta$

$M_{max} = M_C = PL\, \alpha\, \beta$

$\varphi = -\frac{1}{6}\, \frac{PL^2}{EI}\, \alpha\, (1-\alpha^2-3\, \zeta^2)$

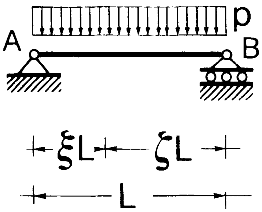
$\varphi_B = -\frac{1}{6}\, \frac{PL^2}{EI}\, \alpha\, (1-\alpha^2)$

$v_B = \frac{1}{6}\, \frac{PL^3}{EI}\, \alpha\, \zeta\, (1-\alpha^2-\zeta^2)$

*(segue)*

*(seguito tabella 6)*

8.




$Y_A = Y_B = \frac{1}{2}\ pL$

$T = \frac{1}{2}\ pL\ (1 - 2\ \xi)$

$M = \frac{1}{2}\ pL^2\ \zeta\ \xi$

$\xi = \frac{1}{2}: \quad M = M_{\max} = \frac{1}{8}\ pL^2$

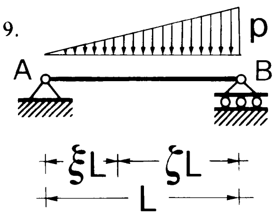
$\varphi = \frac{1}{24}\ \frac{pL^2}{EI}\ [1 + 2\ \xi^2\ (2\ \xi - 3)]$

$\varphi_A = -\ \varphi_B = \frac{1}{24}\ \frac{pL^3}{EI}$

$v = \frac{1}{24}\ \frac{pL^4}{EI}\ \zeta\ \xi\ (1 + \zeta\ \xi)$

$\xi = \frac{1}{2}: \quad v = v_{\max} = \frac{5}{384}\ \frac{PL^4}{EI}$

9.




$Y_A = \frac{1}{6}\ pL; \quad Y_B = \frac{1}{3}\ pL$

$T = \frac{1}{6}\ pL\ (1 - 3\ \xi^2)$

$M = \frac{1}{6}\ pL^2\ \zeta\ \xi\ (1 + \xi)$

$\xi = \frac{\sqrt{3}}{3}: \quad M = M_{\max} = \frac{\sqrt{3}}{27}\ pL^2$

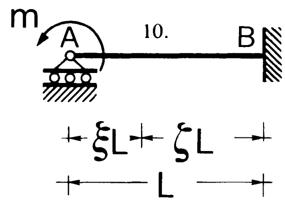
$\varphi = \frac{1}{360}\ \frac{pL^2}{EI}\ [7 - 15\ \xi^2\ (2 - \xi^2)]$

$\varphi_A = \frac{7}{360}\ \frac{pL^3}{EI}$

$\varphi_B = -\ \frac{8}{360}\ \frac{pL^3}{EI}$

$v = \frac{1}{360}\ \frac{pL^4}{EI}\ \zeta\ \xi\ (1 + \xi)\ (7 - 3\ \xi^2)$

10.




$Y_A = -\ Y_B = \frac{3}{2}\ \frac{m}{L}$

$M_B = \frac{1}{2}\ m$

$T = cost = \frac{3}{2}\ \frac{m}{L}$

$M = -\ \frac{1}{2}\ m\ (2 - 3\ \xi)$

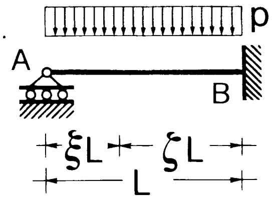
$\varphi = \frac{1}{4}\ \frac{mL}{EI}\ \zeta\ (2 - 3\ \zeta)$

$\varphi_A = -\ \frac{1}{4}\ \frac{mL}{EI}$

$v = -\ \frac{1}{4}\ \frac{mL^2}{EI}\ \xi\ \zeta^2$

$\xi = \frac{1}{3}: \quad v = v_{\min} = -\ \frac{1}{27}\ \frac{mL^2}{EI}$

11.




$Y_A = \frac{3}{8}\ pL; \quad Y_B = \frac{5}{8}\ pL$

$M_B = -\ \frac{1}{8}\ pL^2$

$T = pL\left(\frac{3}{8} - \xi\right)$

$M = \frac{1}{8}\ PL^2\ \xi\ (3 - 4\ \xi)$

$\varphi = \frac{1}{48}\ \frac{pL^3}{EI}\ \zeta\ (1 + \xi - 8\ \xi^2)$

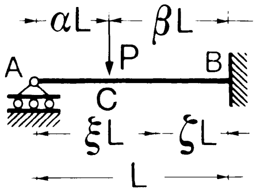
$\varphi_A = \frac{1}{48}\ \frac{pL^3}{EI}$

$v = \frac{1}{48}\ \frac{pL^4}{EI}\ \xi\ \zeta^2\ (1 + 2\ \xi)$

*(segue)*

*(seguito tabella 6)*

12.




– tronco AC:

$Y_A = \frac{1}{2} P \beta^2 (2+\alpha)$

$T = cost = Y_A$

$M = \frac{1}{2} PL \xi \beta^2 (2+\alpha)$

$\varphi = \frac{1}{4} \frac{PL^2}{EI} \beta^2 [\alpha - (2+\alpha) \xi^2]$;

$\varphi_A = \frac{1}{4} \frac{PL^2}{EI} \alpha \beta^2$

$v = \frac{1}{12} \frac{PL^3}{EI} \xi\beta^2 [3\alpha - (2+\alpha) \xi^2]$

– tronco CB:

$Y_B = \frac{1}{2} P \alpha (3-\alpha^2)$

$M_B = -\frac{1}{2} PL \alpha (1-\alpha^2)$

$T = cost = -Y_B$

$M = \frac{1}{2} PL \alpha (3 \zeta + \alpha^2 \xi - 1)$

$\varphi = \frac{1}{4} \frac{PL^2}{EI} \alpha\zeta[\alpha^2(2-\zeta) + 3\zeta - 2]$

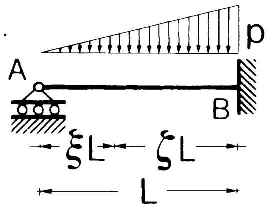
$v = \frac{1}{12} \frac{PL^3}{EI} \alpha\zeta^2 [3(1-\alpha^2) + - (3-\alpha^2)\zeta]$

$M_C = PL \alpha \beta^2 (2+\alpha)/2$

$\varphi_C = \frac{1}{4} \frac{PL^2}{EI} \alpha \beta^2 (1 - 2\alpha - \alpha^2)$

$v_C = \frac{1}{12} \frac{PL^3}{EI} \alpha^2 \beta^3 (3+\alpha)$

13.




$Y_A = \frac{1}{10} pL; \quad Y_B = \frac{2}{5} pL$

$M_B = -\frac{1}{15} pL^2$

$T = \frac{1}{10} pL (1 - 5 \xi^2)$

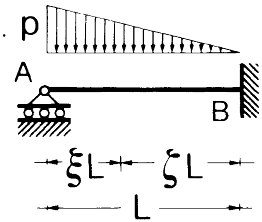
$M = \frac{1}{10} pL^2 \xi (3 - 5 \xi^2)$

$\varphi = \frac{1}{120} \frac{pL^3}{EI} \cdot \zeta (1+\xi) (1-5\xi^2)$

$\varphi_A = \frac{1}{120} \frac{pL^3}{EI}$

$v = \frac{1}{120} \frac{pL^4}{EI} \xi \zeta^2 (1+\xi)^2$

14.




$Y_A = \frac{11}{40} pL; \quad Y_B = \frac{9}{40} pL$

$M_B = -\frac{7}{120} pL^2$

$T = \frac{1}{40} pL (20 \zeta^2 - 9)$

$M = \frac{1}{120} pL^2 \xi (20\zeta^2 + 20\zeta - 7)$

$\varphi = \frac{1}{240} \frac{pL^3}{EI} \zeta (27 \zeta + - 10 \zeta^3 - 14)$

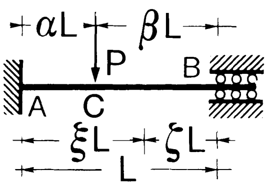
$\varphi_A = \frac{1}{80} \frac{pL^3}{EI}$

$v = \frac{1}{240} \frac{pL^4}{EI} \xi\zeta^2 (7 - 2 \zeta - 2\zeta^2)$

*(segue)*

*(seguito tabella 6)*

**15.**




– tronco AC:

$Y_A = P\ \beta^2\ (1 + 2\ \alpha)$

$M_A = -\ PL\ \alpha\ \beta^2$

$T = cost = Y_A$

$M = PL\ \beta^2\ [\xi\ (1 + 2\ \alpha) - \alpha]$

$\varphi = \frac{1}{2}\ \frac{PL^2}{EI}\ \beta^2\xi\ [2\alpha - \xi\ (1 + 2\alpha)]$

$\varphi_c = \frac{1}{2}\ \frac{PL^2}{EI}\ \alpha^2\ \beta^2\ (1 - 2\ \alpha)$

$v = \frac{1}{6}\ \frac{PL^3}{EI}\ \beta^2\xi^2\ [3\alpha - \xi\ (1 + 2\alpha)]$

$v_C = \frac{1}{3}\ \frac{PL^3}{EI}\ \alpha^3\ \beta^3$

– tronco CB:

$Y_B = P\ \alpha^2\ (1 + 2\ \beta)$

$M_B = -\ PL\ \alpha^2\ \beta$

$T = cost = -\ Y_B$

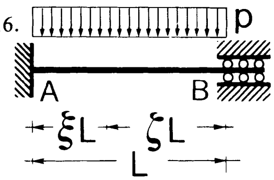
$M = PL\ \alpha^2\ [\zeta\ (1 + 2\ \beta) - \beta]$

$\varphi = -\ \frac{1}{2}\ \frac{PL^2}{EI}\ \alpha^2\zeta\ [2\beta - \zeta\ (1 + 2\beta)]$

$v = \frac{1}{6}\ \frac{PL^3}{EI}\ \alpha^2\zeta^2\ [3\beta - \zeta\ (1 + 2\beta)]$

$M_C = 2\ PL\ \alpha^2\ \beta^2$

**16.**




$Y_A = Y_B = \frac{1}{2}\ pL$

$M_A = M_B = -\ \frac{1}{12}\ pL^2$

$T = \frac{1}{2}\ pL\ (1 - 2\ \xi)$

$M = -\ \frac{1}{12}\ pL^2\ (1 - 6\ \xi\ \zeta)$

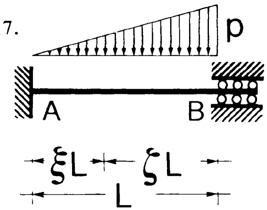
$\xi = \frac{1}{2}: \quad M = \frac{1}{24}\ pL^2$

$\varphi = \frac{1}{12}\ \frac{pL^2}{EI}\ \xi\ \zeta\ (1 - 2\ \xi)$

$v = \frac{1}{24}\ \frac{pL^4}{EI}\ \xi^2\ \zeta^2$

$\xi = \frac{1}{2}: \quad v = \frac{1}{384}\ \frac{pL^4}{EI}$

**17.**




$Y_A = \frac{3}{20}\ pL; \quad Y_B = \frac{7}{20}\ pL$

$M_A = -\ \frac{1}{30}\ pL^2; \quad M_B = \frac{1}{20}\ pL^2$

$T = \frac{1}{20}\ pL\ (3 - 10\ \xi^2)$

$M = -\ \frac{1}{60}\ pL^2\ (2 - 9\ \xi + 10\ \xi^3)$

$\varphi = \frac{1}{120}\ \frac{pL^3}{EI}\ \xi\zeta\ (4 - 5\ \xi - 5\ \xi^2)$

$v = \frac{1}{120}\ \frac{pL^4}{EI}\ \xi^2\ \zeta^2\ (2 + \xi)$