LEZIONE 9-10

Vortice Forzato (moto rigido)

$$\boxed{\frac{dh_0}{dr} - T\frac{ds}{dr} = V_a\frac{dV_a}{dr} + \frac{V_t}{r}\frac{d}{dr}\left(V_t \cdot r\right)}$$

$$\boxed{\frac{dh_0}{dr} - T\frac{ds}{dr} = 0}$$

$$\boxed{\frac{V_t}{r} = \text{cost}}$$

$$\frac{V_t}{r} = \frac{V_{ti}}{r_i} \qquad \rightarrow \qquad V_t = V_{ti}\frac{r}{r_i}$$

Vortice Forzato (moto rigido)

$$\boxed{\frac{dh_0}{dr} - T\frac{ds}{dr} = V_a\frac{dV_a}{dr} + \frac{V_t}{r}\frac{d}{dr}\left(V_t \cdot r\right)}$$

$$\boxed{\frac{dh_0}{dr} - T\frac{ds}{dr} = 0}$$

$$\boxed{\frac{d}{dr}\left(\frac{V_a^2}{2}\right) + \frac{V_{ti}}{r_i}\frac{d}{dr}\left(\frac{V_{ti}}{r_i}r^2\right) = 0}$$

$$\frac{d}{dr}\left(\frac{V_a^2}{2}\right) = -2\left(\frac{V_{ti}}{r_i}\right)^2 \cdot r$$

$$\left(\frac{V_a^2}{2}\right) = -2V_{ti}^2\left(\frac{r}{r_i}\right)^2 + C$$

Vortice Forzato (moto rigido)

$$\boxed{\frac{dh_0}{dr} - T\frac{ds}{dr} = V_a\frac{dV_a}{dr} + \frac{V_t}{r}\frac{d}{dr}\left(V_t \cdot r\right)}$$

$$\boxed{\frac{dh_0}{dr} - T\frac{ds}{dr} = 0}$$

$$V_a = V_{ai}$$

$$r = r_i$$

$$\Rightarrow C = \frac{V_{ai}^2}{2} + V_{ti}^2$$

$$\frac{V_{ti}}{V_{ai}} = tg\alpha_i$$

$$\boxed{\left(\frac{V_a}{V_{ai}}\right)^2 = 1 - 2\tan^2\alpha_i\left[\left(\frac{r}{r_i}\right)^2 - 1\right]}$$

Vortice Forzato

$$h = h_i + \frac{V_i^2 - V^2}{2}$$

$$\frac{h}{h_i} = 1 + \frac{1}{2h_i}\left(V_{ti}^2 - V_t^2 + \cancel{V_{ai}^2} - \cancel{V_a^2}\right)$$

$$\boxed{\frac{V_t}{r} = \text{cost}}$$

$$\boxed{\frac{h}{h_i} = 1 + \frac{V_{ti}^2}{2h_i}\left[1 - \left(\frac{V_t}{V_{ti}}\right)^2\right] + \frac{V_{ai}^2}{2h_i}\left[1 - \left(\frac{V_a}{V_{ai}}\right)^2\right]}$$

Vortice Forzato

$$\boxed{\frac{h}{h_i} = 1 + \frac{V_{ti}^2}{2h_i}\left[1 - \left(\frac{V_t}{V_{ti}}\right)^2\right] + \frac{V_{ai}^2}{2h_i}\left[1 - \left(\frac{V_a}{V_{ai}}\right)^2\right]}$$

$$\frac{V_t}{V_{ti}} = \frac{r}{r_i} \qquad \frac{V_{ti}}{V_{ai}} = tg\,\alpha_i$$

$$\boxed{\left(\frac{V_a}{V_{ai}}\right)^2 = 1 - 2\tan^2\alpha_i\left[\left(\frac{r}{r_i}\right)^2 - 1\right]}$$

$$\boxed{\frac{h}{h_i} = 1 + \frac{V_{ti}^2}{2h_i}\left[\left(\frac{r}{r_i}\right)^2 - 1\right]}$$

Vortice Forzato

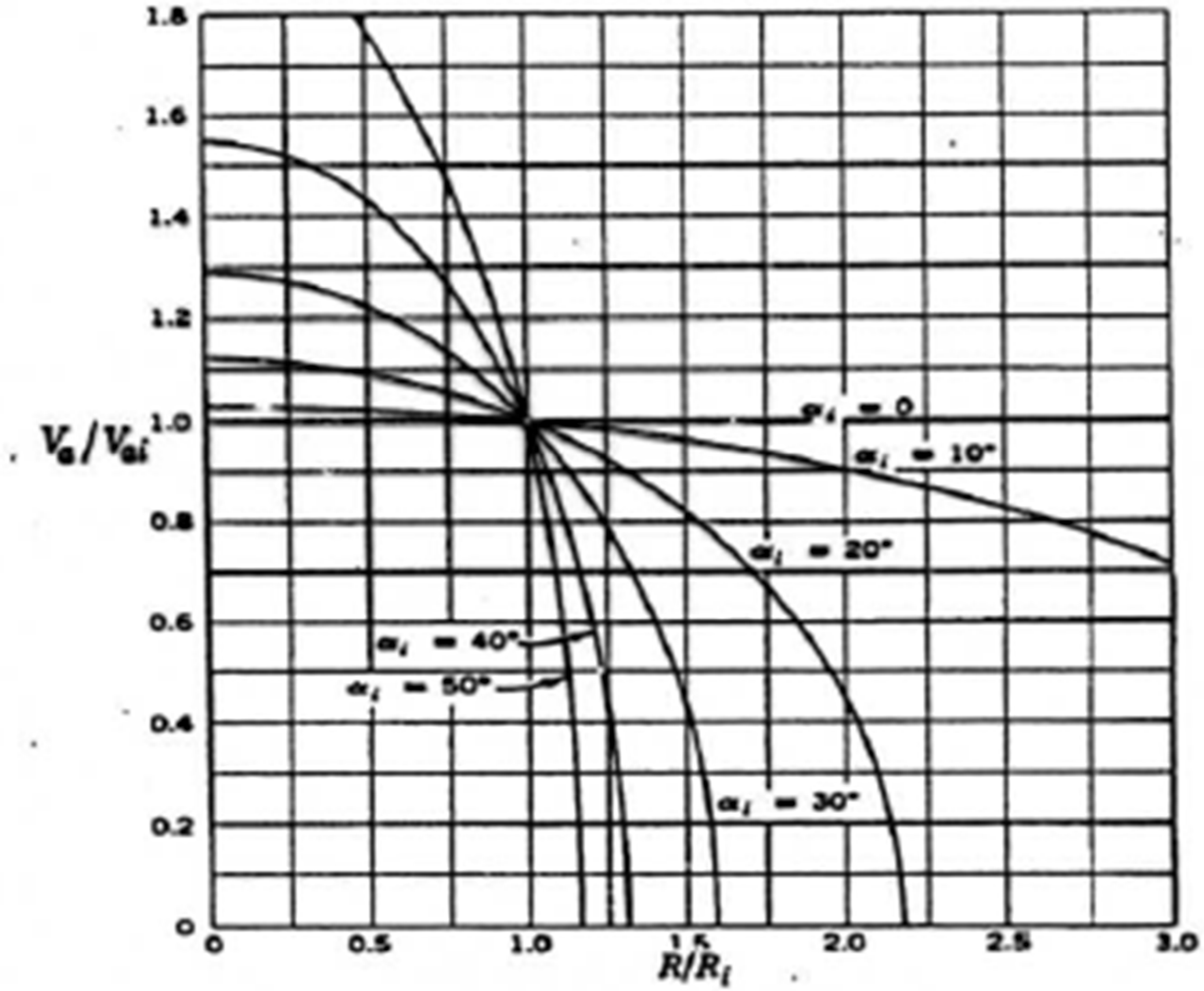
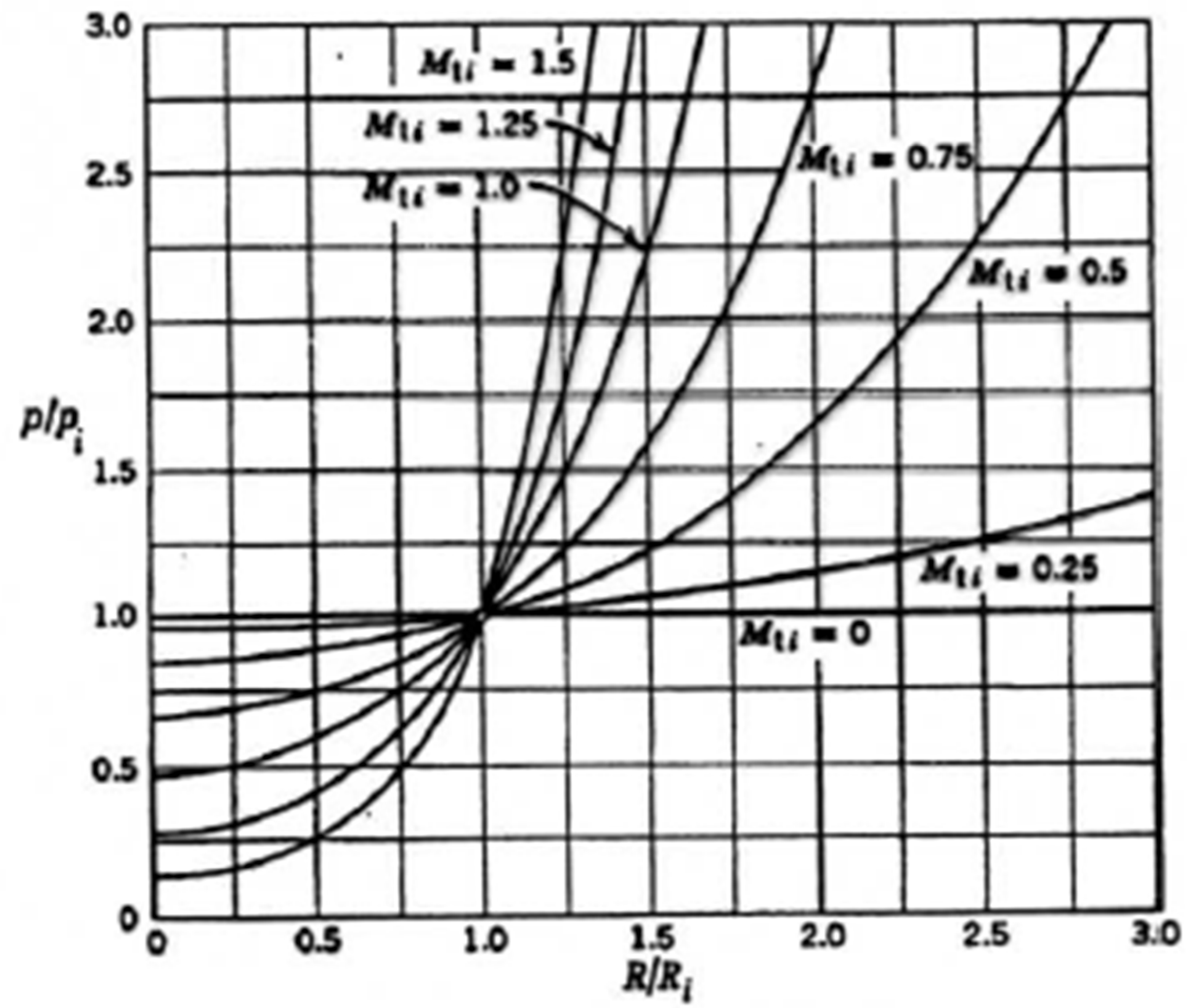


Figura 5.24: *Velocità assiale in funzione del raggio per un flusso a vortic forzato*

Vortice Forzato



.25: *Pressione in funzione del raggio per un flusso a vortice forzato*

$$\frac{p}{p_i} = \left(\frac{h}{h_i}\right)^{\frac{k}{k-1}}$$

$$\frac{h}{h_i} = 1 + \frac{k-1}{2} \mathrm{Mti}^2 \left[\left(\frac{r}{r_i}\right)^2 - 1 \right]$$

Vortice Forzato

Imponendo: $\frac{dh_0}{dr} = 0$

cosa accade nella sezione a valle?

Sezione 1

$$h_{01} = \cos t$$

$$V_{t1} = K_1 \cdot r$$

$$\frac{V_t}{r} = K$$

Sezione 2

$$V_{t2} = K_2 \cdot r$$

Vortice Forzato

$$\boxed{\frac{dh_0}{dr} - T\frac{ds}{dr} = V_a\frac{dV_a}{dr} + \frac{V_t}{r}\frac{d}{dr}\left(V_t \cdot r\right)}$$

Lavoro scambiato:

$$h_{02} - h_{01} = u\left(V_{t2} - V_{t1}\right) = \omega r\left(K_2 r - K_1 r\right) =$$

$$= \omega\left(K_2 - K_1\right)r^2 = \Delta h_{012}$$

ovvero: $\Delta h \propto r^2$ $\qquad h_{02} \neq \cos t$

$$\frac{d\Delta h_{012}}{dr} = \frac{d\left(h_{02} - h_{01}\right)}{dr} = \frac{dh_{02}}{dr} = V_a\frac{dV_a}{dr} + \frac{V_t}{r}\frac{d}{dr}\left(V_t r\right) =$$

$$= 2\omega\left(K_2 - K_1\right)r = \frac{d}{dr}\left(\frac{V_{a2}^2}{2}\right) + K_2\frac{d}{dr}\left(K_2 r^2\right)$$

Vortice Forzato

$$V_{a2}^{2} = -2\left[K_{2}^{2} - \omega\left(K_{2} - K_{1}\right)\right]r^{2} + C$$

il valore di C posso calcolarlo da:

$$\frac{\dot{m}}{2\pi\rho} = \int_{rh}^{rs} V_{a1} r dr = \int_{rh}^{rs} V_{a2} r dr$$

Vortice generico

$$V_{t1} = a \cdot r^n - \frac{b}{r} \qquad V_{t2} = a \cdot r^n + \frac{b}{r}$$

$$L_u = h_{02} - h_{01} = u\left(V_{t2} - V_{t1}\right) = 2 \cdot b \cdot \omega$$

- $n = 0$ “zero power blending”
- $n = 1$ “fast power blending”

Vortice generico

$n=1$ "Palettatura a grado di reazione costante"

$$V_{t1}=a\cdot r-\frac{b}{r}$$

$$V_{t2}=a\cdot r+\frac{b}{r}$$

$$R=1-\frac{V_2^2-V_1^2}{2L_u}=1-\frac{V_{t2}^2+\cancel{V_{a2}^2}-V_{t1}^2-\cancel{V_{a1}^2}}{2L_u}$$

$$V_{a2}\simeq V_{1a}$$

$$R=1-\frac{\cancel{\left(V_{t2}-V_{t1}\right)}\left(V_{t2}+V_{t1}\right)}{2u\cancel{\left(V_{t2}-V_{t1}\right)}}=1-\frac{\left(V_{t2}+V_{t1}\right)}{2u}=1-\frac{a}{\omega}=\text{cost}$$

Angolo palettatura costante

$$\frac{V_t}{V_a} = \tan \alpha = \cos t$$

Angolo costante significa bordo d'attacco rettilineo, come si presenta il deflusso?

Come variano Va e Vt al variare di r rispettando l'equilibrio radiale?

Angolo palettatura costante

$$\frac{V_t}{V_a} = \tan\alpha = \cos t$$

$$\frac{V}{V_i} = \frac{V_a}{V_{ai}} = \frac{V_t}{V_{ti}} = \left(\frac{r_i}{r}\right)^{sen^2\alpha}$$

$$\begin{cases} \dfrac{dh_0}{dr} = 0 \\ T\dfrac{ds}{dr} = 0 \end{cases}$$

$$\frac{d}{dr}\left(\frac{V_a^{\,2}}{2}\right) + \frac{V_t}{r}\frac{d\left(rV_t\right)}{dr} = 0$$

Angolo palettatura costante

$$V_a = V\cos\alpha$$

$$V_t = V sen\alpha$$

$$\frac{d}{dr}\left(\frac{V^2\cos^2\alpha}{2}\right) + \frac{V sen\alpha}{r}\frac{d\left(rV sen\alpha\right)}{dr} = 0$$

$$\boxed{\frac{dV}{dr} + \frac{V}{r} sen^2\alpha = 0}$$

Angolo palettatura costante

$$y' + \varphi(x)\,y + \psi(x) = 0 \qquad\qquad y = e^{-\int \varphi(x)dx}\left[C - \int \psi(x)\,e^{\int \varphi(x)dx}dx\right]$$

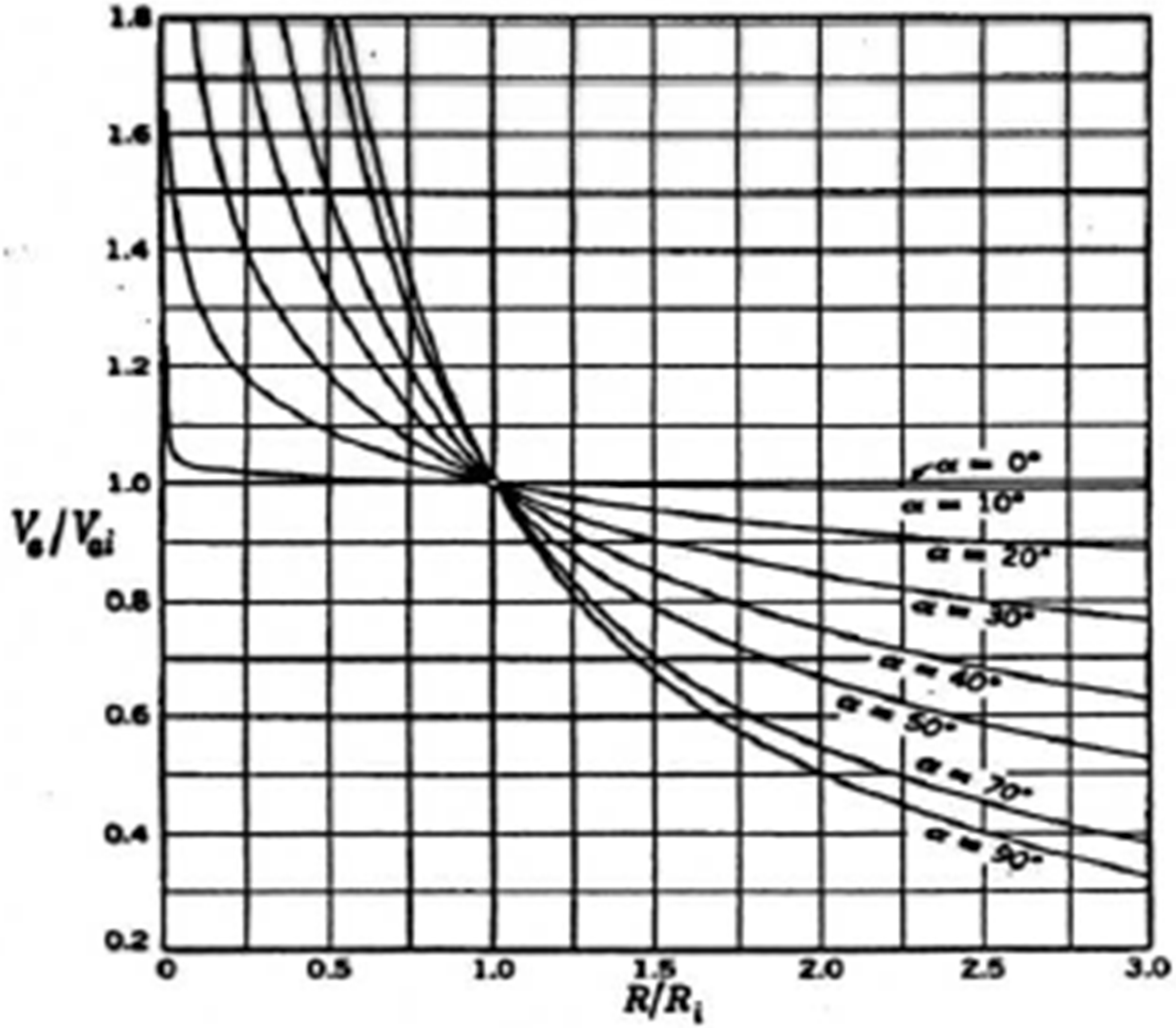
in questo caso: $\varphi(x) = 0$

$$V = e^{-\int \frac{sen^2\alpha}{r}dr} \cdot C = r^{-sen^2\alpha} \cdot C$$

$$V = V_i \quad \text{se} \quad r = r_i \quad \rightarrow \quad C = \frac{V_i}{r_i^{-sen^2\alpha}}$$

$$\boxed{\frac{h}{h_i} = 1 + \frac{V_{ti}^2}{2h_i sen^2\alpha}\left[1 - \left(\frac{r_i}{r}\right)^{2sen^2\alpha}\right]}$$

Angolo palettatura costante



turbina vortice libero

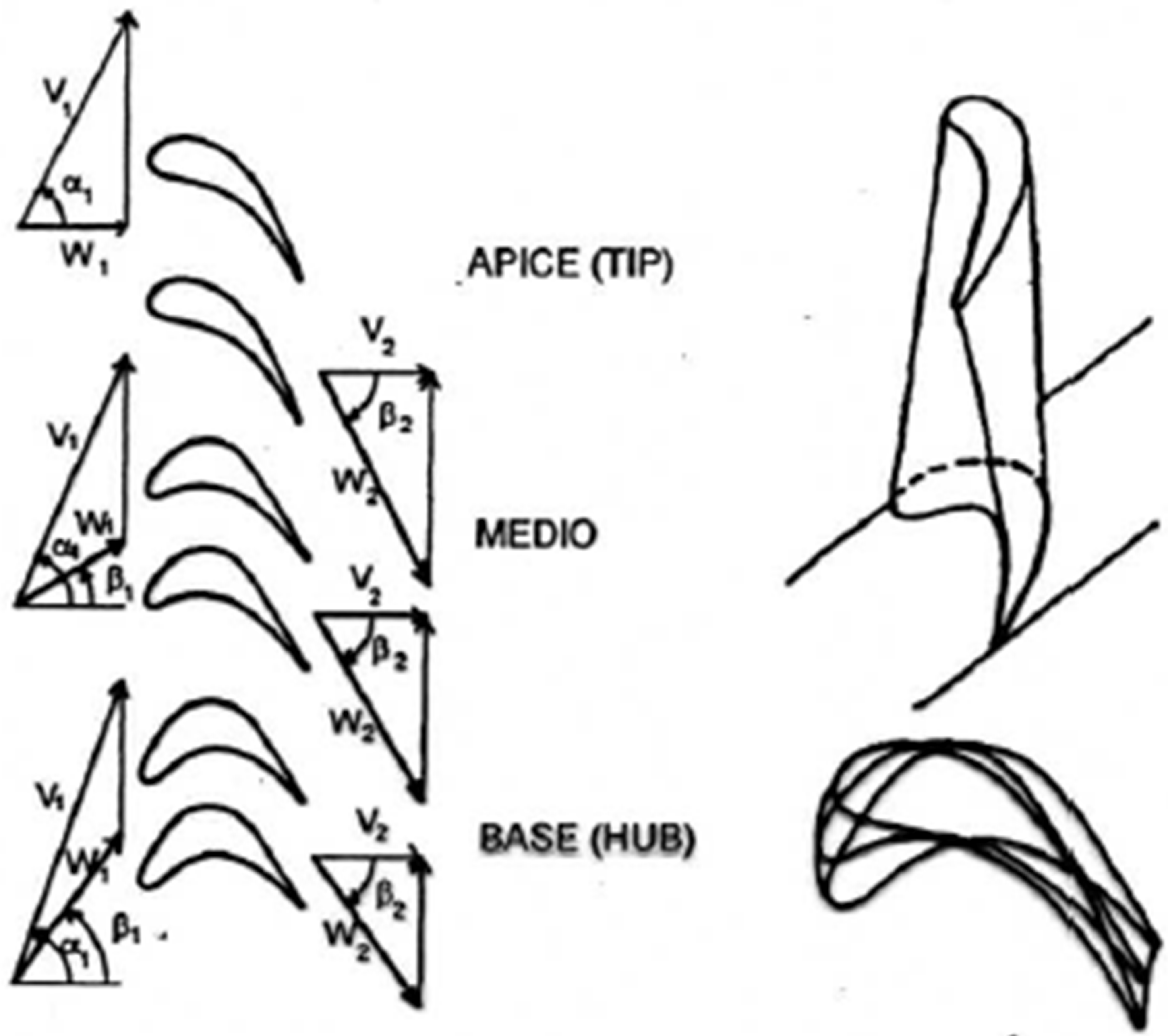


Figura 5.27: *Profili e triangoli delle velocità per il rotore di uno stadio di bassa pressione di turbina a vapore o a gas* ($r_{Tip}/r_{Hub} = 1.4$).

compressore vortice libero

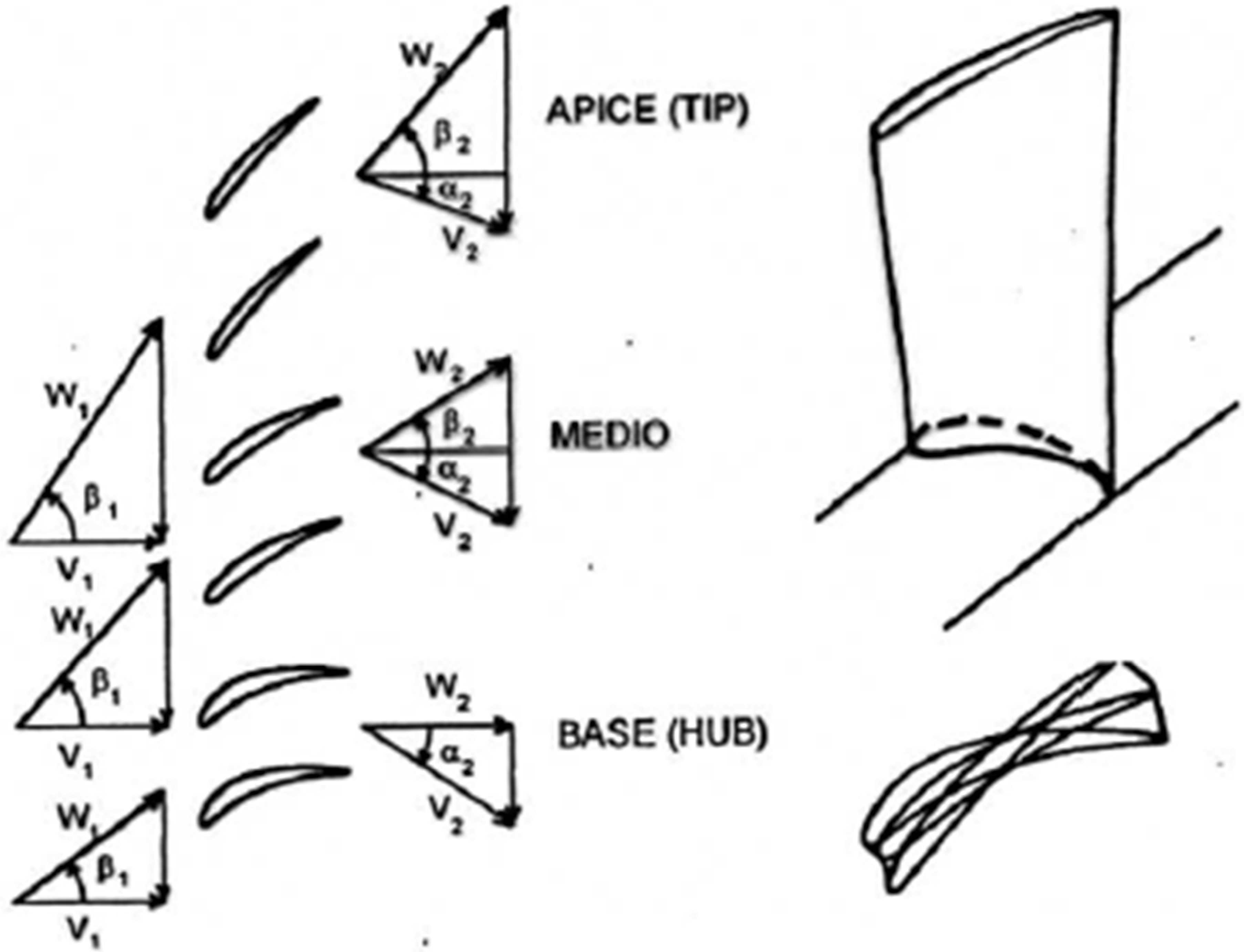


Figura 5.28: *Profili e triangoli delle velocità per un rotore di turbo.fan a vortice libero* ($r_{Tip}/r_{Hub} = 2$).