1-3 DISTINGUISHING ATTRIBUTES OF GEOPHYSICAL FLUID DYNAMICS

Two main ingredients distinguish the discipline from traditional fluid mechanics: the effects of rotation and those of stratification. The controlling influence of one, the other, or both leads to peculiarities exhibited only by geophysical flows. In a nutshell, the present book can be viewed as an account of these peculiarities.

The presence of an ambient rotation, such as that due to the earth's spin about its axis, introduces in the equations of motion two acceleration terms that, in the rotating framework, can be interpreted as forces. They are the Coriolis force and the centrifugal force (Stommel and Moore, 1989). Although the latter is the more palpable of the two, it plays no role in geophysical flows, however surprising this may be to the neophyte. The former and less intuitive of the two turns out to be a crucial factor in geophysical motions.

In anticipation of the following chapters, it can be mentioned here (without explanation) that a major effect of the Coriolis force is to impart a certain vertical rigidity to the fluid. In rapidly rotating, homogeneous fluids, this effect can be so strong that the flow displays strict columnar motions; that is, all particles along the same vertical evolve in concert, thus forever retaining their vertical alignment. The discovery of this property is attributed to Geoffrey I. Taylor, a British physicist famous for his varied contributions to fluid dynamics. (See the short biography at the end of Chapter 4.) It is said that Taylor first arrived at the rigidity property with mathematical arguments alone. Not believing that this could be correct, he then performed laboratory experiments that revealed, much to his amazement, that the theoretical prediction was indeed correct. Drops of dye released in such rapidly rotating, homogeneous fluids form vertical streaks, which, within a few rotations, shear laterally to form spiral sheets of dyed fluid (Figure 1-2). These, called Taylor curtains, are easily created at home by releasing food coloring in a clear water vessel brought to rotation on a turntable. The vertical coherence of these sheets is truly fascinating!

In large-scale atmospheric and oceanic flows, such state of perfect vertical rigidity is not realized, chiefly because the rotation rate is not sufficiently fast and the density is not sufficiently uniform to mask other, ongoing processes. Nonetheless, motions in

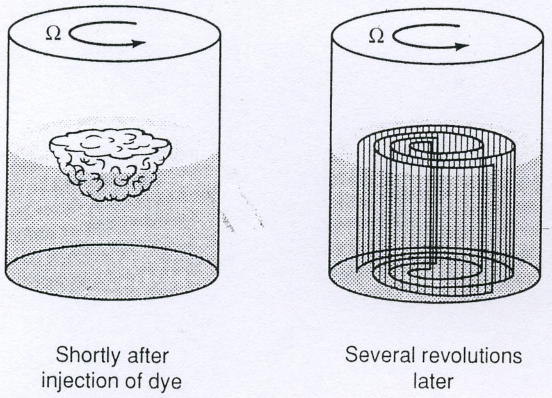

Figure 1-2 Experimental evidence of the apparent rigidity of a rapidly rotating, homogeneous fluid. The initial cloud of dye is transformed over several rotations into perfectly vertical sheets, known as *Taylor curtains*.

the atmosphere, in the oceans, and on other planets manifest a tendency toward columnar behavior. For example, currents in the western North Atlantic have been observed to extend vertically over 4000 m without significant change in amplitude and direction (Schmitz, 1980).

Stratification, the other distinguishing attribute of geophysical fluid dynamics, arises because naturally occurring flows typically involve fluids of different densities (e.g., warm and cold air masses, fresh and saline waters). Here, the gravitational force is of great importance, for it tends to lower the heaviest fluid and to raise the lightest. Under equilibrium conditions, the fluid is stably stratified, consisting of vertically stacked horizontal layers. Fluid motions, however, disturb this equilibrium, which gravity systematically strives to restore. Small perturbations generate internal waves, the three-dimensional analogue of surface waves, with which we are all familiar. Large perturbations, especially those maintained over time, may cause mixing and convection. For example, the prevailing winds in our atmosphere are manifestations of the planetary convection driven by the pole-to-equator temperature difference.

It is worth mentioning the perplexing situation in which a boat may experience strong resistance to forward motion while sailing under apparently calm conditions. This phenomenon, called *dead waters* by mariners, was first documented by the Norwegian oceanographer Fridtjof Nansen, famous for his epic expedition on the *Fram* through the Arctic Ocean, begun in 1893. Nansen reported the problem to his Swedish colleague Vagn Walfrid Ekman who, after performing laboratory simulations (Ekman, 1904), affirmed that internal waves were to blame. The scenario is as follows: During times of dead waters, Nansen must have been sailing in a layer of relatively fresh water capping the more saline oceanic waters and of thickness, coincidently, comparable to the ship draft; the ship created a wake of internal waves along the interface (Figure 1-3), unseen at the surface but radiating considerable energy and causing the noted resistance to the forward motion of the ship.

1-4 SCALES OF MOTION

To discern whether a physical process is dynamically important in any particular situation, geophysical fluid dynamicists introduce *scales of motion*. These are dimensional quantities expressing the overall magnitude of the variables under consideration. They are estimates rather than precisely defined quantities and are understood solely as *orders of magnitude* of physical variables. In most situations, the key scales are those for time, length, and velocity. For example, in the dead-water situation investigated by Ekman (Figure 1-3), fluid motions comprise a series of waves whose dominant wavelength is about the length of the submerged ship hull; this length is the natural choice for the length scale L of the problem; likewise, the ship speed provides a reference velocity that can be taken as the velocity scale U; finally, the time taken for the ship to travel the distance L at its speed U is the natural choice of time scale: $T = L/U$.

As a second example, consider Hurricane Hugo during its course off the southeastern coast of the United States in late September 1989 (Figure 1-1). The satellite

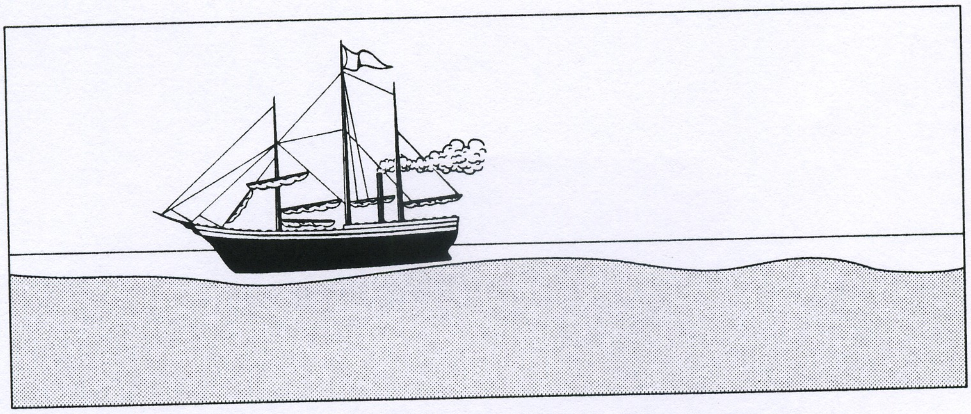

Figure 1-3 A laboratory experiment by V. W. Ekman (1904) showing internal waves generated by a model ship in a tank filled with two fluids of different densities. The heavier fluid has been colored to make the interface visible. The model ship (the superstructure of which has been drawn onto the original picture to depict Fridtjof Nansen's *Fram*) is towed from right to left, causing a wake of waves on the interface. The energy consumed by the generation of those waves produces a drag that, for a real ship, would translate into a resistance to forward motion. The absence of any significant surface wave has prompted sailors to call such situations *dead waters*. (From Ekman, 1904, as adapted by Gill, 1982.)

picture reveals an almost circular feature spanning approximately 3° of latitude (333 km), whereas the track displays appreciable changes in direction and speed of propagation over 2-day intervals. Finally, sustained surface wind speeds of level-5 hurricanes such as Hugo exceed 70 m/s. All this suggests the following choice of scales: $L = 300$ km, $T = 2 \times 10^5$ s (= 55.6 h), and $U = 70$ m/s.

As a last example, consider the famous Great Red Spot in Jupiter's atmosphere, known to have existed at least several hundred years (Figure 1-4). The structure is an elliptical vortex centered at 22°S and spanning approximately 12° in latitude and 25° in longitude; it exhibits wind speeds slightly exceeding 110 m/s and slowly drifts zonally at a speed of 3 m/s (Ingersoll et al., 1979; Dowling and Ingersoll, 1988). Knowing that the planet's equatorial radius is 71,400 km, we determine the vortex semimajor and semiminor axes (14,400 km and 7500 km, respectively) and deem $L = 10{,}000$ km to be an appropriate length scale. A natural velocity scale for the fluid is $U = 100$ m/s. The selection of a time scale is somewhat problematic in view of the nearly steady state of the vortex; one choice is the time taken by a fluid particle to cover the distance L at the speed U ($T = L/U = 10^5$ s), whereas another is the time taken by the vortex to drift zonally over a distance equal to its longitudinal extent ($T = 10^7$ s). Additional information on the physics of the problem is clearly needed before selecting a time scale. Such ambiguity is not uncommon because many natural phenomena vary on different temporal scales (e.g., the earth's atmosphere exhibits daily weather variation as well as decadal climatic variations, among others). The selection of a time scale then reflects the particular choice of physical processes being investigated in the system.

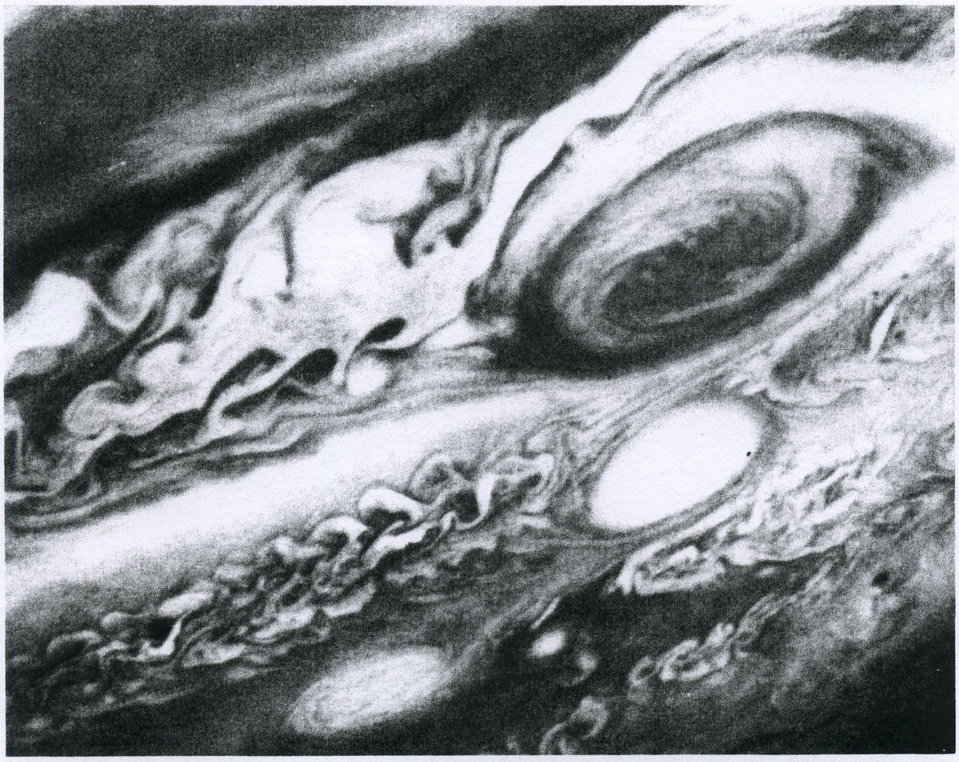

Figure 1-4 The Great Red Spot of Jupiter as seen by the spacecraft *Voyager 1.* (Image processed at the Jet Population Laboratory, USA.)

As the novice to geophysical fluid dynamics has already realized, the selection of scales for any given problem is more an art than a science. Choices are rather subjective. The trick is to choose quantities that are relevant to the problem, yet simple to establish. There is freedom. Fortunately, small inaccuracies are inconsequential because the scales are meant only to guide in the clarification of the problem, whereas grossly inappropriate scales will usually lead to flagrant contradictions. Practice, which forms intuition, is necessary to build confidence.

Before closing this section, it is worth mentioning three additional scales that play important roles in analyzing geophysical fluid problems. As we mentioned earlier, geophysical fluids generally exhibit a certain degree of density heterogeneity, called stratification. The important parameters are then the average density ρ_0, the range of density variations $\Delta\rho$, and the height H over which such density variations occur. In the ocean, the weak compressibility of water under changes of pressure, temperature, and salinity translates into values of $\Delta\rho$ always much less than ρ_0, whereas the compressibility of air renders the selection of $\Delta\rho$ in atmospheric flows somewhat delicate. Since geophysical flows are generally bounded in the vertical direction, the total depth of the fluid may be substituted for the height scale H. Usually, the smaller of the two height scales is selected.

As an example, the density and height scales in the dead-water problem (Figure 1-3) can be chosen as follows: $\rho_0 = 1025\ \mathrm{kg/m^3}$, the density of either fluid layer (almost the same); $\Delta\rho = 1\ \mathrm{kg/m^3}$, the density difference between lower and upper layers (much smaller than ρ_0); and $H = 5$ m, the depth of the upper layer.

1-5 IMPORTANCE OF ROTATION

Naturally, we may wonder at which scales the ambient rotation becomes an important factor in controlling the fluid motions. To answer this question, we must first determine the ambient rotation rate. Let us denote it by Ω:

$$\Omega = \frac{2\pi \text{ radians}}{\text{time of one revolution}}, \tag{1-1}$$

which, for our planet Earth is, $2\pi/24\ \mathrm{h} + 2\pi/1\ \mathrm{y} = 7.29 \times 10^{-5}\ \mathrm{s^{-1}}$.

If fluid motions evolve on a time scale comparable to or longer than the time of one rotation, we can anticipate that the fluid will feel the effect of the ambient rotation. We thus define the dimensionless quantity

$$\omega = \frac{\text{time of one revolution}}{\text{motion time scale}} = \frac{2\pi/\Omega}{T} = \frac{2\pi}{\Omega T}, \tag{1-2}$$

where T is used to denote the time scale of the flow. Our criterion is as follows: If ω is on the order of or less than unity ($\omega \lesssim 1$), rotation effects should be considered. On Earth, this occurs when T exceeds 24 h.

A second and usually more useful criterion results from considering the velocity and length scales of the motion. Let us denote these by U and L, respectively. Naturally, if a particle traveling at the speed U covers the distance L in a time interval greater than or comparable to a rotation period, we expect the trajectory to be influenced by the ambient rotation, and so we write

$$\varepsilon = \frac{\text{time of one revolution}}{\text{time taken by particle to cover distance } L \text{ at speed } U} = \frac{2\pi/\Omega}{L/U} = \frac{2\pi U}{\Omega L}. \tag{1-3}$$

If ε is on the order of or less than unity ($\varepsilon \lesssim 1$), we conclude that rotation is important.

Let us now consider a variety of possible length scales, using the value Ω for Earth. The corresponding velocity criteria are listed in Table 1-1.

Obviously, in most engineering applications (such as the flow of water at a speed of 5 m/s in a turbine 1 m in diameter or the air flow past a 5-m wing on an airplane flying at 100 m/s), the inequality is not met, and the effects of rotation can be ignored. On the contrary, geophysical flows (such as an ocean current flowing at 10 cm/s and meandering over a distance of 10 km or a wind blowing at a speed of 10 m/s in a

TABLE 1-1 LENGTH AND VELOCITY SCALES OF MOTIONS IN WHICH ROTATION EFFECTS ARE IMPORTANT

$L = 1$ m	$U \leq 0.012$ mm/s
$L = 10$ m	$U \leq 0.12$ mm/s
$L = 100$ m	$U \leq 1.2$ mm/s
$L = 1$ km	$U \leq 1.2$ cm/s
$L = 10$ km	$U \leq 12$ cm/s
$L = 100$ km	$U \leq 1.2$ m/s
$L = 1000$ km	$U \leq 12$ m/s
$L =$ Earth radius $= 6371$ km	$U \leq 74$ m/s

1000-km-wide anticyclonic formation) do meet the inequality. This demonstrates that rotation is usually important in geophysical flows.

6 IMPORTANCE OF STRATIFICATION

The next question concerns the condition under which stratification effects are expected to play an important dynamic role. Geophysical fluids typically consist of fluid masses of different densities, which under gravitational action tend to arrange themselves in vertical stacks, corresponding to a state of minimal potential energy. But, motions continuously disturb this equilibrium, tending to raise dense fluid and lower light fluid. The corresponding increase of potential energy must be at the expense of kinetic energy. Therefore, the dynamical importance of stratification can be evaluated by comparing potential and kinetic energies.

If $\Delta\rho$ is the scale of density variations in the fluid and H is its height scale, a prototypical perturbation to the stratification consists of raising a fluid element of density $\rho_0 + \Delta\rho$ over the height H and, in order to conserve volume, lowering a lighter fluid element of density ρ_0 over the same height. The corresponding change in potential energy, per unit volume, is $(\rho_0 + \Delta\rho)gH - \rho_0 gH = \Delta\rho gH$. With a typical fluid velocity U, the kinetic energy available per unit volume is $\frac{1}{2}\rho_0 U^2$. We therefore construct the comparative ratio

$$\sigma = \frac{\frac{1}{2}\rho_0 U^2}{\Delta\rho gH}, \tag{1-4}$$

to which we can give the following interpretation. If σ is on the order of unity ($\sigma \sim 1$), a typical potential-energy increase necessary to perturb the stratification consumes a sizable portion of the available kinetic energy, thereby modifying the flow field substantially. Stratification is then important. If σ is much less than unity ($\sigma \ll 1$), there is insufficient kinetic energy to perturb significantly the stratification, and the latter greatly constrains the flow. Finally, if σ is much greater than unity ($\sigma \gg 1$), potential-energy modifications occur at very little cost to the kinetic energy, and stratification hardly affects the flow. In conclusion, stratification effects cannot be ignored in the first two cases—that is, when the dimensionless ratio defined in (1-4) is on the order of or much less than unity ($\sigma \lesssim 1$). In other words, σ is to stratification what the number ε, defined in (1-3), is to rotation.

A most interesting situation arises in geophysical fluids when rotation and stratification effects are simultaneously important, yet neither is dominant. Mathematically, this occurs when $\varepsilon \sim 1$ and $\sigma \sim 1$ and yields the following relations among the various scales:

$$L \sim \frac{U}{\Omega} \quad \text{and} \quad U \sim \sqrt{\frac{\Delta\rho}{\rho_0} gH}\,. \tag{1-5}$$

(The factors 2π and $\frac{1}{2}$ have been omitted because they are secondary in a scale analysis.) Elimination of the velocity U yields a fundamental length scale:

$$L \sim \frac{1}{\Omega}\sqrt{\frac{\Delta\rho}{\rho_0} gH}\,. \tag{1-6}$$

In a given fluid, of mean density ρ_0 and density variation $\Delta\rho$, occupying a height H on a planet rotating at rate Ω and exerting a gravitational acceleration g, the scale L arises as a preferential length over which motions will take place. On Earth ($\Omega = 7.29 \times 10^{-5}$ $\mathrm{s^{-1}}$ and $g = 9.81\ \mathrm{m/s^2}$), typical conditions in the atmosphere ($\rho_0 = 1.2\ \mathrm{kg/m^3}$, $\Delta\rho = 0.03\ \mathrm{kg/m^3}$, $H = 5000$ m) and in the ocean ($\rho_0 = 1028\ \mathrm{kg/m^3}$, $\Delta\rho = 2\ \mathrm{kg/m^3}$, $H = 1000$ m) yield the following natural length and velocity scales:

$$L_{\text{atmosphere}} \sim 500\ \text{km} \qquad U_{\text{atmosphere}} \sim 30\ \text{m/s} \tag{1-7}$$

$$L_{\text{ocean}} \sim 60\ \text{km} \qquad U_{\text{ocean}} \sim 4\ \text{m/s}. \tag{1-8}$$

Although these estimates are relatively crude, we can easily recognize here the typical size and wind speed of weather patterns in the lower atmosphere and the typical width and speed of major currents in the upper ocean.

1-7 IMPORTANT DISTINCTIONS BETWEEN THE ATMOSPHERE AND OCEANS

Generally, motions of the air in our atmosphere and of seawater in the oceans that fall under the scope of geophysical fluid dynamics occur on scales of several kilometers up to the size of the earth. Atmospheric phenomena comprise the coastal sea breeze, local to regional processes associated with topography, the cyclones, anticyclones, and fronts that form our daily weather, the general atmospheric circulation, and climatic variations. Oceanic phenomena of interest include coastal upwelling and other processes associated with the presence of a coast, large eddies and fronts, major ocean currents such as the Gulf Stream, and the large-scale circulation. Table 1-2 lists the typical velocity, length, and time scales of these motions. As we can readily see, the general rule is that oceanic motions are slower and more confined than their atmospheric counterparts. Also, the ocean tends to evolve more slowly than the atmosphere.

TABLE 1-2 LENGTH, VELOCITY, AND TIME SCALES IN THE EARTH'S ATMOSPHERE AND OCEANS

Phenomenon	Length scale L	Velocity scale U	Time scale T
Atmosphere:			
Sea breeze	5–50 km	1–10 m/s	12 h
Mountain waves	10–100 km	1–20 m/s	Days
Weather patterns	100–5000 km	1–50 m/s	Days to weeks
Prevailing winds	Global	5–50 m/s	Seasons to years
Climatic variations	Global	1–50 m/s	Decades and beyond
Ocean:			
Internal waves	1–20 km	0.05–0.5 m/s	Minutes to hours
Coastal upwelling	1–10 km	0.1–1 m/s	Several days
Large eddies, fronts	10–200 km	0.1–1 m/s	Days to weeks
Major currents	50–500 km	0.5–2 m/s	Weeks to seasons
Large-scale gyres	Basin scale	0.01–0.1 m/s	Decades and beyond

Besides notable scale disparities, the earth's atmosphere and oceans also have their own peculiarities. For example, a number of oceanic processes are caused by the presence of lateral boundaries (continents, islands), a constraint practically nonexistent in the atmosphere. On the other hand, atmospheric motions are sometimes strongly dependent on the moisture content of the air (clouds, precipitation), a characteristic without oceanic counterpart.

Flow patterns in the atmosphere and oceans are generated by vastly different mechanisms. By and large, the atmosphere is thermodynamically driven, that is, its primary source of energy is the solar radiation. Briefly, this shortwave solar radiation traverses the air layer to be partially absorbed by the continents and oceans, which in turn reemit a radiation at longer wavelengths. This secondhand radiation effectively heats the atmosphere from below, and the resulting convection drives the winds.

In contrast, the oceans are forced by a variety of mechanisms. In addition to the periodic gravitational forces of the moon and sun that generate the tides, the ocean surface is subjected to a wind stress that drives most ocean currents. Finally, local differences between air and sea temperatures generate heat fluxes, evaporation, and precipitation, which in turn act as thermodynamical forcings capable of modifying the wind-driven currents or of producing additional currents.

In passing, while we are contrasting the atmosphere with the oceans, it is appropriate to mention an enduring difference in terminology. Because meteorologists and laypeople alike are generally interested in knowing from where the winds are blowing, it is common in meteorology to refer to air velocities by their direction or origin, such as easterly (from the east—that is, toward the west). On the contrary, sailors and navigators are interested in knowing where ocean currents may take them. Hence, oceanographers designate currents by their downstream direction, such as westward (from the east or to the west). Meteorologists and oceanographers agree, however, on the terminology for vertical motions: upward or downward.