COSTRUZIONI NAVALI II

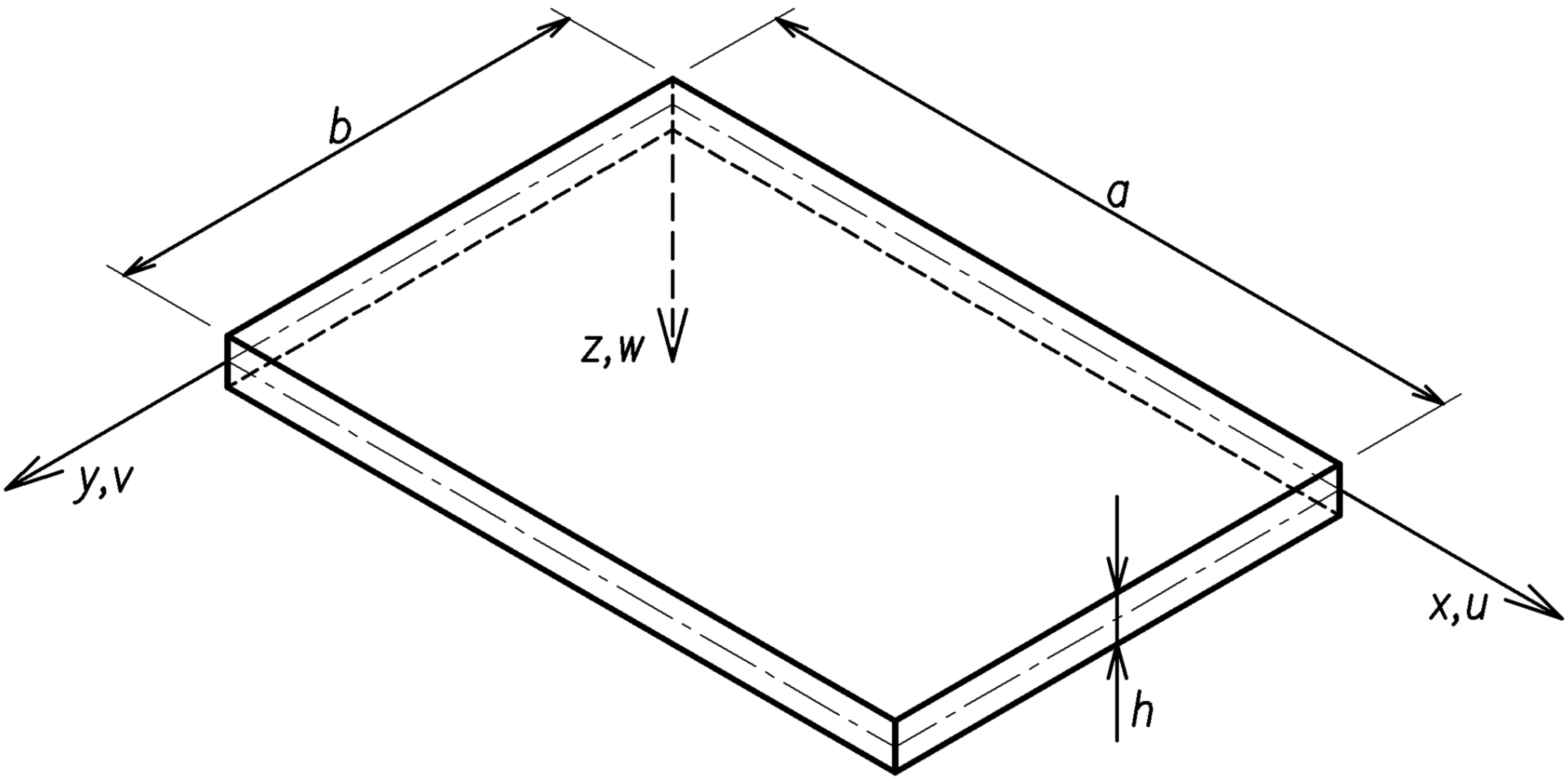
5 (teoria delle piastre)

Joseph Louis Lagrange
(1736-1813)

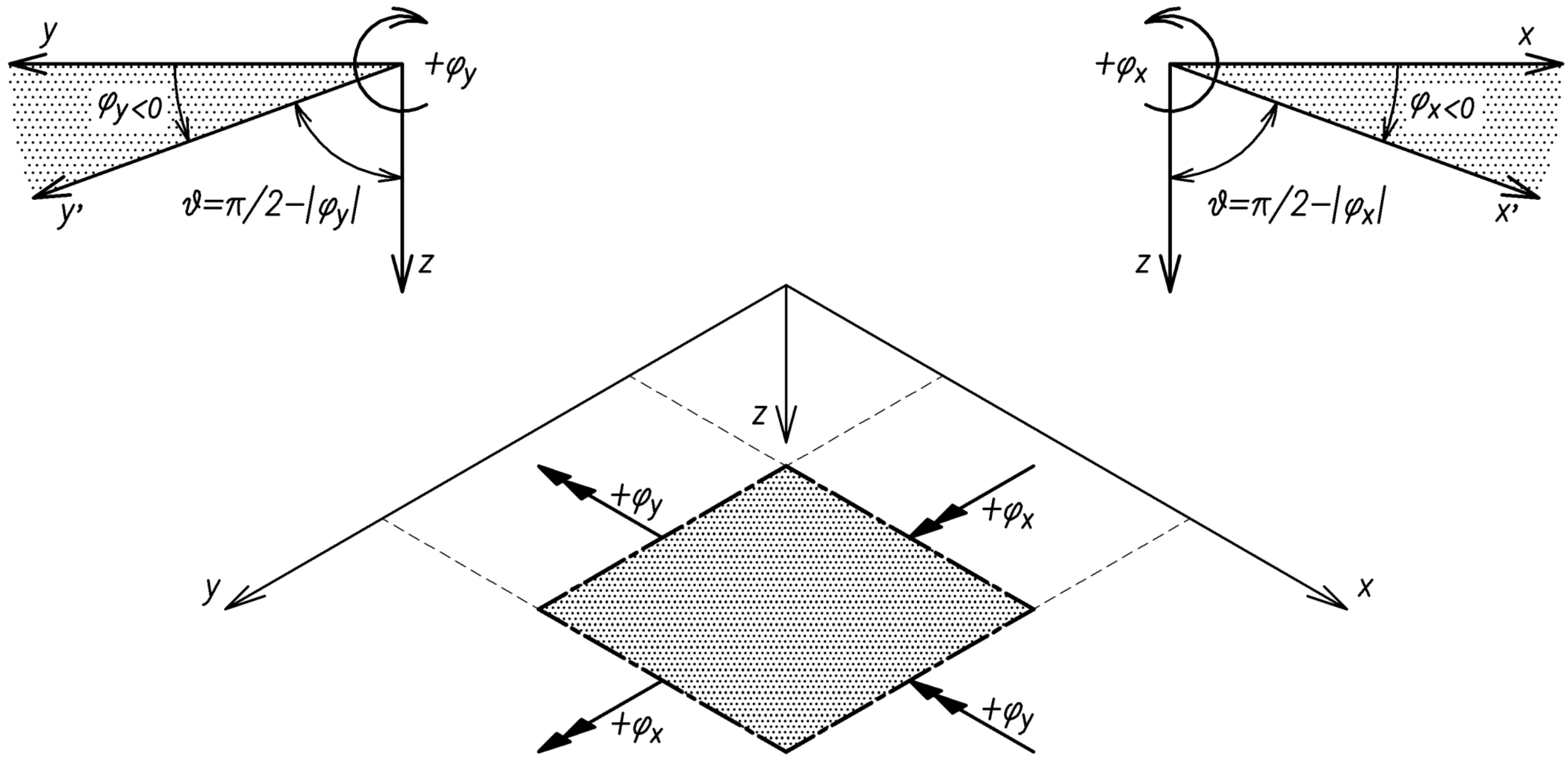
Marie Sophie Germain
(1776-1831)

Gustav Robert Kirchhoff
(1824-1887)

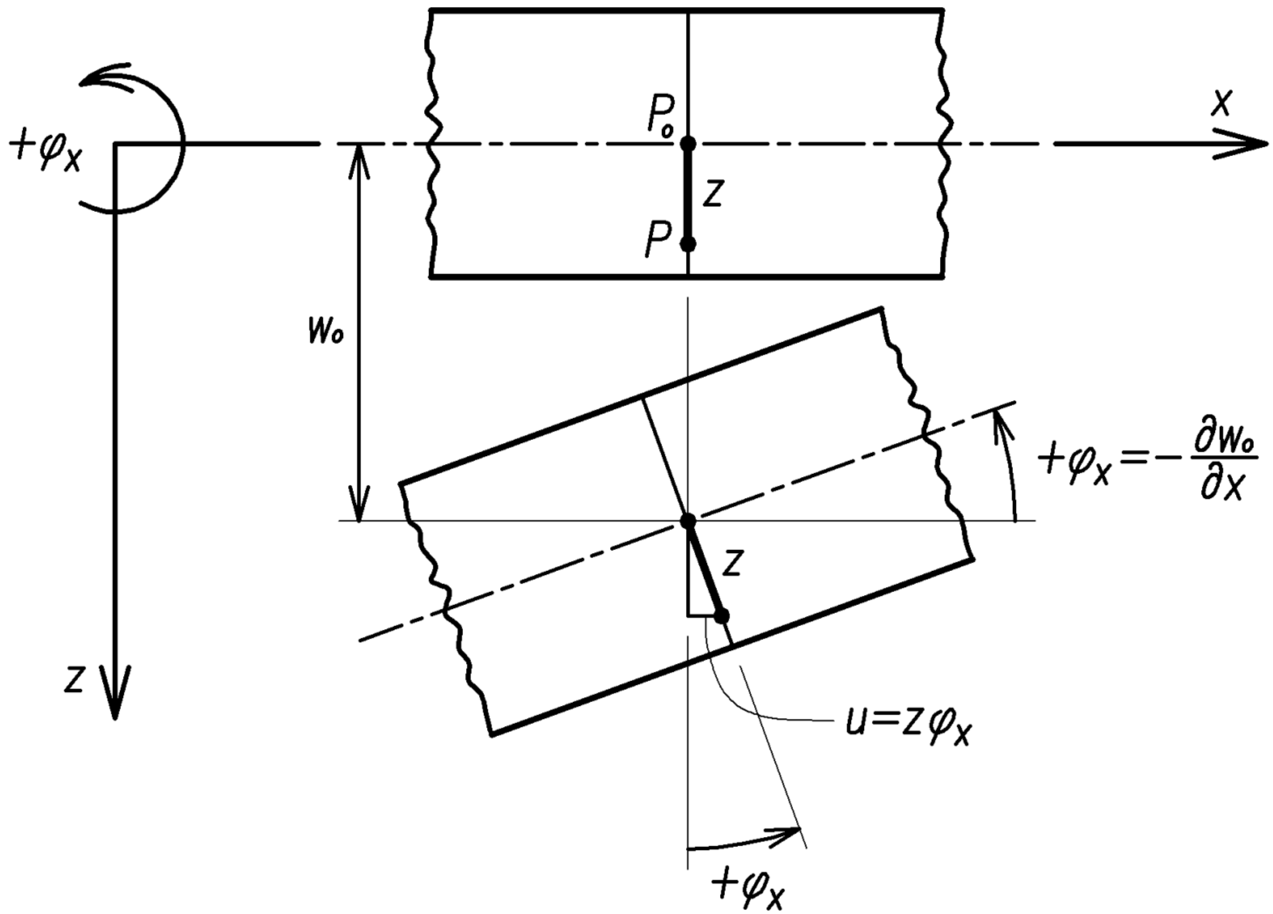
Il riferimento geometrico per piastre rettangolari



Convenzioni di segno per le rotazioni nei piani coordinati trasversali



Componenti di spostamento nel piano xz



Il campo degli spostamenti

$$u = z\,\varphi_x = -z\,\frac{\partial w_o}{\partial x}$$

$$v = z\,\varphi_y = -z\,\frac{\partial w_o}{\partial y}$$

$$w = w_o$$

Il campo delle deformazioni

$$\chi_x = -\frac{\partial^2 w}{\partial x^2}$$

$$\chi_y = -\frac{\partial^2 w}{\partial y^2}$$

$$\chi_{xy} = -\frac{\partial^2 w}{\partial x \partial y}$$

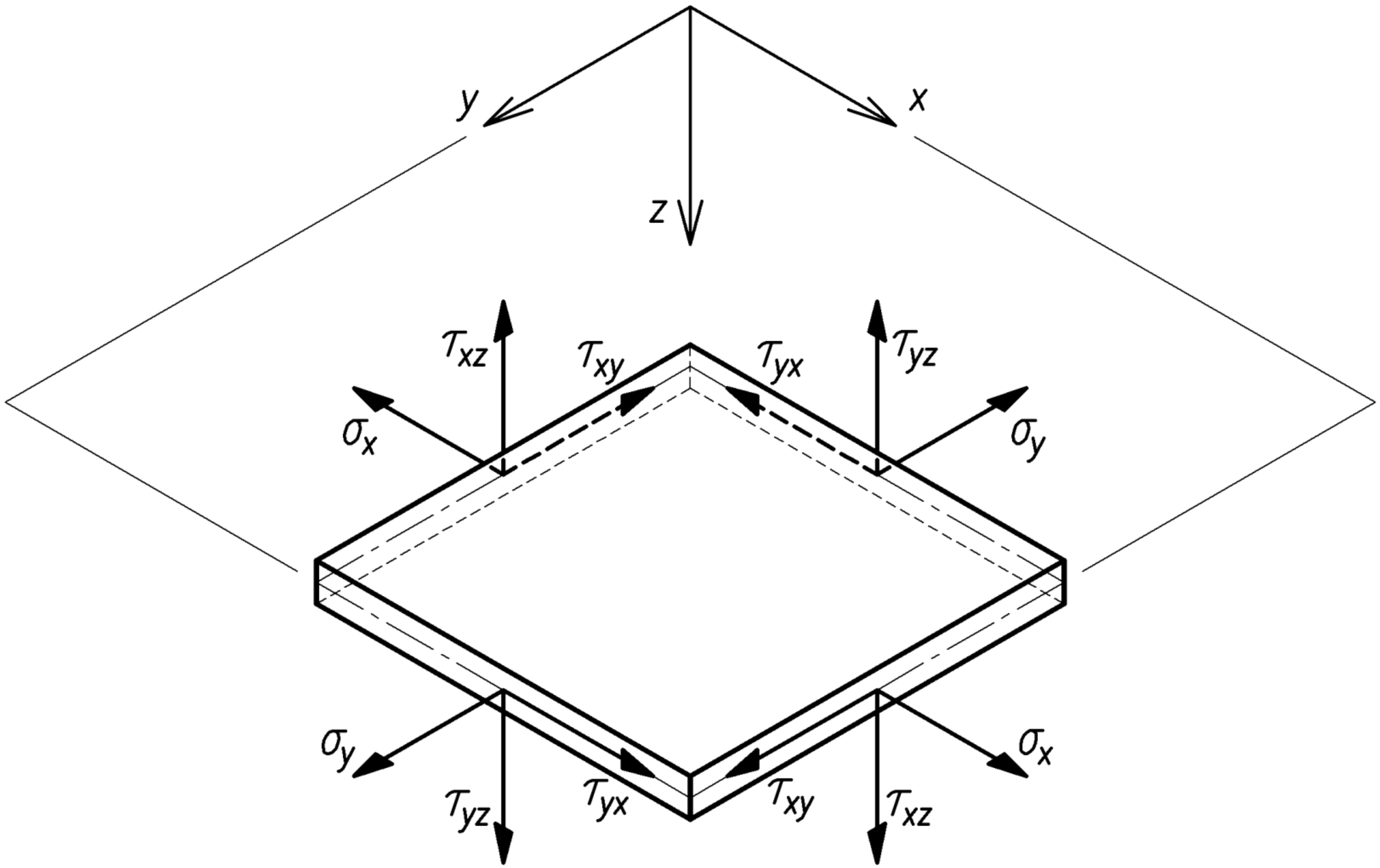
$$\varepsilon_x = z\,\chi_x = -z\,\frac{\partial^2 w}{\partial x^2}$$

$$\varepsilon_y = z\,\chi_y = -z\,\frac{\partial^2 w}{\partial y^2}$$

$$\gamma_{xy} = 2\,z\,\chi_{xy} = -2\,z\,\frac{\partial^2 w}{\partial x \partial y}$$

$$\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0$$

Convenzioni di segno per le tensioni



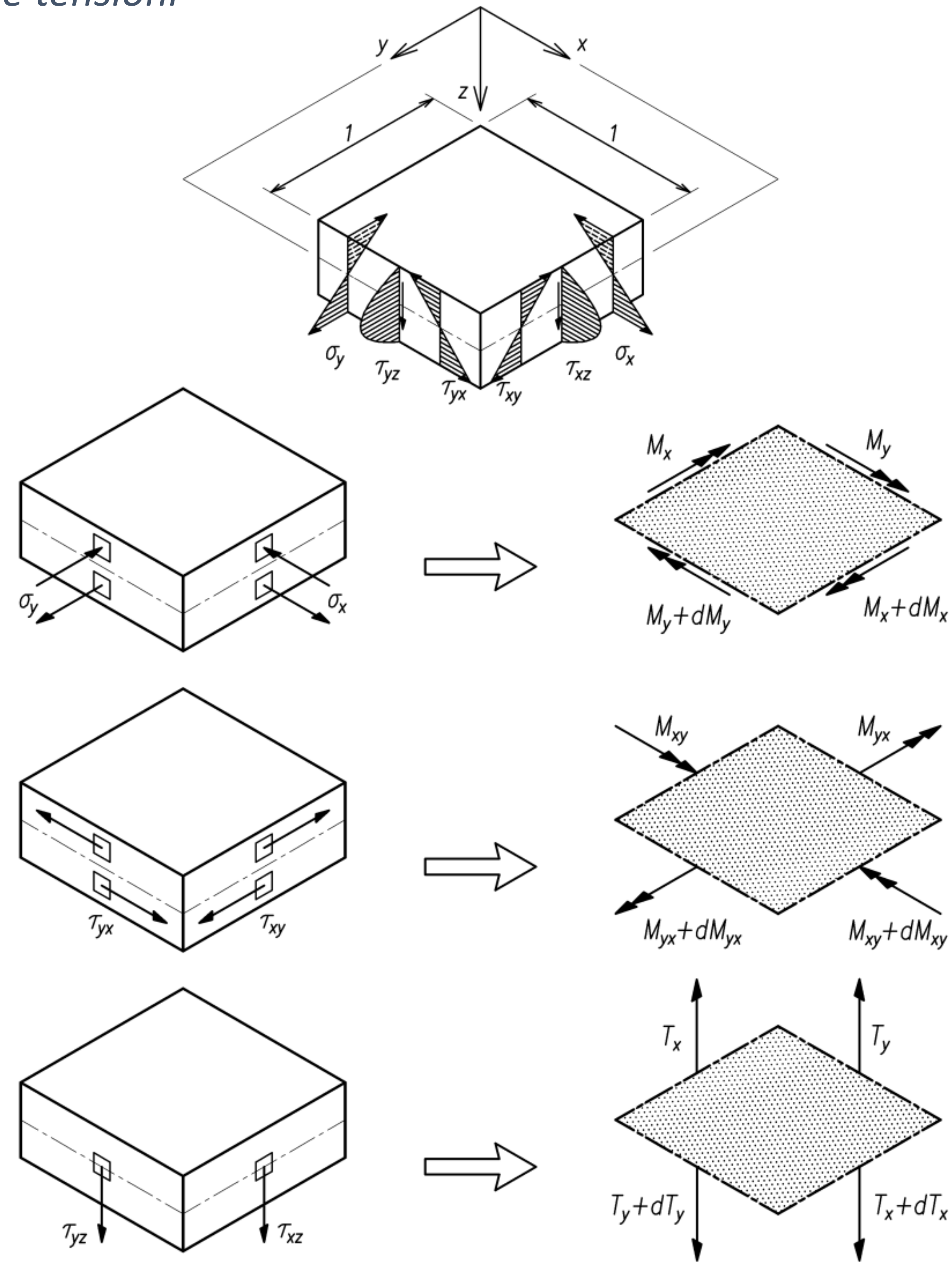
Il campo delle tensioni

$$\sigma_x = \frac{E}{1-\nu^2} z \left(\chi_x + \nu \chi_y\right) = -\frac{E}{1-\nu^2} z \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right)$$

$$\sigma_y = \frac{E}{1-\nu^2} z \left(\chi_y + \nu \chi_x\right) = -\frac{E}{1-\nu^2} z \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right)$$

$$\tau_{xy} = 2 G z \chi_{xy} = \frac{E}{1+\nu} z \chi_{xy} = -\frac{E}{1+\nu} z \frac{\partial^2 w}{\partial x \partial y}$$

Convenzioni di segno per le risultanti delle tensioni



Caratteristiche unitarie di sollecitazione (risultanti delle tensioni)

$$M_x = \int_{-h/2}^{h/2} \sigma_x \, z \, \mathrm{d}z$$

$$M_y = \int_{-h/2}^{h/2} \sigma_y \, z \, \mathrm{d}z$$

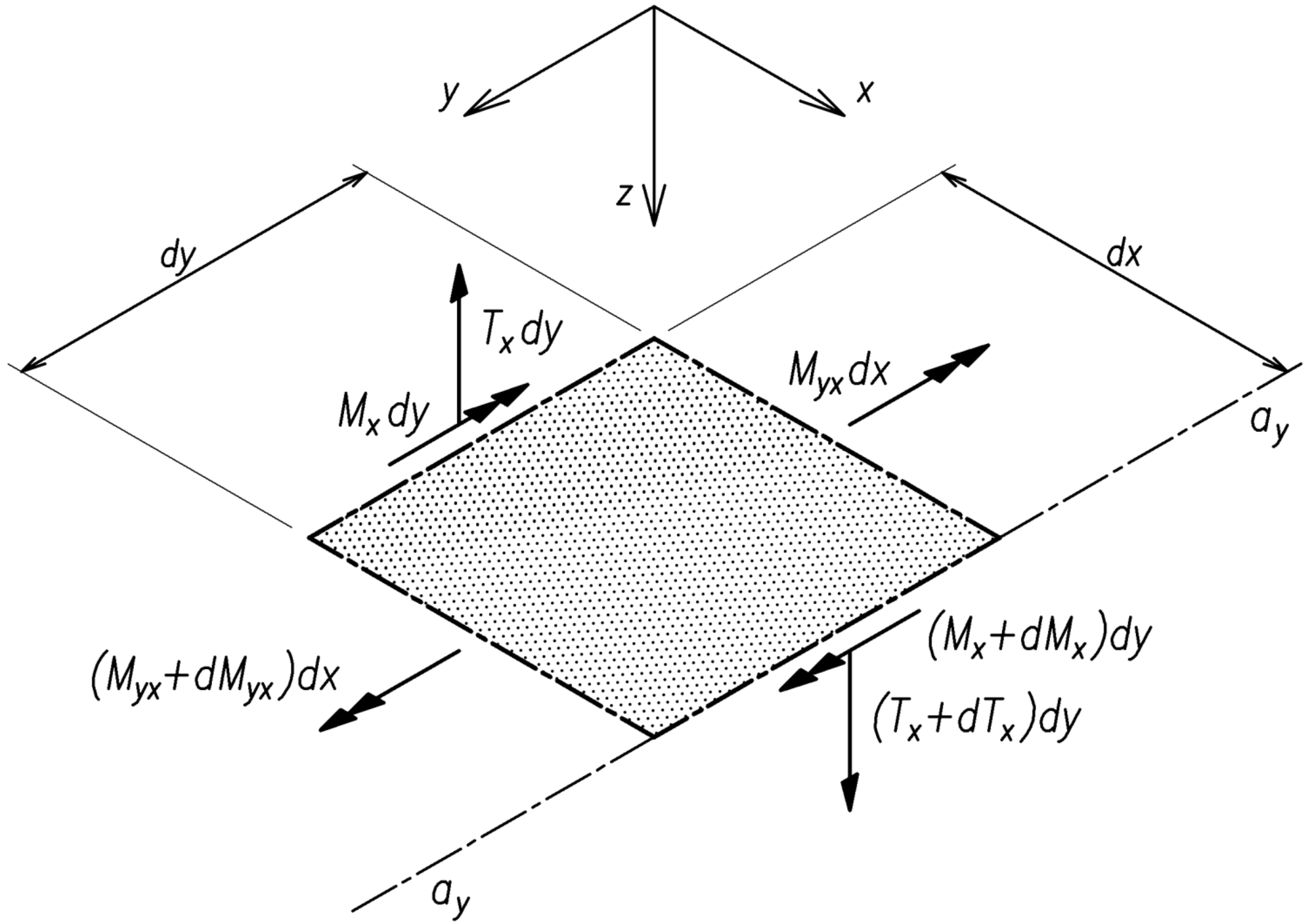
$$M_{xy} = \int_{-h/2}^{h/2} \tau_{xy} \, z \, \mathrm{d}z$$

$$M_{xy} = M_{yx}$$

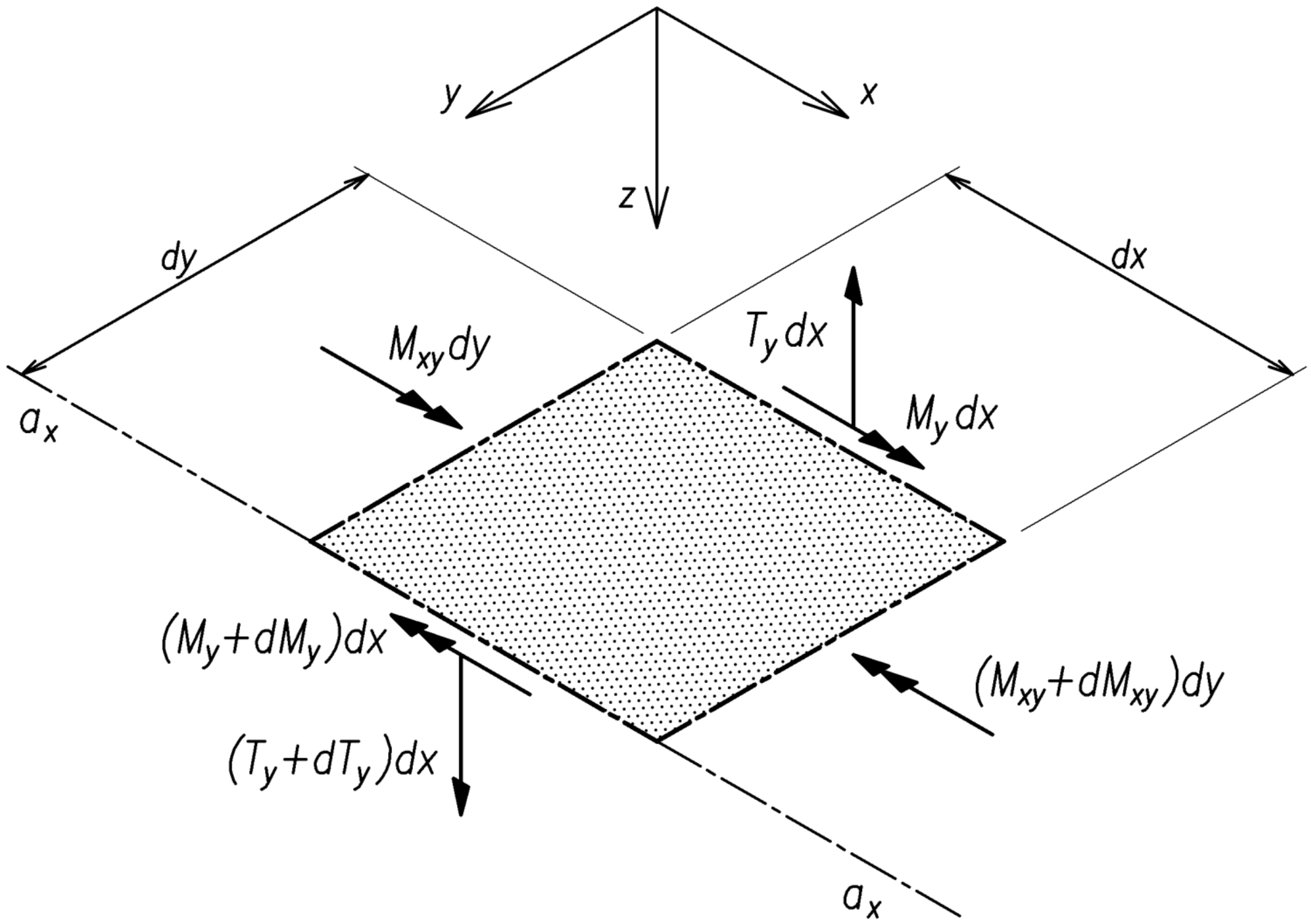
$$T_x = \int_{-h/2}^{h/2} \tau_{xz} \, \mathrm{d}z$$

$$T_y = \int_{-h/2}^{h/2} \tau_{yz} \, \mathrm{d}z$$

Equilibrio alla rotazione dell'elemento di controllo attorno all'asse a_y//y



Equilibrio alla rotazione dell'elemento di controllo attorno all'asse a_x//x



Caratteristiche unitarie di sollecitazione: momenti flettenti e torcente Mx , My e Mxy

$$M_x = D\,(\chi_x + \nu\,\chi_y) = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\,\frac{\partial^2 w}{\partial y^2}\right)$$

$$M_y = D\,(\chi_y + \nu\,\chi_x) = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\,\frac{\partial^2 w}{\partial x^2}\right)$$

$$M_{xy} = (1-\nu)\,D\,\chi_{xy} = -(1-\nu)\,D\,\frac{\partial^2 w}{\partial x \partial y}$$

Caratteristiche unitarie di sollecitazione: taglio trasversale Tx e Ty

$$T_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y}$$

$$T_x = D\,\frac{\partial}{\partial x}\left(\chi_x + \nu\,\chi_y\right) + D\,\frac{\partial}{\partial y}\left[(1-\nu)\,\chi_{yx}\right]$$

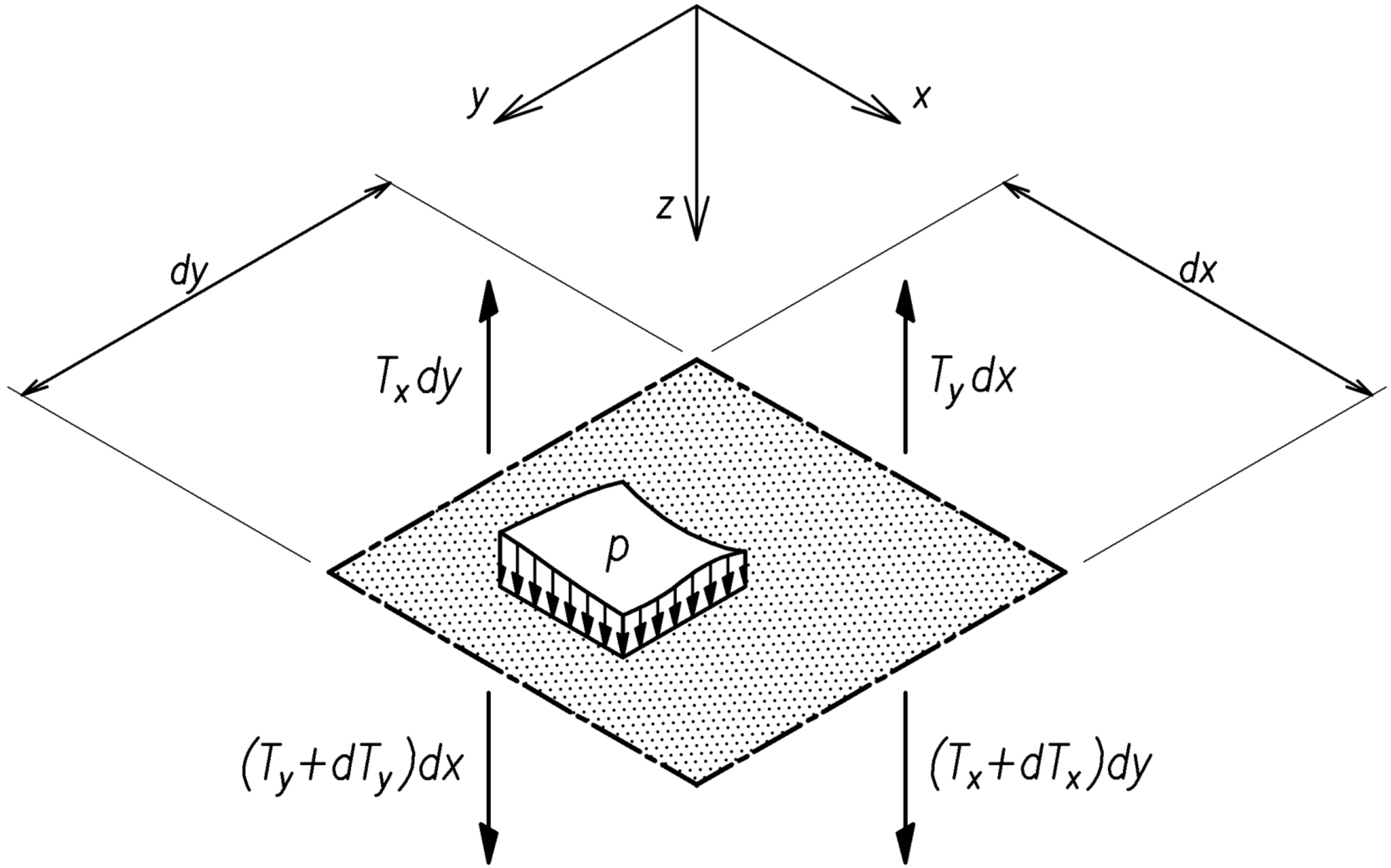
$$T_x = -D\,\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = -D\,\frac{\partial}{\partial x}\left(\nabla^2 w\right)$$

$$T_y = \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x}$$

$$T_y = D\,\frac{\partial}{\partial y}\left(\chi_y + \nu\,\chi_x\right) + D\,\frac{\partial}{\partial x}\left[(1-\nu)\,\chi_{xy}\right]$$

$$T_y = -D\,\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = -D\,\frac{\partial}{\partial y}\left(\nabla^2 w\right)$$

Equilibrio alla traslazione dell'elemento di controllo secondo z



Equazione indefinita di equilibrio in funzione delle caratteristiche di momento

$$T_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y}$$

$$T_y = \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x}$$

$$\frac{\partial T_x}{\partial x} + \frac{\partial T_y}{\partial y} = -p$$

$$\frac{\partial^2 M_x}{\partial x^2} + 2\,\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -p$$

Equazione indefinita di equilibrio in funzione delle curvature

$$\frac{\partial^2 M_x}{\partial x^2} + 2\,\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -p$$

$$\begin{aligned} M_x &= D\,(\chi_x + \nu\,\chi_y) \\ M_y &= D\,(\chi_y + \nu\,\chi_x) \\ M_{xy} &= (1-\nu)\,D\,\chi_{xy} \end{aligned}$$

$$\frac{\partial^2 \chi_y}{\partial x^2} = \frac{\partial^2 \chi_x}{\partial y^2} = \frac{\partial^2 \chi_{xy}}{\partial x \partial y}$$

$$\frac{\partial^2 \chi_x}{\partial x^2} + 2\,\frac{\partial^2 \chi_{xy}}{\partial x \partial y} + \frac{\partial^2 \chi_y}{\partial y^2} = -\frac{p}{D}$$

Equazione indefinita di equilibrio in funzione della superficie elastica: equazione di Lagrange

$$\frac{\partial^2 \chi_x}{\partial x^2} + 2\,\frac{\partial^2 \chi_{xy}}{\partial x \partial y} + \frac{\partial^2 \chi_y}{\partial y^2} = -\frac{p}{D}$$

$$\chi_x = -\frac{\partial^2 w}{\partial x^2}$$

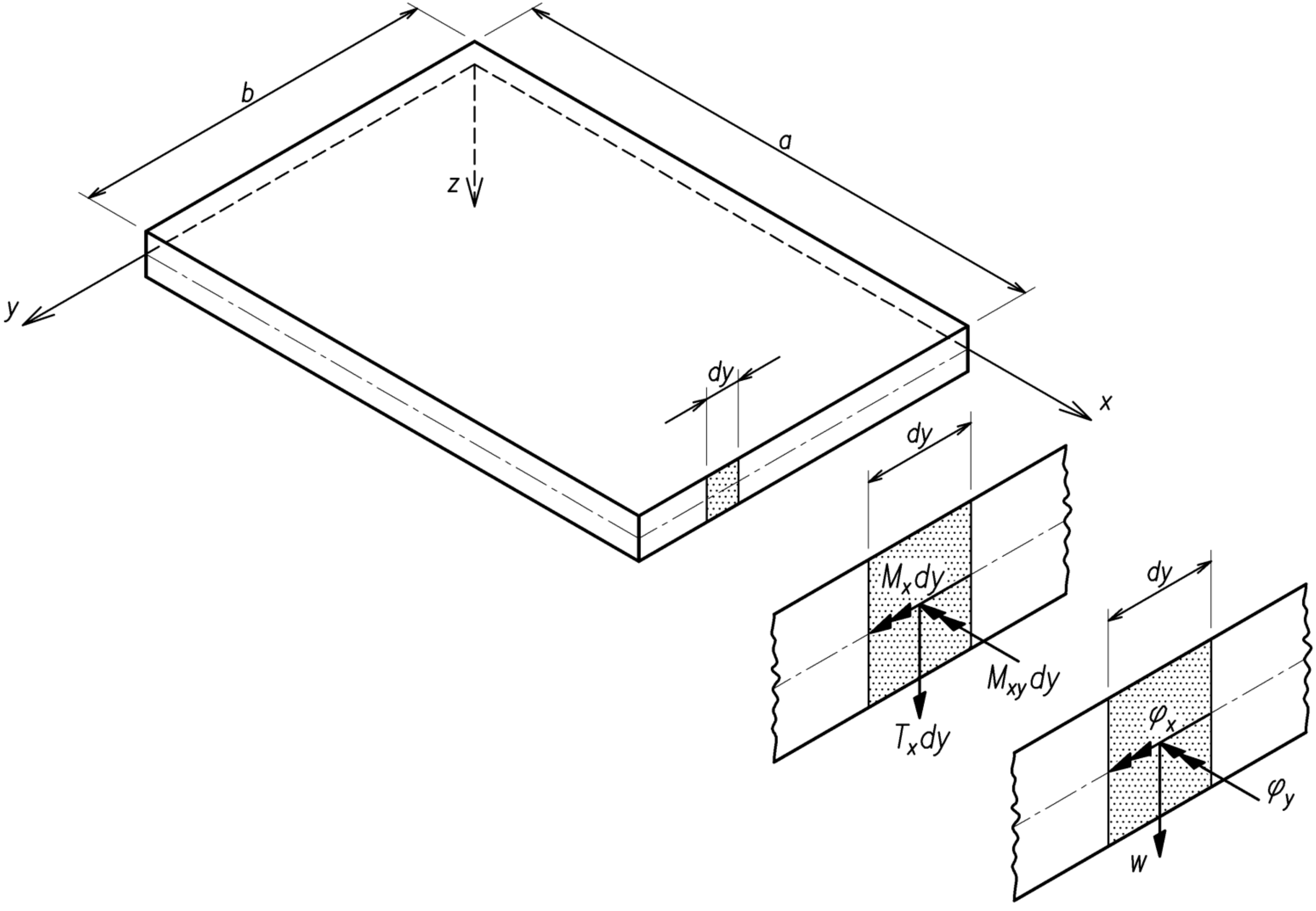
$$\chi_y = -\frac{\partial^2 w}{\partial y^2}$$

$$\chi_{xy} = -\frac{\partial^2 w}{\partial x \partial y}$$

$$\frac{\partial^4 w}{\partial x^4} + 2\,\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D}$$

$$\nabla^4 w = \frac{p}{D}$$

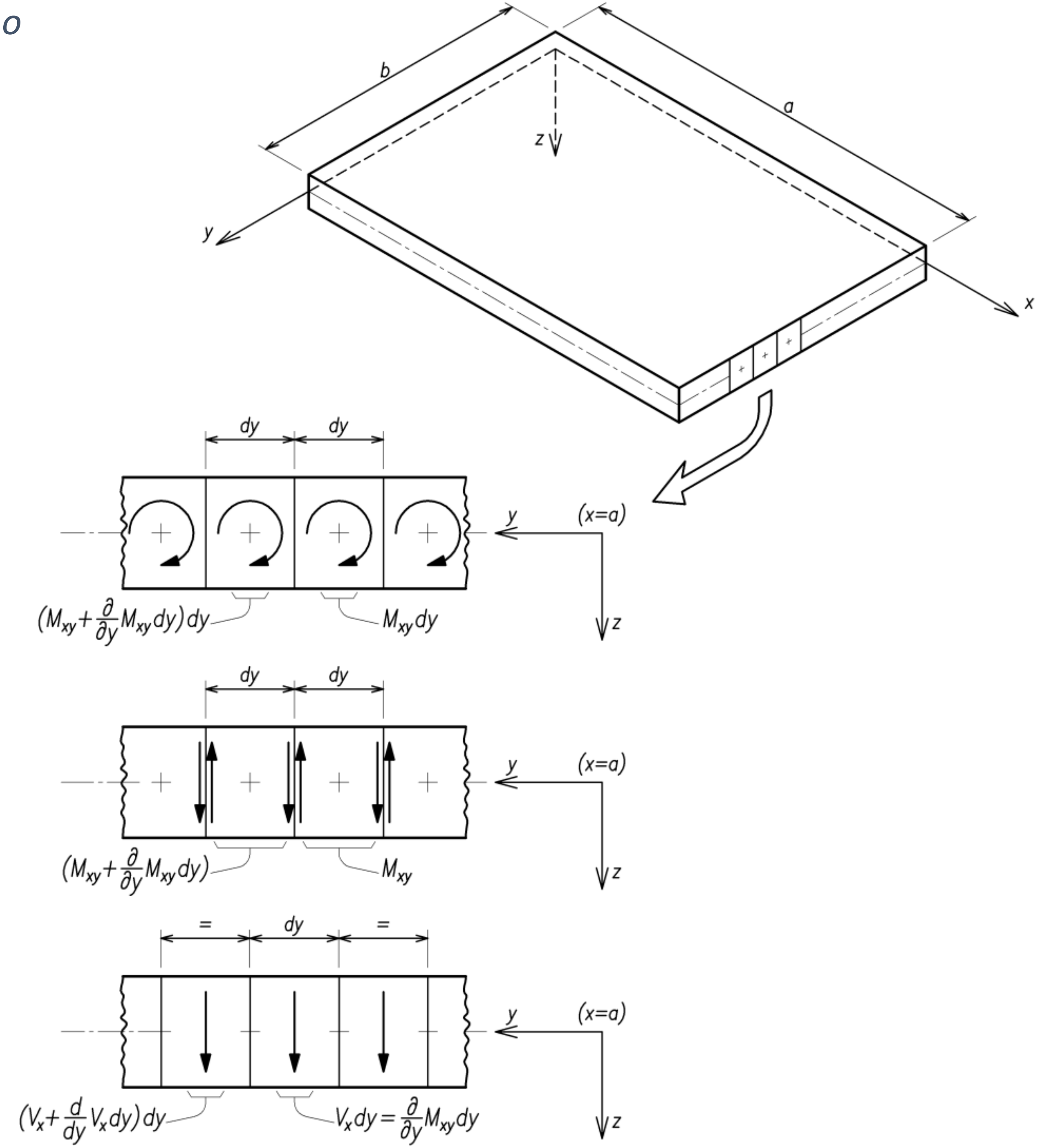
Grandezze statiche e cinematiche sul contorno con x=a



Forze di sostituzione sul contorno

$$V_x = \frac{\partial M_{xy}}{\partial y}$$

$$V_y = \frac{\partial M_{yx}}{\partial x}$$



Forze di taglio effettive sul contorno

$$\overline{T}_x = T_x + V_x = T_x + \frac{\partial M_{xy}}{\partial y} \qquad \text{(sul contorno di normale } x\text{)}$$

$$\overline{T}_y = T_y + V_y = T_y + \frac{\partial M_{yx}}{\partial x} \qquad \text{(sul contorno di normale } y\text{)}$$

$$\overline{T}_x = T_x + \frac{\partial M_{xy}}{\partial y} = -D\,\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + \frac{\partial}{\partial y}\left[-(1-\nu)\,D\,\frac{\partial^2 w}{\partial x \partial y}\right] =$$

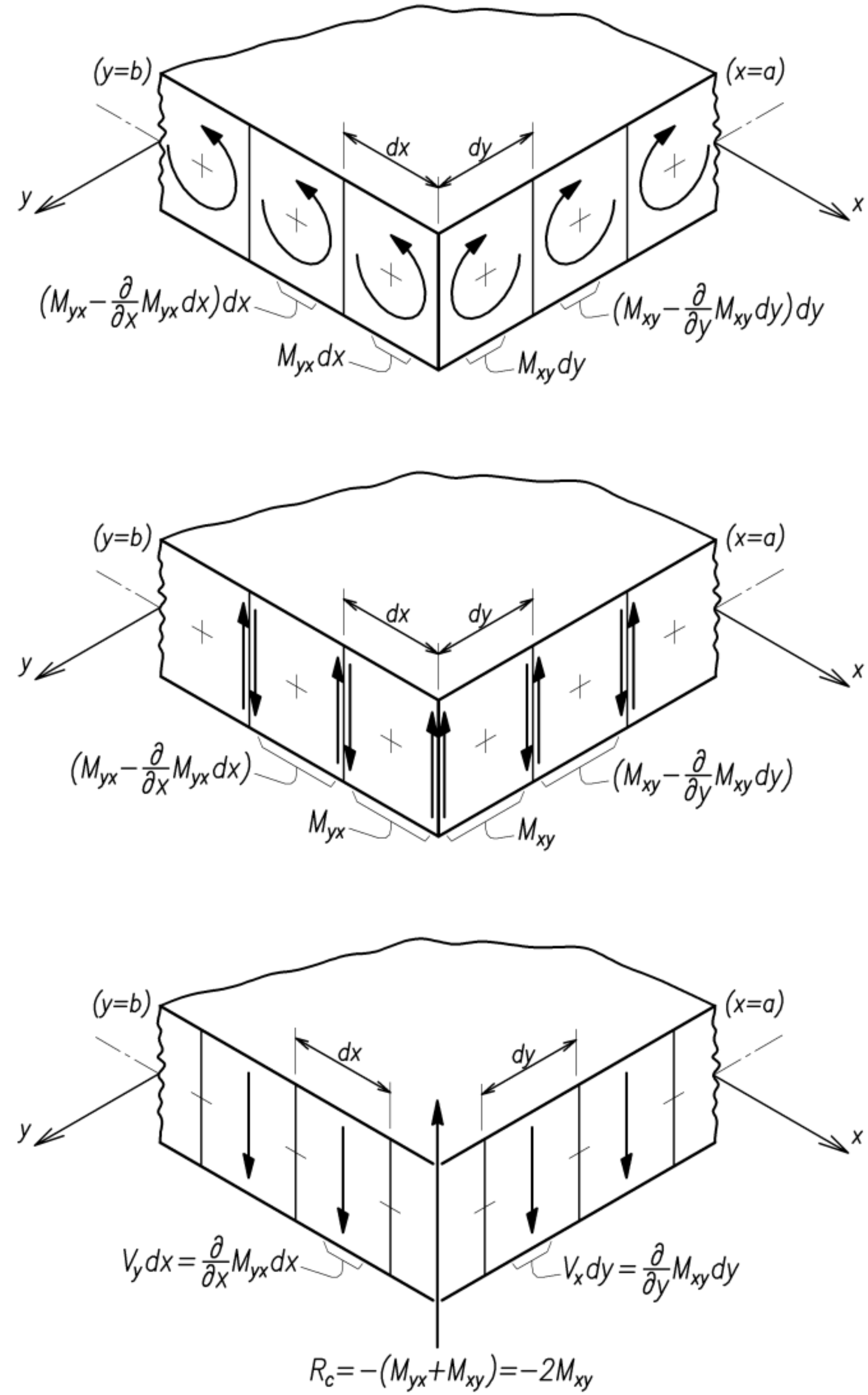
$$= -D\left[\frac{\partial^3 w}{\partial x^3} + (2-\nu)\,\frac{\partial^3 w}{\partial x \partial y^2}\right]$$

$$\overline{T}_y = T_y + \frac{\partial M_{yx}}{\partial x} = -D\,\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + \frac{\partial}{\partial x}\left[-(1-\nu)\,D\,\frac{\partial^2 w}{\partial x \partial y}\right] =$$

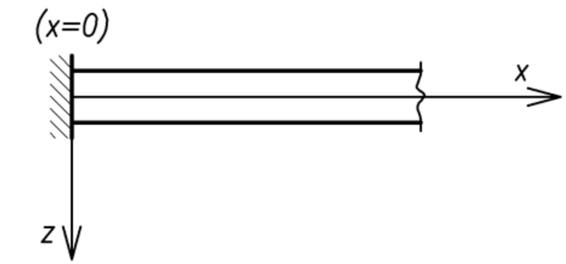
$$= -D\left[\frac{\partial^3 w}{\partial y^3} + (2-\nu)\,\frac{\partial^3 w}{\partial y \partial x^2}\right]$$

Forza al vertice (a, b) del contorno

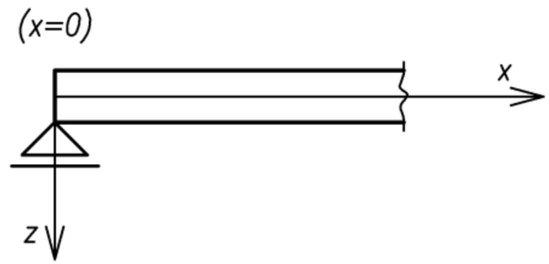
$$R_C = -2\,M_{xy} = 2\,(1-\nu)\,D\,\frac{\partial^2 w}{\partial x \partial y}$$



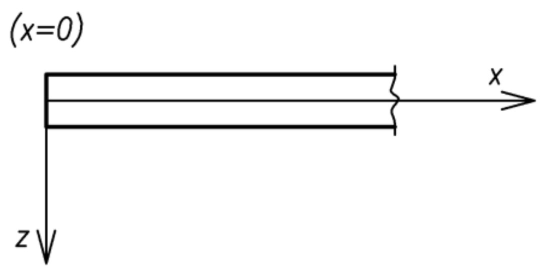
Condizioni al contorno di casi fondamentali



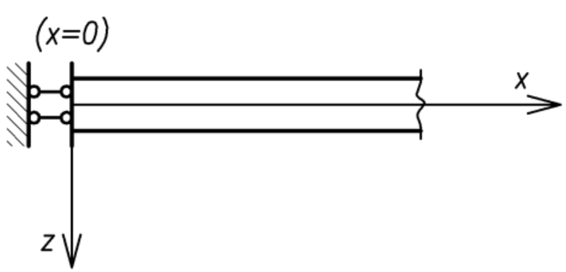
$$w = 0$$
$$\frac{\partial w}{\partial x} = 0$$



$$w = 0$$
$$\frac{\partial^2 w}{\partial x^2} = 0$$

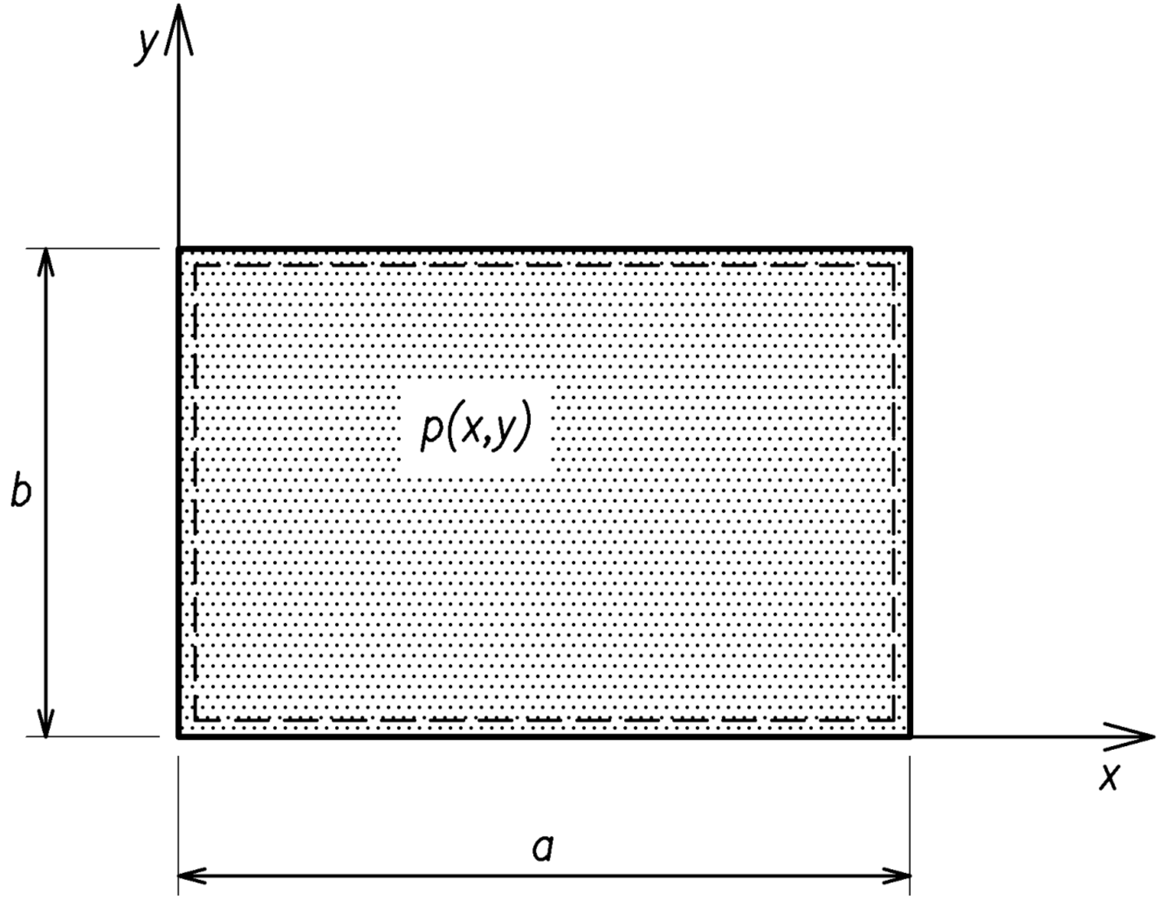


$$\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} = 0$$
$$\frac{\partial^3 w}{\partial x^3} + (2-\nu) \frac{\partial^3 w}{\partial x \partial y^2} = 0$$

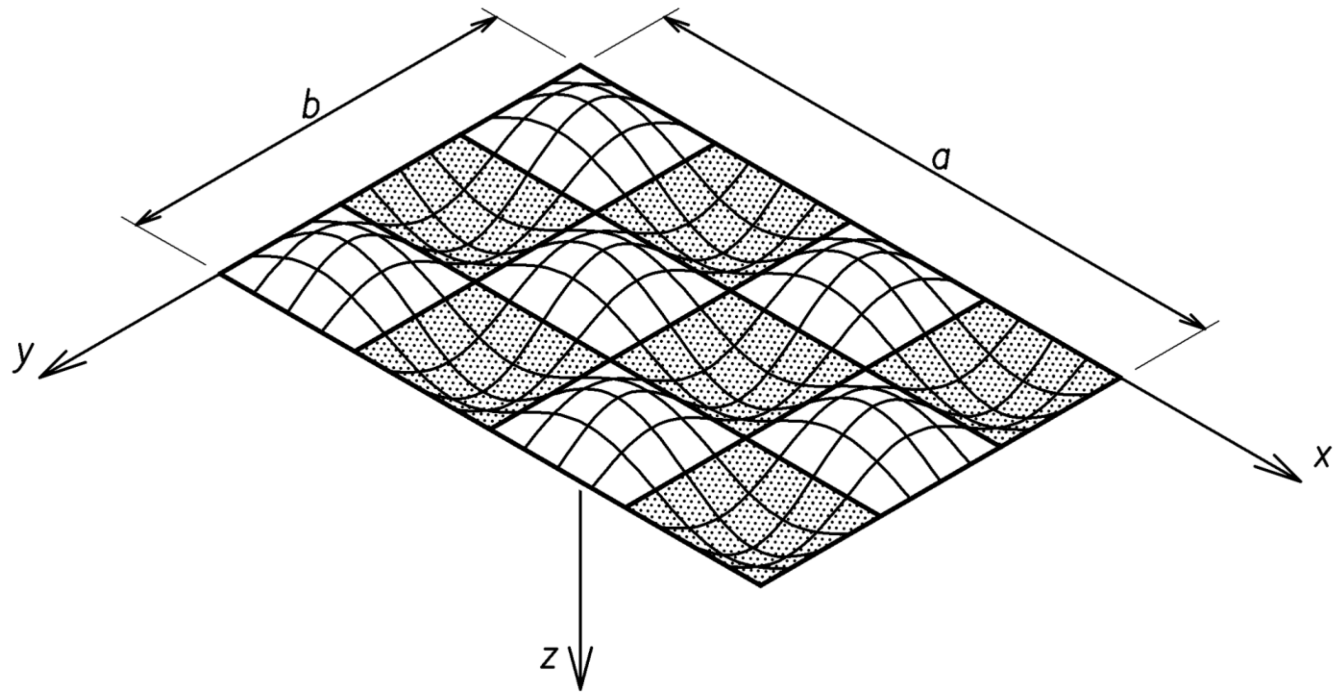


$$\frac{\partial^3 w}{\partial x^3} + (2-\nu) \frac{\partial^3 w}{\partial x \partial y^2} = 0$$
$$\frac{\partial w}{\partial x} = 0$$

Piastra inflessa appoggiata al contorno soggetta a carico distribuito p(x, y)



Carico sinusoidale e superficie elastica affine per m=4 e n=3



$$p(x,y) = p_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$w(x,y) = w_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

Carico sinusoidale e superficie elastica affine per m=4 e n=3

$$w = \quad w_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$\frac{\partial w}{\partial x} = \quad w_{mn} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \left(\frac{m\pi}{a}\right)$$

$$\frac{\partial^2 w}{\partial x^2} = -w_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \left(\frac{m\pi}{a}\right)^2$$

$$\frac{\partial^3 w}{\partial x^3} = -w_{mn} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \left(\frac{m\pi}{a}\right)^3$$

$$\frac{\partial^4 w}{\partial x^4} = \quad w_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \left(\frac{m\pi}{a}\right)^4$$

$$\frac{\partial w}{\partial y} = \quad w_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \left(\frac{n\pi}{b}\right)$$

$$\frac{\partial^2 w}{\partial y^2} = -w_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \left(\frac{n\pi}{b}\right)^2$$

$$\frac{\partial^3 w}{\partial y^3} = -w_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \left(\frac{n\pi}{b}\right)^3$$

$$\frac{\partial^4 w}{\partial y^4} = \quad w_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \left(\frac{n\pi}{b}\right)^4$$

$$\frac{\partial^3 w}{\partial x^2 \partial y} = -w_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \left(\frac{m\pi}{a}\right)^2 \left(\frac{n\pi}{b}\right)$$

$$\frac{\partial^4 w}{\partial x^2 \partial y^2} = \quad w_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \left(\frac{m\pi}{a}\right)^2 \left(\frac{n\pi}{b}\right)^2$$

$$\frac{\partial^4 w}{\partial x^4} + 2\,\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D}$$

$$w_{mn}\left[\left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4\right] = \frac{p_{mn}}{D}$$

$$\boxed{w_{mn} = \frac{p_{mn}}{\pi^4 D\left[(m/a)^2 + (n/b)^2\right]^2}}$$

Legge p=p(x,y) sviluppata in serie doppia di Fourier

$$p(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} p_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$p_{mn} = \frac{4}{a\,b} \int_0^b \int_0^a p(x,y)\, \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \mathrm{d}x\, \mathrm{d}y$$

$$w(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$w_{mn} = \frac{p_{mn}}{\pi^4 D \left[(m/a)^2 + (n/b)^2\right]^2}$$

$$w(x,y) = \frac{1}{\pi^4 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{p_{mn}}{\left[(m/a)^2 + (n/b)^2\right]^2} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

Caratteristiche unitarie di sollecitazione in funzione di w(x,y)

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\,\frac{\partial^2 w}{\partial y^2}\right)$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\,\frac{\partial^2 w}{\partial x^2}\right)$$

$$M_{xy} = -D\,(1-\nu)\,\frac{\partial^2 w}{\partial x \partial y}$$

$$T_x = -D\,\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

$$T_y = -D\,\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

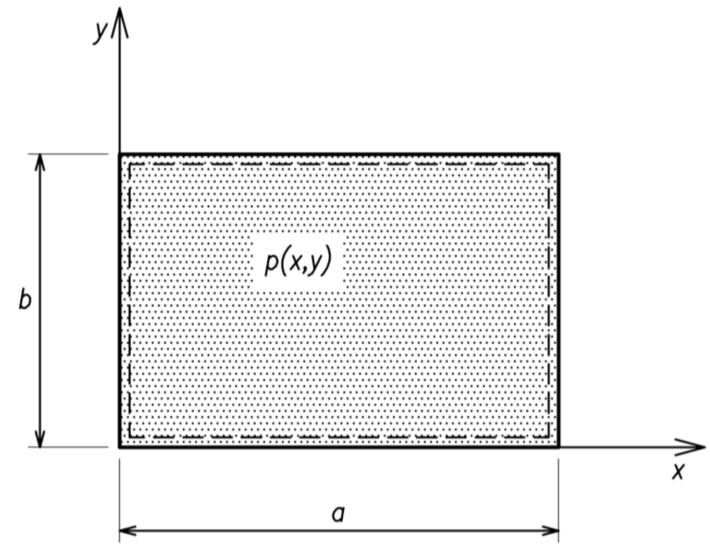
$$\overline{T}_x = -D\left[\frac{\partial^3 w}{\partial x^3} + (2-\nu)\,\frac{\partial^3 w}{\partial x \partial y^2}\right]$$

$$\overline{T}_y = -D\left[\frac{\partial^3 w}{\partial y^3} + (2-\nu)\,\frac{\partial^3 w}{\partial y \partial x^2}\right]$$

$$R_C = 2\,D\,(1-\nu)\,\frac{\partial^2 w}{\partial x \partial y}$$

Carico uniformemente ripartito su tutta la piastra

$$p(x,y) = p_o$$

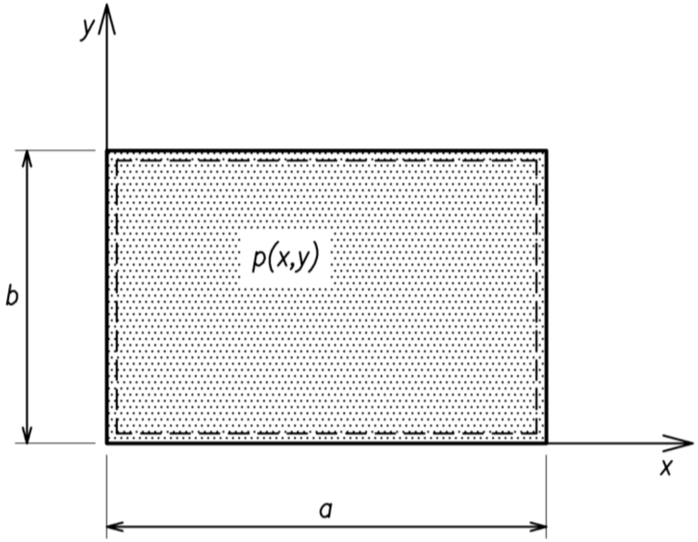


$$p_{mn} = \frac{16\,p_o}{\pi^2 m\,n} \qquad \text{con} \qquad (m,n = 1,3,5,\cdots)$$

$$w(x,y) = \frac{16\,p_o}{\pi^6 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin(m\pi x/a)\,\sin(n\pi y/b)}{m\,n\left[(m/a)^2 + (n/b)^2\right]^2} \qquad \text{con} \qquad (m,n = 1,3,5,\cdots)$$

$$w_{max} = \frac{16\,p_o}{\pi^6 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(-1)^{(m+n-2)/2}}{m\,n\left[(m/a)^2 + (n/b)^2\right]^2} \qquad \text{con} \qquad (m,n = 1,3,5,\cdots)$$

Carico uniformemente ripartito su tutta la piastra



$$\frac{\partial w}{\partial x} = \frac{16\,p_o}{\pi^6 D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\cos(m\pi x/a)\,\sin(n\pi y/b)}{m\,n\left[(m/a)^2+(n/b)^2\right]^2}\left(\frac{m\pi}{a}\right)$$

$$\frac{\partial^2 w}{\partial x^2} = -\frac{16\,p_o}{\pi^6 D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin(m\pi x/a)\,\sin(n\pi y/b)}{m\,n\left[(m/a)^2+(n/b)^2\right]^2}\left(\frac{m\pi}{a}\right)^2$$

$$\frac{\partial w}{\partial y} = \frac{16\,p_o}{\pi^6 D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin(m\pi x/a)\,\cos(n\pi y/b)}{m\,n\left[(m/a)^2+(n/b)^2\right]^2}\left(\frac{n\pi}{b}\right)$$

$$\frac{\partial^2 w}{\partial y^2} = -\frac{16\,p_o}{\pi^6 D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin(m\pi x/a)\,\sin(n\pi y/b)}{m\,n\left[(m/a)^2+(n/b)^2\right]^2}\left(\frac{n\pi}{b}\right)^2$$

$$\frac{\partial^2 w}{\partial x\partial y} = \frac{16\,p_o}{\pi^6 D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\cos(m\pi x/a)\,\cos(n\pi y/b)}{m\,n\left[(m/a)^2+(n/b)^2\right]^2}\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)$$

$$\frac{\partial^3 w}{\partial x^3} = -\frac{16\,p_o}{\pi^6 D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\cos(m\pi x/a)\,\sin(n\pi y/b)}{m\,n\left[(m/a)^2+(n/b)^2\right]^2}\left(\frac{m\pi}{a}\right)^3$$

$$\frac{\partial^3 w}{\partial y^3} = -\frac{16\,p_o}{\pi^6 D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin(m\pi x/a)\,\cos(n\pi y/b)}{m\,n\left[(m/a)^2+(n/b)^2\right]^2}\left(\frac{n\pi}{b}\right)^3$$

$$\frac{\partial^3 w}{\partial x\partial y^2} = -\frac{16\,p_o}{\pi^6 D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\cos(m\pi x/a)\,\sin(n\pi y/b)}{m\,n\left[(m/a)^2+(n/b)^2\right]^2}\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)^2$$

$$\frac{\partial^3 w}{\partial y\partial x^2} = -\frac{16\,p_o}{\pi^6 D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin(m\pi x/a)\,\cos(n\pi y/b)}{m\,n\left[(m/a)^2+(n/b)^2\right]^2}\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)$$

Carico uniformemente ripartito su tutta la piastra

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\,\frac{\partial^2 w}{\partial y^2}\right) =$$

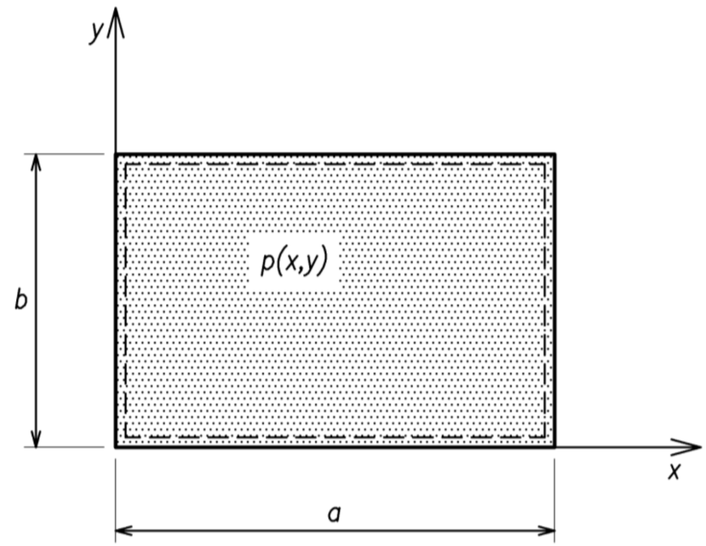
$$= \frac{16\,p_o}{\pi^4}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(m/a)^2 + \nu\,(n/b)^2}{m\,n\left[(m/a)^2 + (n/b)^2\right]^2}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\,\frac{\partial^2 w}{\partial x^2}\right) =$$

$$= \frac{16\,p_o}{\pi^4}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\nu\,(m/a)^2 + (n/b)^2}{m\,n\left[(m/a)^2 + (n/b)^2\right]^2}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$M_{xy} = -D\,(1-\nu)\,\frac{\partial^2 w}{\partial x \partial y} =$$

$$= -\frac{16\,p_o(1-\nu)}{\pi^4 a\,b}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{\left[(m/a)^2 + (n/b)^2\right]^2}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$



$$T_x = -D\left[\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}\right] =$$

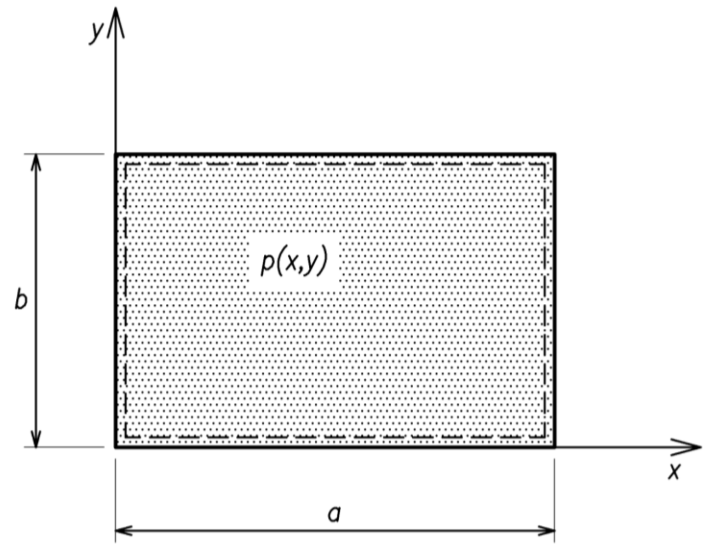
$$= \frac{16\,p_o}{\pi^3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(m/a)}{m\,n\left[(m/a)^2 + (n/b)^2\right]}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$T_y = -D\left[\frac{\partial^3 w}{\partial y \partial x^2} + \frac{\partial^3 w}{\partial y^3}\right] =$$

$$= \frac{16\,p_o}{\pi^3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(n/b)}{m\,n\left[(m/a)^2 + (n/b)^2\right]}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

con $m, n = 1, 3, 5, \cdots$

Carico uniformemente ripartito su tutta la piastra



$$\overline{T}_x = -D\left[\frac{\partial^3 w}{\partial x^3} + (2-\nu)\,\frac{\partial^3 w}{\partial x \partial y^2}\right] =$$

$$= \frac{16\,p_o}{\pi^3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(m/a)\Big[(m/a)^2 + (2-\nu)\,(n/b)^2\Big]}{m\,n\,\Big[(m/a)^2 + (n/b)^2\Big]^2}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$\overline{T}_y = -D\left[\frac{\partial^3 w}{\partial y^3} + (2-\nu)\,\frac{\partial^3 w}{\partial y \partial x^2}\right] =$$

$$= \frac{16\,p_o}{\pi^3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(n/b)\Big[(2-\nu)\,(m/a)^2 + (n/b)^2\Big]}{m\,n\,\Big[(m/a)^2 + (n/b)^2\Big]^2}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

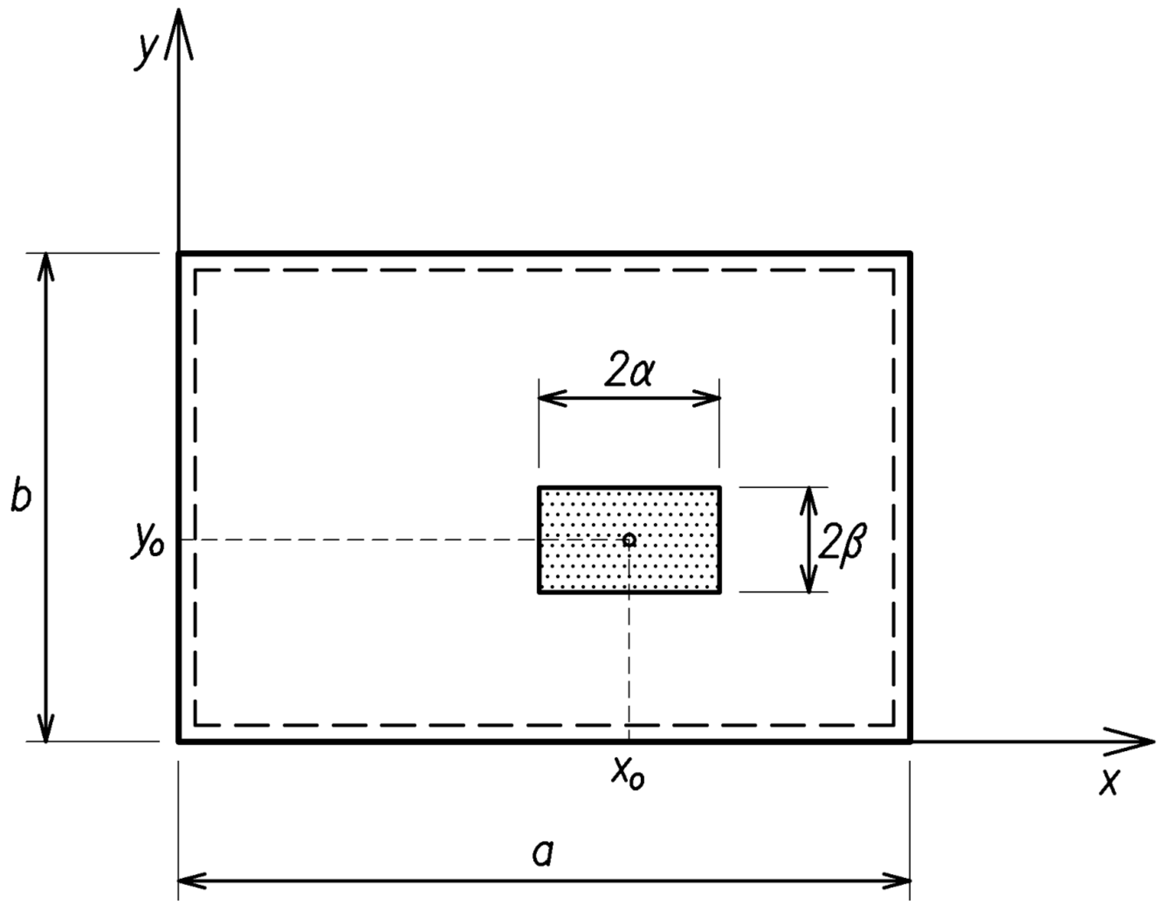
con $m, n = 1, 3, 5, \cdots$

$$R_C = 2\,D\,(1-\nu)\,\frac{\partial^2 w}{\partial x \partial y} =$$

$$= \frac{32\,p_o(1-\nu)}{\pi^4 a\,b}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\cos(m\pi x/a)\cos(n\pi y/b)}{\Big[(m/a)^2 + (n/b)^2\Big]^2}$$

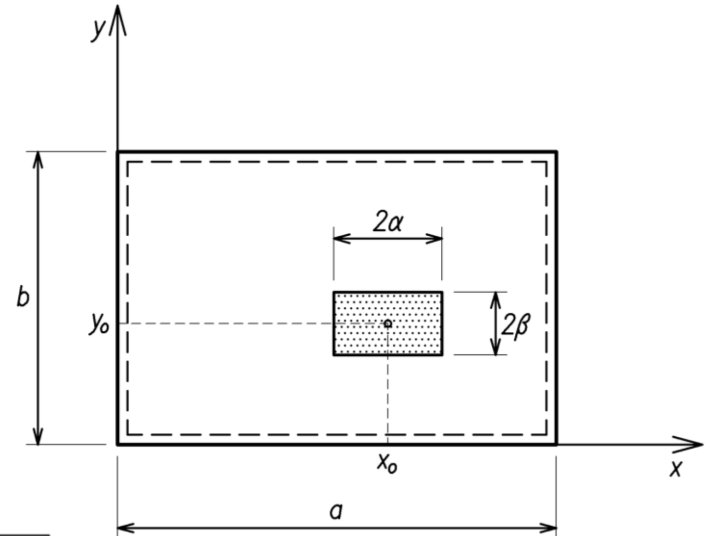
$$R_C = \frac{32\,p_o(1-\nu)}{\pi^4 a\,b}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{\Big[(m/a)^2 + (n/b)^2\Big]^2} \qquad \text{con} \qquad (m, n = 1, 3, 5, \cdots)$$

Piastra inflessa appoggiata al contorno soggetta a carico d'impronta



Carico d'impronta

$$\begin{cases} p(x,y) = \dfrac{P}{4\,\alpha\,\beta} & \text{in} \quad (2\alpha \times 2\beta) \\ p(x,y) = 0 & \text{altrove} \end{cases}$$



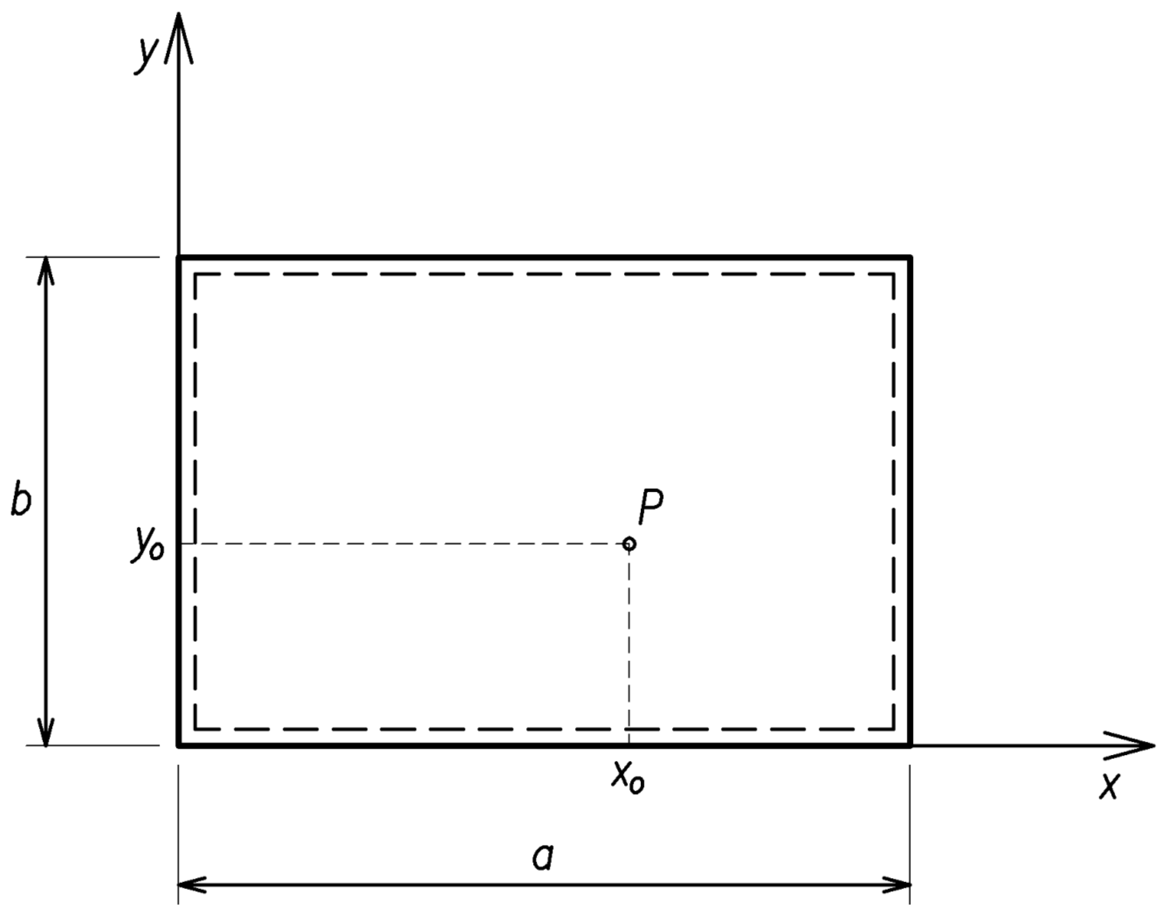
$$p_{mn} = \frac{4}{a\,b}\int_{y_o-\beta}^{y_o+\beta}\int_{x_o-\alpha}^{x_o+\alpha}\frac{P}{4\,\alpha\,\beta}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\mathrm{d}x\,\mathrm{d}y$$

$$p_{mn} = \frac{4\,P}{\alpha\,\beta\,m\,n\,\pi^2}\sin\left(\frac{m\pi x_o}{a}\right)\sin\left(\frac{m\pi\alpha}{a}\right)\sin\left(\frac{n\pi y_o}{b}\right)\sin\left(\frac{n\pi\beta}{b}\right)$$

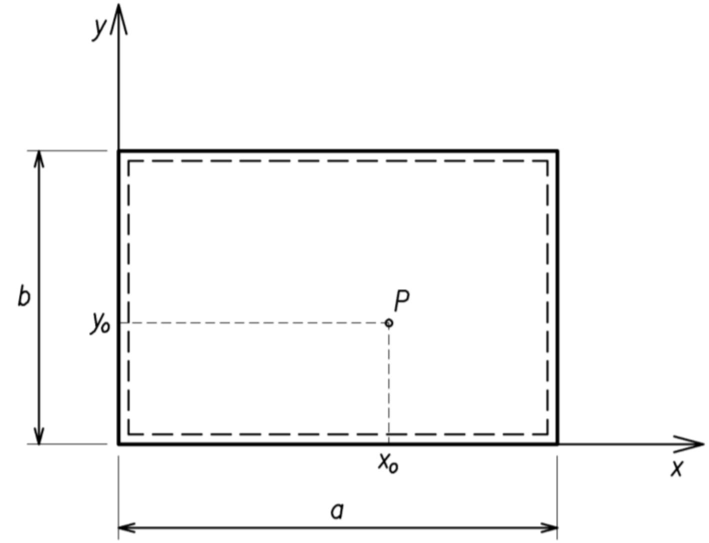
$$w(x,y) = \frac{4\,P}{\pi^6 D\,\alpha\,\beta}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin(m\pi x/a)\sin(n\pi y/b)}{m\,n\left[(m/a)^2+(n/b)^2\right]^2}\cdot$$
$$\cdot\sin\left(\frac{m\pi x_o}{a}\right)\sin\left(\frac{m\pi\alpha}{a}\right)\sin\left(\frac{n\pi y_o}{b}\right)\sin\left(\frac{n\pi\beta}{b}\right)$$

con $m, n = 1, 3, 5, \cdots$

Piastra inflessa appoggiata al contorno soggetta a carico concentrato



Carico concentrato



$$p_{mn} = \lim_{\substack{\alpha \to 0 \\ \beta \to 0}} \frac{4\,P}{\alpha\,\beta\,m\,n\,\pi^2} \sin\left(\frac{m\pi x_o}{a}\right) \sin\left(\frac{m\pi\alpha}{a}\right) \sin\left(\frac{n\pi y_o}{b}\right) \sin\left(\frac{n\pi\beta}{b}\right)$$

$$\boxed{p_{mn} = \frac{4\,P}{a\,b} \sin\left(\frac{m\pi x_o}{a}\right) \sin\left(\frac{n\pi y_o}{b}\right)}$$

$$\boxed{w(x,y) = \frac{4\,P}{\pi^4 D\,a\,b} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin(m\pi x/a)\sin(n\pi y/b)}{\left[(m/a)^2 + (n/b)^2\right]^2} \sin\left(\frac{m\pi x_o}{a}\right) \sin\left(\frac{n\pi y_o}{b}\right)}$$

con $\;m, n = 1, 3, 5, \cdots$

Esercitazione: Calcolo della freccia massima in piastre inflesse

Sia data una piastra rettangolare in acciaio (E = 206 GPa) avente spessore h = 10 mm e lato corto b = 500 mm, appoggiata sui bordi e uniformemente caricata con pressione $p(x,y) = p_o$ = 1.5 bar.

Calcolare la freccia massima w_{max} al variare del rapporto di allungamento nel campo $a/b = 1 \div 10$.

Si risolva il problema mediante l'impiego di serie doppie di Fourier.

Si esegua altresì il confronto con quanto ottenibile tramite un'analisi semplificata basata sulle ipotesi di Grashof.

Esporre le soluzioni ottenute anche in forma grafica.