Esercizi

Calcolare:

- $\iint_E xy\,dx\,dy$, $\quad E=\{(x,y):\ x>0,\ x+2\le \frac{y}{x}\le 4-x\}$
- $\iint_E \frac{1}{x}\,dx\,dy$, $\quad E=\{(x,y):\ x>0,\ 2x\le y\le 3x,\ 1\le 2xy\le 2\}$.
- $\iint_E \frac{y}{x^2+y^2}\,dx\,dy$, $\quad E=\{(x,y):\ x^2+y^2<1,\ y>|x|\}$
- $\iint_E (x+2y)\,dx\,dy$, $\quad E=\{(x,y):\ x^2-2x+y^2<0\}$
- $\iint_E x\,dx\,dy$, $\quad E=\{(x,y):(x-1)^2+y^2\ge 1,\ (x-2)^2+y^2\le 4\}$
- $m_2(E)$, $\quad E=\{(x,y):\ |\log x|\le 1,\ |y-x\log x|\le 1\}$
- $\iint_E \frac{x^2}{y}e^{xy}\,dx\,dy$, $\quad E=\{(x,y):\ x>0,\ \frac{1}{2x}\le y\le \frac{1}{x},\ 2x^2\le y\le 3x^2\}$
- $m_2(E)$, $\quad E=\{(x,y):\ (x^2+y^2)^3\le 4x^2y^2,\ x\ge 0,\ y\ge 0\}$
- $m_2(E)$, $\quad E=\{(x,y):\ x^2+y^2=t^2,\ y=x\cdot \mathrm{tg}\,t,\ \text{con } t\in]0,\frac{\pi}{2}[\}$
- $m_2(E)$, $\quad E=\{(x,y):\ x^2+y^2=(1+\cos t)^2,\ y=x\cdot\mathrm{tg}\,t \text{ con } t\in]-\frac{\pi}{2},\frac{\pi}{2}[\}$
- baricentro di $\quad E=\{(x,y,z):\ x^2+y^2\le z^2-1,\ 0\le z\le 2-x^2-y^2\}$
- $\iiint_E |z|\,dx\,dy\,dz$, $\quad E=\{(x,y,z):\ (x^2+y^2+z^2)^2\le x^2+y^2-z^2\}$
- $m_3(E)$, $\quad E=\{(x,y,z):\ z^4\le (x^2+y^2)^2\le x^2+y^2\}$
- momento d'inerzia rispetto all'asse z di $\quad E=\{(x,y,z):\sqrt{x^2+y^2}\le z\le 2-\sqrt{x^2+y^2}\}$
- $\lim_{t\to 0^+}\iint_{E_t}\frac{x+1}{y}\,dx\,dy$, $\quad E_t=\{(x,y):\ x^2+y^2\ge t^2,\ \sqrt{x}\le y\le 1\}$
- $\lim_{n\to+\infty}\iint_{E_n}\frac{1}{(1+(nx+y)^2+(x+ny)^2)^2}\,dx\,dy$, $\quad E_n=\{(x,y):\ x^2+y^2\ge n^2,\ x\ge 0,\ y\ge 0\}$
- $\iint_J \frac{xy}{(1+x^2+y^2)^{5/2}}\,dx\,dy$, $\quad J=\{(x,y):\ 0<x<y\}$
- $\iiint_J z\,dx\,dy\,dz$, $\quad J=\{(x,y,z):\ x^2+y^2\le e^{-z},\ z\ge 0\}$
- $m_3(J)$, $\quad J=\{(x,y,z):\ \max\{|y|,|z|\}\le e^{-|x|}\}$

Dire se esistono al variare di α:

- $\iint_J \frac{y}{(x^2+y^2)^\alpha}\,dx\,dy$, $\quad J=\{(x,y):\ (x-1)^2+y^2\le 1\ \vee\ x^2+y^2\ge 4,\ y\ge 0\}$
- $\iint_J x^\alpha\,dx\,dy$, $\quad J=\{(x,y):\ \frac{x}{1+x^2}\le y\le \frac{2x}{1+x^2}\}$