Intertemporal choice: discounted utility model

Historical development of Intertemporal choice

- relative value people assign to two or more payoffs at different points in time.
- John Rae (1834) in the "Sociological Theory of Capital“:
  - Effective desire of accumulation
    - Promoting factors: bequest motive
    - Limiting factors: uncertainty of human life

- Eugen von Böhm-Bawerk (1889) in “Capital and interest”
  - tendency to underestimate future
  - Intertemporal choice as other economic tradeoffs

- Irving Fisher (1930) in “The theory of interest”
  - Formalization of Böhm-Bawerk treatment of intertemporal choice
  - Two good indifference diagram
    - Time preference
    - Marginal utility

The Discounted Utility (DU) Model

- Paul Samuelson (1937) "A Note on Measurement of Utility.“
  - generalized model of intertemporal Choice applicable to multiple time periods
  - the discount rate
  - intertemporal preferences over consumption profiles $(c_t,\ldots,c_T)$
  - preferences can be represented by an intertemporal utility function $U_t(c_t,\ldots,c_T)$

$$U_t(c_t, \ldots\ldots c_T) = \sum_{k=0}^{T-t} D(k)\, u(c_{t+k})$$

$$\text{where } D(k) = \left(\frac{1}{1+\rho}\right)^k$$

- *$u(c_{t+k})$ instantaneous utility function*
- *$D(k)$ discount function*
- *ρ discount rate*

Useful results

$$\delta = \frac{1}{1+r} \text{ where } r > 0$$

$$\sum_{t=0}^{\infty} \delta^t = \frac{1}{1+\delta}$$

$$\sum_{t=1}^{\infty} \delta^t = \frac{\delta}{1+\delta}$$

$$\sum_{t=2}^{\infty} \delta^t = \frac{\delta^2}{1+\delta}$$

$$\sum_{t=x}^{\infty} \delta^t = \frac{\delta^x}{1+\delta}$$

$$\sum_{t=0}^{y} \delta^t = \frac{1-\delta^{y+1}}{1+\delta}$$

$$\sum_{t=1}^{y} \delta^t = \frac{\delta-\delta^{y+1}}{1+\delta}$$

$$\sum_{t=2}^{y} \delta^t = \frac{\delta^2-\delta^{y+1}}{1+\delta}$$

$$\sum_{t=x}^{y} \delta^t = \frac{\delta^x-\delta^{y+1}}{1+\delta}$$

Characteristics of DU model

- ***Integration of New Alternatives with Existing Plans***
  - new alternatives by integrating them with existing plans
  - *Example: pay x$ today to receive y$ in the future*
- ***Utility Independence***
  - the distribution of *utility across time makes no difference among utility profiles with equal discounted value.*
  - *Example: a flat profile versus a decreasing profile*

- ***Consumption Independence***
  - preferences over consumption profiles are not affected by the nature of consumption in periods in which consumption is identical in the two profiles
- ***Stationary Instantaneous Utility***
  - utility function u(c) is constant across time

- ***Independence of Discounting from Consumption***
  - the discount function is invariant across all forms of consumption.
- ***Diminishing Marginal Utility and Positive Time Preference***
  - diminishing marginal → to spread consumption over time
  - while positive time preference → to concentrate consumption in the present

- ***Constant Discounting and Time Consistency***

$$D(k) = \prod_{n=0}^{k-1} \frac{1}{1+\rho_n}$$

– *where* ρ_n *per-period discount rate*

$$D(k) = \left(\frac{1}{1+\rho}\right)^k$$

implies $\rho_n = \rho \; \forall n$

Continuous compounding

$$P_t = F_{t+x} e^{-x\rho}$$

Discounting Utility Model‘s Anomalies

Empirical researches describes some deviation from the behavior described by the DU model:

1. Gains are discounted more than losses
2. Small amounts are discounted more that large ones
3. Delay speedup asymmetry
4. Preference for improving sequences
5. Date/delay effect
6. Violation of independence
7. Subadditive discounting
8. Discount rates are not constant

Gains are discounted more than losses

<u>Thaler (1981)</u>

He asked to subjects how much they would be willing to pay for a traffic ticket if payment could be delayed for periods of 3 months, 1 years, 3 years.

Result: lower discount rates than in situations where monetary gains were involved.

Extreme case: many people prefer to incur a loss immediately rather than delay it. This implies a discount rate equal to zero.

Small amounts are discounted more that large ones

Thaler (1981)

people are indifferent between:

1. 15 now – 60 in 1 year → 139%
2. 250 now – 350 in 1 year → 34%
3. 3000 now - 4000 in 1 year → 29%

Delay speedup asymmetry

Loewenstein (1988)

Delivery time:

An acceleration is evaluated at lower discount rate than a delay

Preference for improving sequences

- The discounted utility model predict that, total undiscounted utility being equal, individual will prefer a declining sequence to an increasing one.
- Research examining preferences over sequences of outxomes found that people prefer improving sequences

Date/delay effect

LeBoeuf (2006)

He ask to subject the following two questions (on february 15):

1) How much money you would receive in 8 months to be equivalent to receive 100 $ now?
2) How much money you would receive on october 15 to be equivalent to receive 100 $ now?

Result: larger amounts in the answers to 1.

Violation of independence

- Loewenstein and Prelec (1993)

Imagine that over the next five weekends you must decide how to spend your Saturday nights. From each pair of sequences of dinners below, circle the one you would prefer. "Fancy French" refers to a dinner at a fancy French restaurant. "Fancy Lobster" refers to an exquisite lobster dinner at a four-star restaurant. Ignore scheduling considerations (e.g., your current plans).

first weekend	second weekend	third weekend	fourth weekend	fifth weekend	
		Option A			
Fancy French	Eat at home	Eat at home	Eat at home	Eat at home	[11%]
		Option B			
Eat at home	Eat at home	Fancy French	Eat at home	Eat at home	[89%]
		Option C			
Fancy French	Eat at home	Eat at home	Eat at home	Fancy Lobster	[49%]
		Option D			
Eat at home	Eat at home	Fancy French	Eat at home	Fancy Lobster	[51%]

Subadditive discounting

- total amount of discounting over a temporal interval increases as the interval is more finely partitioned
- Read (2001)
  - He elicited discount rates for a two year (24-month) interval and for its three constituent intervals (each of 8 months)
  - Discount rate of 2 years interval was lower than the coumpoded average discount rate over the 3 intervals.

Discount rates are not constant

- *Thaler (1981)*
- amount of money people want to receive after either 1 month or 1 year or 10 years to be indifferent to receive $15 now.
- median responses were $20 - $50 - $l00
- That correspond to an average (annual) discount rate of:
  - 345 % over a one-month horizon,
  - 120 % over a one-year horizon,
  - 19 % over a ten-year horizon.
  - ..or 345% between now and one month
  - 100% between 1 month and 1 year,
  - 7,7% between 1 year and 10 year

Discount rates are not constant (2)

- *Preference reversal*
- preferences between two delayed rewards can reverse in favor of the more proximate reward as the time to both rewards diminishes
- Example:
- Individuals may prefer
  - x \$ in t' days over y \$ in t days where $y < x$ and $t' > t$ *and*
  - y \$ now over x \$ in t'-t days

Discount rates are not constant (3)

Across studies

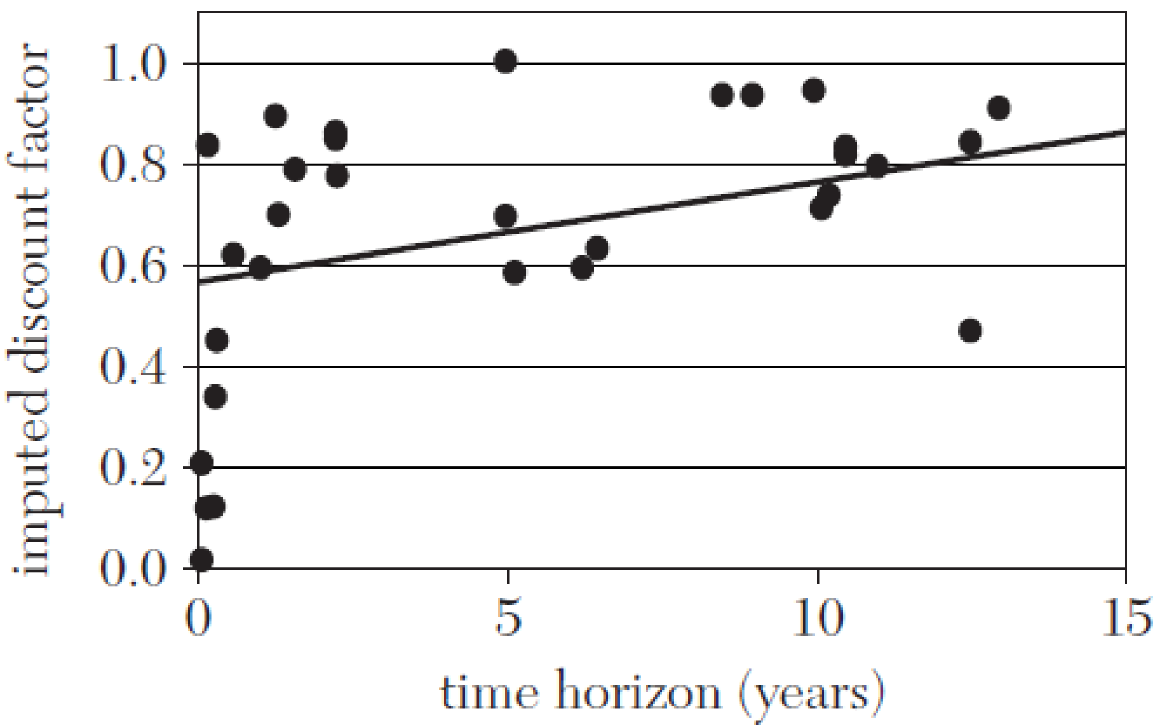


Figure 1a. Discount Factor as a Function of Time Horizon (all studies)

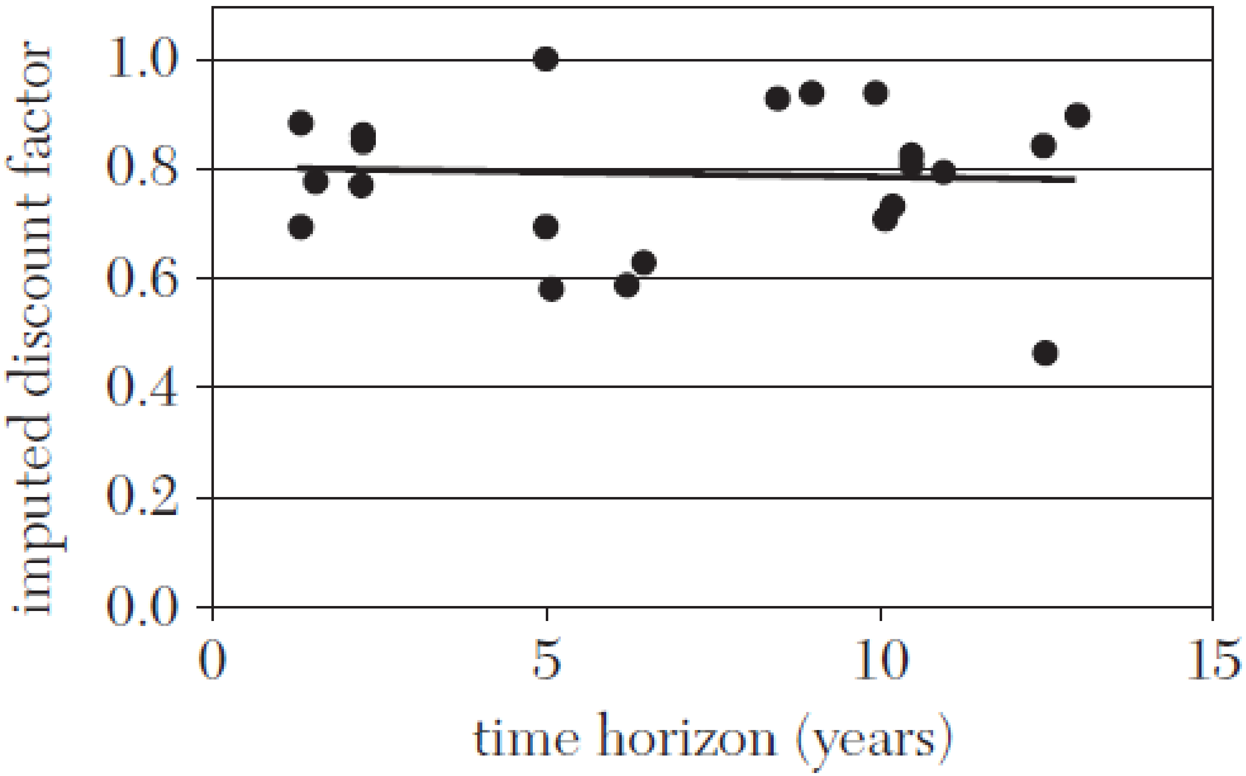


Figure 1b. Discount Factor as a Function of Time Horizon (studies with avg. horizons > 1 year)

Model of Hyperbolic Discounting

Strotz (1955-56) was the first to consider alternatives to exponential discounting,

There is "no reason why an individual should have such a special discount function" (p. 172).

any discount function other than exponential, a person would have time-inconsistent preferences.

He proposed two strategies that might be employed by a person who foresees how her preferences will change over time:

•the "strategy of precommitment“

•the "strategy of consistent planning“

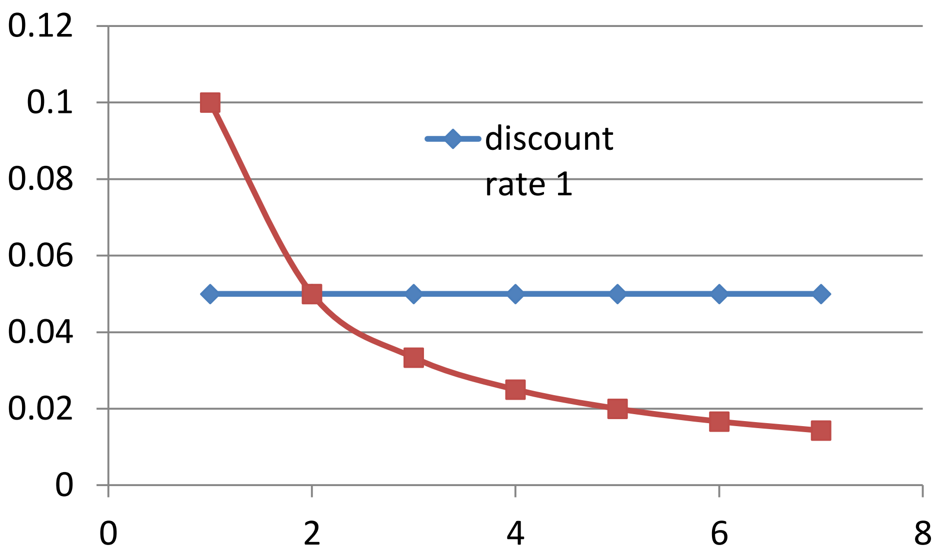
Strotz did not posit any specific alternative functional forms

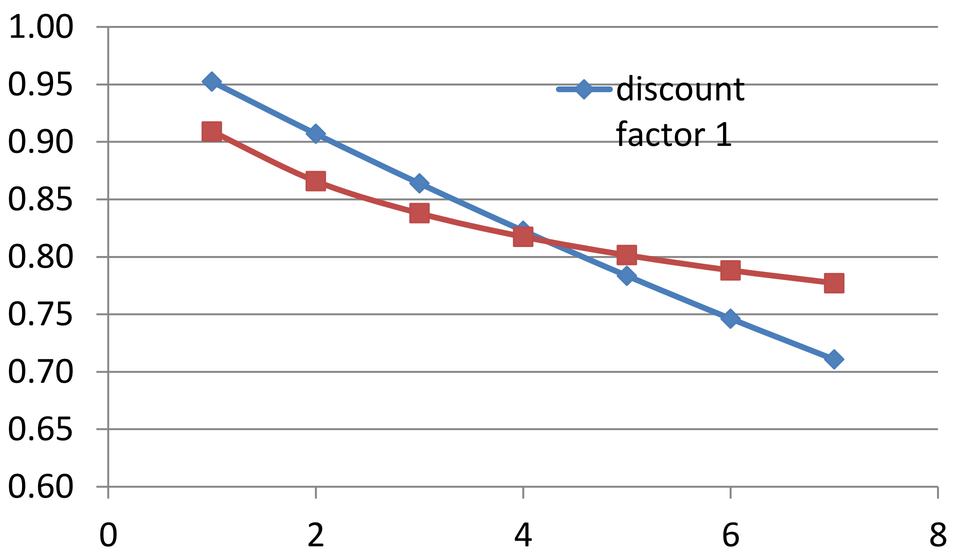
"special attention" be given to the case of declining discount rates.

Hyperbolic discount

- This term is used to indicate that people have a declining rate of time preference, that is $\square_n$ is declining in *n*.

period	discount rate 1	discount rate 2	discount factor 1	discount factor 2
1	0.05	0.10	0.95	0.91
2	0.05	0.05	0.91	0.87
3	0.05	0.03	0.86	0.84
4	0.05	0.03	0.82	0.82
5	0.05	0.02	0.78	0.80
6	0.05	0.02	0.75	0.79
7	0.05	0.01	0.71	0.78


0.12
0.1
0.08
0.06
0.04
0.02
0
0
2
4
6
8
discount rate 1


1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0
2
4
6
8
discount factor 1

Functional forms

- Hyperbolic functional forms fit the data better than the exponential functional forms
  - $D(t) = 1/t$
  - $D(t) = 1/(1 + \alpha t)$
  - $D(t) = 1/(1 + \alpha t)^{\beta/\alpha}$

The (β, δ) discounting

Simple functional form capturing the essence of hyperbolic discounting
Phelps and Pollak (1966)

$$D(k) = \begin{cases} 1 \; if \; k = 0 \\ \beta\delta^k \; if \; k > 0 \end{cases}$$

The discount rate between now and the next period is:

$$\rho = \frac{1 - \beta\delta}{\beta\delta}$$

Per period discount rate between any 2 future period is:

$$\rho = \frac{1 - \delta}{\delta}$$

To do or not to do an homework

$\delta = 1 \quad \beta = \frac{1}{2}$

Cost is immediate

Benefits are delayed of one period

- homework gives utility 3, has cost 2.
- planning to do homework later (costs and benefits merged)

$$u(HW_L) = \beta\delta\,(\delta \cdot 3 - 2) = \frac{1}{2}\,(3 - 2) > 0$$

- do homework now (costs and benefits separated)

$$u(HW_N) = \beta\delta \cdot 3 - 2 = \frac{1}{2} \cdot 3 - 2 < 0$$

- Doing homework later:

	costs	benefits
Now	–	–
Later	2	3

- Doing homework now:

	costs	benefits
Now	2	–
Later	–	3

- Time-consistent agents:
  - no self-control problems, i.e. $\beta = 1$, correctly predict future behavior: future selves behave in the future as they currently want to behave in the future.
- Naive agents:
  - unaware of self-control problems, i.e. $\beta < 1$, incorrectly expect future selves to behave in the future as they currently want to behave in the future.
- Sophisticated agents:
  - aware of self-control problems, i.e. $\beta < 1$, correctly predict future behavior (may deviate from how they currently want to behave in the future).

- Must do homework by time T.
  Immediate cost 2, gives utility 3 at time T.
  If HW is done at T, extra cost $c = 1$.

  - At time $t = T - 1$, Naifs evaluate doing HW at T as

    $$u(HW_L) = \beta\delta\,(3 - 2 - c) = 0$$

    whereas doing homework "now" (at time $T - 1$) is

    $$u(HW_N) = \beta\delta \cdot 3 - 2 = \frac{1}{2} \cdot 3 - 2 = -\frac{1}{2}$$

    If naif waits till last moment $T - 1$, he procrastinates till T, and pays cost c

  - At time t, Naifs evaluate doing HW at $t + 1$ as

    $$u(HW_L) = \beta\delta\,(3 - 2) = \frac{1}{2}$$

    So naif always procrastinates till T, and pays cost c.

- Sophisticates are aware that their $\beta < 1$.
- They know that if, they wait till $t = T - 1$, they will do HW at T and pay cost c.
- Intuitively:
  - they prefer to avoid cost (ex-post)
  - So sophisticate would like to commit to doing HW at earlier date.