Chapter 10
Market Power: Monopoly and Monopsony

- Review Questions

1. A monopolist is producing at a point at which marginal cost exceeds marginal revenue. How should it adjust its output to increase profit?

When marginal cost is greater than marginal revenue, the cost of producing the last unit is greater than the additional revenue from the sale of the last unit, so the firm loses money on that unit. The firm would increase profit by not producing as many units. It should reduce production, thereby decreasing marginal cost and increasing marginal revenue, until marginal cost is equal to marginal revenue. The diagram shows this situation. The firm is producing an output like \( Q' \), where \( MC > MR \). The firm should decrease output until it reaches the profit-maximizing output \( Q^* \).

![Diagram of price and output with MC and MR curves]

2. We write the percentage markup of prices over marginal cost as \( \frac{P - MC}{P} \). For a profit-maximizing monopolist, how does this markup depend on the elasticity of demand? Why can this markup be viewed as a measure of monopoly power?

Equation 10.1 on page 363 shows that the markup percentage is equal to the negative inverse of the price elasticity of demand.

\[
\frac{P - MC}{P} = -\frac{1}{E_d}.
\]

Therefore, as demand becomes more elastic (\( E_d \) becomes more negative), the markup percentage becomes smaller. For example, if \( E_d \) changes from \(-2\) to \(-5\), the markup decreases from \(0.5\) to \(0.2\). This tells us that the firm has less power to mark up its price above marginal cost when it faces a more elastic demand. Therefore, the markup percentage can be viewed as a measure of monopoly power.

3. Why is there no market supply curve under conditions of monopoly?
The monopolist’s output decision depends not only on marginal cost, but also on the demand curve. Shifts in demand do not trace out a series of prices and quantities that we can identify as the supply curve for the firm. Instead, shifts in demand lead to changes in price, output, or both. Thus there is no one-to-one correspondence between the price and the seller’s quantity; therefore, a monopolized market lacks a supply curve.

4. Why might a firm have monopoly power even if it is not the only producer in the market?
   A firm can have some monopoly power if its product is differentiated from other firms’ products, and if some consumers prefer its product to other firms’ products. A firm might also be located more conveniently for some consumers. These differences allow the firm to charge a price above its marginal cost and different from its rivals.

5. What are some of the different types of barriers to entry that give rise to monopoly power? Give an example of each.
   There are several types of barriers to entry, including exclusive rights (e.g., patents, copyrights, and licenses), control of an essential resource, and economies of scale. Exclusive rights are legally granted property rights to produce or distribute a good or service. Complete control over an essential raw material such as bauxite to produce aluminum or oil to produce gasoline prevents other firms from producing the same product. Large economies of scale lead to “natural monopolies” because the largest producer can charge a lower price, driving competition from the market. For example, in the production of aluminum, there is evidence to suggest that there are scale economies in the conversion of bauxite to alumina. (See U.S. v. Aluminum Company of America, 148 F.2d 416 [1945], discussed in Exercise 10, below.)

6. What factors determine the amount of monopoly power an individual firm is likely to have? Explain each one briefly.
   Three factors determine the firm’s elasticity of demand and hence its market power: (1) the elasticity of market demand, (2) the number of firms in the market, and (3) the interaction among firms in the market. The elasticity of market demand depends on the uniqueness of the product, i.e., how easy it is for consumers to substitute for the product. As the number of firms in the market increases, the demand elasticity facing each firm increases because customers have more choices. The number of firms in the market is determined by how easy it is to enter the industry (the height of barriers to entry). Finally, the ability to raise price above marginal cost depends on how other firms react to the firm’s price changes. If other firms match price changes, customers have little incentive to switch to another supplier, and this increases market power.

7. Why is there a social cost to monopoly power? If the gains to producers from monopoly power could be redistributed to consumers, would the social cost of monopoly power be eliminated? Explain briefly.
   When the firm exploits its monopoly power by charging a price above marginal cost, consumers buy less at the higher price, and consumer surplus decreases. Some of the lost consumer surplus is not captured by the seller, however, because the quantity produced and consumed decreases at the higher price, and this is a deadweight loss to society. Therefore, if the gains to producers were redistributed to consumers, society would still suffer the deadweight loss.
8. Why will a monopolist’s output increase if the government forces it to lower its price? If the government wants to set a price ceiling that maximizes the monopolist’s output, what price should it set?

By restricting price to be below the monopolist’s profit-maximizing price, the government can change the shape of the firm’s marginal revenue curve. When a price ceiling is imposed, \( MR \) is equal to the price ceiling for all quantities less than or equal to the quantity demanded at the price ceiling. For example, in the diagram the price ceiling is set at \( P' \), which is below the profit-maximizing price \( P^* \). The \( MR \) curve becomes the line \( PA \) and then jumps down to \( B \) and follows the original \( MR \) curve beyond that point. The optimal output for the monopolist is then \( Q' \), which is greater than the profit-maximizing output.

If the government wants to maximize output, it should set a price ceiling at the point where the demand curve and the marginal cost curve intersect, point \( C \) in the diagram. Then, when the firm produces where \( MR = MC \), it will be producing the output level at which \( P = MC \), where \( P \) is the price ceiling. In this way, the government can induce the monopolist to produce the competitive level of output. If the price ceiling is set below this point, the monopolist will decrease output below the competitive level.

9. How should a monopsonist decide how much of a product to buy? Will it buy more or less than a competitive buyer? Explain briefly.

The marginal expenditure is the change in the total expenditure as the purchased quantity changes. For a firm competing with many firms for inputs, the marginal expenditure is equal to the average expenditure (price). For a monopsonist, the marginal expenditure curve lies above the average expenditure curve because the decision to buy an extra unit raises the price that must be paid for all units, including the last unit. All firms should buy inputs so that the marginal value of the last unit is equal to the marginal expenditure on that unit. This is true for both the competitive buyer and the monopsonist. However, because the monopsonist’s marginal expenditure curve lies above the average expenditure curve and because the marginal value curve is downward sloping, the monopsonist buys less than a firm would buy in a competitive market.

10. What is meant by the term “monopsony power”? Why might a firm have monopsony power even if it is not the only buyer in the market?

Monopsony power refers to a buyer’s ability to affect the price of a good and to purchase the good for a lower price than in a competitive market. Any buyer facing an upward-sloping supply curve has some monopsony power. In a competitive market, the seller faces a perfectly elastic market demand curve and the buyer faces a perfectly elastic market supply curve. Thus, any characteristic of the market (e.g., a small number of buyers or buyers who engage in collusive behavior) that leads to a
less-than-perfectly-elastic supply curve gives the buyer some monopsony power, even if it is not the only buyer in the market.

11. **What are some sources of monopsony power? What determines the amount of monopsony power an individual firm is likely to have?**

   The individual firm’s monopsony power depends on the characteristics of the “buying-side” of the market. There are three characteristics that enhance monopsony power: (1) the elasticity of market supply, (2) the number of buyers, and (3) how the buyers interact. First, if market supply is very inelastic, then the buyer will enjoy more monopsony power. When supply is very elastic, marginal expenditure and average expenditure do not differ by much, so price will be closer to the competitive price. Second, the fewer the number of buyers, the greater the monopsony power. Third, if buyers are able to collude and/or they do not compete very aggressively with each other, then each will enjoy more monopsony power.

12. **Why is there a social cost to monopsony power? If the gains to buyers from monopsony power could be redistributed to sellers, would the social cost of monopsony power be eliminated? Explain briefly.**

   With monopsony power, the price is lower and the quantity is less than under competitive buying conditions. Because of the lower price and reduced sales, sellers lose revenue. Only part of this lost revenue is transferred to the buyer as consumer surplus, and the net loss in total surplus is deadweight loss. Even if the consumer surplus could be redistributed to sellers, the deadweight loss persists. This inefficiency will remain because quantity is reduced below the level where price is equal to marginal cost.

13. **How do the antitrust laws limit market power in the United States? Give examples of major provisions of these laws.**

   Antitrust laws limit market power by proscribing a firm’s behavior in attempting to maximize profit. Section 1 of the Sherman Act prohibits every restraint of trade, including any attempt to fix prices by buyers or sellers. Section 2 of the Sherman Act prohibits behavior that leads to monopolization. The Clayton Act, with the Robinson-Patman Act, prohibits price discrimination and exclusive dealing (sellers prohibiting buyers from buying goods from other sellers). The Clayton Act also limits mergers when they could substantially lessen competition. The Federal Trade Commission Act makes it illegal to use unfair or deceptive practices.

14. **Explain briefly how the U.S. antitrust laws are actually enforced.**

   Antitrust laws are enforced in three ways: (1) through the Antitrust Division of the Justice Department whenever firms violate federal statutes, (2) through the Federal Trade Commission whenever firms violate the Federal Trade Commission Act, and (3) through civil suits. The Justice Department can seek to impose fines or jail terms on managers or owners, or it can seek to reorganize a firm, as it did in its case against AT&T. The FTC can seek a voluntary understanding to comply with the law or a formal Commission order. Individuals or companies can sue in federal court for awards equal to three times the damage arising from anticompetitive behavior.

### Exercises

1. **Will an increase in the demand for a monopolist’s product always result in a higher price? Explain.**

2. **Will an increase in the supply facing a monopsonist buyer always result in a lower price? Explain.**
As illustrated in Figure 10.4b in the textbook, an increase in demand for a monopolist’s product need not always result in a higher price. Under the conditions portrayed in Figure 10.4b, the monopolist supplies different quantities at the same price.

Similarly, an increase in supply facing a monopsonist need not always result in a higher price. Suppose the average expenditure curve shifts from $AE_1$ to $AE_2$, as illustrated in the figure. With the shift in the average expenditure curve, the marginal expenditure curve shifts from $ME_1$ to $ME_2$. The $ME_1$ curve intersects the marginal value curve (demand curve) at $Q_1$, resulting in a price of $P$. When the $AE$ curve shifts, the $ME_2$ curve intersects the marginal value curve at $Q_2$ resulting in the same price at $P$.

2. Caterpillar Tractor, one of the largest producers of farm machinery in the world, has hired you to advise it on pricing policy. One of the things the company would like to know is how much a 5% increase in price is likely to reduce sales. What would you need to know to help the company with this problem? Explain why these facts are important.

As a large producer of farm equipment, Caterpillar Tractor has some market power and should consider the entire demand curve when choosing prices for its products. As their advisor, you should focus on the determination of the elasticity of demand for the company’s tractors. There are at least four important factors to be considered. First, how similar are the products offered by Caterpillar’s competitors? If they are close substitutes, a small increase in price could induce customers to switch to the competition. Second, how will Caterpillar’s competitors respond to a price increase? If the other firms are likely to match Caterpillar’s increase, Caterpillar’s sales will not fall nearly as much as they would were the other firms not to match the price increase. Third, what is the age of the existing stock of tractors? With an older population of tractors, farmers will want to replace their aging stock, and their demands will be less elastic. In this case, a 5% price increase induces a smaller drop in sales than would occur with a younger stock of tractors that are not in need of replacement. Finally, because farm tractors are a capital input in agricultural production, what is the expected profitability of the agricultural sector? If farm incomes are expected to fall, an increase in tractor prices would cause a greater decline in sales than would occur if farm incomes were high.

3. A monopolist firm faces a demand with constant elasticity of $-2.0$. It has a constant marginal cost of $20 per unit and sets a price to maximize profit. If marginal cost should increase by 25%, would the price charged also rise by 25%?
The monopolist’s pricing rule is: \( \frac{P - MC}{P} = -\frac{1}{E_d} \), or alternatively, \( P = \frac{MC}{1 + \left( \frac{1}{E_d} \right)} \). Therefore, price should be set so that \( P = \frac{MC}{1 + \left( \frac{1}{E_d} \right)} = 2MC \). With \( MC = 20 \), the optimal price is \( P = 2(20) = 40 \) cents.

If \( MC \) increases by 25% to $25, the new optimal price is \( P = 2(25) = 50 \) cents, a 25% increase. So if marginal cost increases by 25%, the price also increases by 25%.

4. A firm faces the following average revenue (demand) curve:

\[
P = 120 - 0.02Q
\]

where \( Q \) is weekly production and \( P \) is price, measured in cents per unit. The firm’s cost function is given by \( C = 60Q + 25,000 \). Assume that the firm maximizes profits.

a. What is the level of production, price, and total profit per week?

The profit-maximizing output is found by setting marginal revenue equal to marginal cost. Given a linear demand curve in inverse form, \( P = 120 - 0.02Q \), we know that the marginal revenue curve has the same intercept and twice the slope of the demand curve. Thus, the marginal revenue curve for the firm is \( MR = 120 - 0.04Q \). Marginal cost is the slope of the total cost curve. The slope of \( TC = 60Q + 25,000 \) is 60, so \( MC \) is constant and equal to 60. Setting \( MR = MC \) to determine the profit-maximizing quantity:

\[
120 - 0.04Q = 60, \text{ or } Q = 1500.
\]

Substituting the profit-maximizing quantity into the inverse demand function to determine the price:

\[
P = 120 - (0.02)(1500) = 90 \text{ cents}.
\]

Profit equals total revenue minus total cost:

\[
\pi = (90)(1500) - (25,000 + (60)(1500)), \text{ so } \pi = 20,000 \text{ cents per week, or } $200 \text{ per week}.
\]

b. If the government decides to levy a tax of 14 cents per unit on this product, what will be the new level of production, price, and profit?

Suppose initially that consumers must pay the tax to the government. Since the total price (including the tax) that consumers would be willing to pay remains unchanged, we know that the demand function is

\[
P^* + t = 120 - 0.02Q, \text{ or } P^* = 120 - 0.02Q - t,
\]

where \( P^* \) is the price received by the suppliers and \( t \) is the tax per unit. Because the tax increases the price consumers pay for each unit, total revenue for the monopolist decreases by \( tQ \). You can see this most easily by expressing \( R = P^*Q \), which means \( tQ \) is subtracted from revenue. Marginal revenue, the revenue on each additional unit, decreases by \( t \):
\[ MR = 120 - 0.04Q - t \]

where \( t = 14 \) cents. To determine the profit-maximizing level of output with the tax, equate marginal revenue with marginal cost:

\[ 120 - 0.04Q - 14 = 60, \text{ or} \]

\[ Q = 1150 \text{ units.} \]

Substituting \( Q \) into the demand function to determine price:

\[ P^* = 120 - (0.02)(1150) - 14 = 83 \text{ cents.} \]

Profit is total revenue minus total cost:

\[ = (83)(1150) - [(60)(1150) + 25,000] = 1450 \text{ cents, or} \]

\[ $14.50 \text{ per week.} \]

**Note:** The price facing the consumer after the imposition of the tax is \( 83 + 14 = 97 \) cents. Compared to the 90-cent price before the tax is imposed, consumers and the monopolist each pay 7 cents of the tax.

If the monopolist had to pay the tax instead of the consumer, we would arrive at the same result. The monopolist’s cost function would then be

\[ TC = 60Q + 25,000 + tQ = (60 + t)Q + 25,000. \]

The slope of the cost function is \( (60 + t) \), so \( MC = 60 + t \). We set this \( MC \) equal to the marginal revenue function from part a:

\[ 120 - 0.04Q = 60 + 14, \text{ or} \]

\[ Q = 1150. \]

Thus, it does not matter who sends the tax payment to the government. The burden of the tax is shared by consumers and the monopolist in exactly the same way.

5. **The following table shows the demand curve facing a monopolist who produces at a constant marginal cost of $10:**

<table>
<thead>
<tr>
<th>Price</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
</tr>
<tr>
<td>14</td>
<td>8</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>28</td>
</tr>
<tr>
<td>2</td>
<td>32</td>
</tr>
<tr>
<td>0</td>
<td>36</td>
</tr>
</tbody>
</table>
a. **Calculate the firm’s marginal revenue curve.**

To find the marginal revenue curve, we first derive the inverse demand curve. The intercept of the inverse demand curve on the price axis is 18. The slope of the inverse demand curve is the change in price divided by the change in quantity. For example, a decrease in price from 18 to 16 yields an increase in quantity from 0 to 4. Therefore, the slope of the inverse demand is

\[ \frac{\Delta P}{\Delta Q} = \frac{-2}{4} = -0.5, \]

and the demand curve is therefore

\[ P = 18 - 0.5Q. \]

The marginal revenue curve corresponding to a linear demand curve is a line with the same intercept as the inverse demand curve and a slope that is twice as steep. Therefore, the marginal revenue curve is

\[ MR = 18 - Q. \]

b. **What are the firm’s profit-maximizing output and price? What is its profit?**

The monopolist’s profit-maximizing output occurs where marginal revenue equals marginal cost. Marginal cost is a constant $10. Setting \( MR = MC \) to determine the profit-maximizing quantity:

\[ 18 - Q = 10, \]

or \( Q = 8. \)

To find the profit-maximizing price, substitute this quantity into the demand equation:

\[ P = 18 - (0.5)(8) = 14. \]

Total revenue is price times quantity:

\[ TR = (14)(8) = 112. \]

The profit of the firm is total revenue minus total cost, and total cost is equal to average cost times the level of output produced. Since marginal cost is constant, average variable cost is equal to marginal cost. Ignoring any fixed costs, total cost is \( 10Q \) or 80, and profit is

\[ \pi = 112 - 80 = 32. \]

c. **What would the equilibrium price and quantity be in a competitive industry?**

For a competitive industry, price would equal marginal cost at equilibrium. Setting the expression for price equal to a marginal cost of 10:

\[ 18 - 0.5Q = 10, \]

so that \( Q = 16 \) and \( P = 10. \)

Note the increase in the equilibrium quantity and decrease in price compared to the monopoly solution.

d. **What would the social gain be if this monopolist were forced to produce and price at the competitive equilibrium? Who would gain and lose as a result?**

The social gain arises from the elimination of deadweight loss. When price drops from $14 to $10, consumer surplus increases by area \( A + B + C = 8(14 - 10) + (0.5)(16 - 8)(14 - 10) = 48. \)

Producer surplus decreases by area \( A + B = 8(14 - 10) = 32. \) So consumers gain $48 while
producers lose $32. Deadweight loss decreases by the difference, $48 – 32 = $16. Thus the social gain if the monopolist were forced to produce and price at the competitive level is $16.

6. Suppose that an industry is characterized as follows:

\[
\begin{align*}
C &= 100 + 2q^2 \quad \text{each firm’s total cost function} \\
MC &= 4q \quad \text{firm’s marginal cost function} \\
P &= 90 - 2Q \quad \text{industry demand curve} \\
MR &= 90 - 4Q \quad \text{industry marginal revenue curve}
\end{align*}
\]

a. If there is only one firm in the industry, find the monopoly price, quantity, and level of profit.

If there is only one firm in the industry, then the firm will act like a monopolist and produce at the point where marginal revenue is equal to marginal cost:

\[
90 - 4Q = 4Q
\]

\[
Q = 11.25.
\]

For a quantity of 11.25, the firm will charge a price \( P = 90 - 2(11.25) = $67.50. \) 

\[
\]

b. Find the price, quantity, and level of profit if the industry is competitive.

If the industry is competitive, price will equal marginal cost. Therefore \( 90 - 2Q = 4Q, \) or \( Q = 15. \)

At a quantity of 15, price is equal to \( P = 90 - 2(15) = $60. \) The industry’s profit is \( = 60(15) - [100 + 2(15)^2] = $350. \)

c. Graphically illustrate the demand curve, marginal revenue curve, marginal cost curve, and average cost curve. Identify the difference between the profit level of the monopoly and the profit level of the competitive industry in two different ways. Verify that the two are numerically equivalent.

The graph below illustrates the demand curve, marginal revenue curve, and marginal cost curve. The average cost curve is not shown because it makes the diagram too cluttered. \( AC \) reaches its minimum value of $28.28 and intersects the marginal cost curve at a quantity of 7.07. The profit that is lost by having the firm produce at the competitive solution as compared to the monopoly solution is the difference of the two profit levels as calculated in parts a and b: \( $406.25 - 350 = $56.25. \) On the graph below, this difference is represented by the lost profit area, which is the triangle below the marginal cost curve and above the marginal revenue curve, between the quantities of 11.25 and 15. This is lost profit because for each of these 3.75 units, extra revenue earned was less than extra cost incurred. This area is \( (0.5)(3.75)(60-30) = $56.25. \) Another way to find this difference is to use the fact that the change in producer surplus equals the change in
profit. Going from the monopoly price to the competitive price, producer surplus is reduced by areas $A + B$ and increased by area $C$. $A + B$ is a rectangle with area $(11.25)(67.50 - 60) = 84.375$. Area $C$ equals $(0.5)(3.75)(60 - 45) = 28.125$. The difference is $84.375 - 28.125 = 56.25$. A final method of graphically illustrating the difference in the two profit levels is to draw in the average cost curve and identify the two profit rectangles, one for the monopoly output and the other for the competitive output. The area of each profit rectangle is the difference between price and average cost multiplied by quantity, $(P - AC)Q$. The difference between the areas of the two profit rectangles is $56.25$.

![Graph of monopoly and competitive pricing with areas A, B, and C highlighted.]

7. Suppose a profit-maximizing monopolist is producing 800 units of output and is charging a price of $40 per unit.

   a. If the elasticity of demand for the product is $-2$, find the marginal cost of the last unit produced.

      The monopolist’s pricing rule as a function of the elasticity of demand is:

      \[
      \frac{(P - MC)}{P} = -\frac{1}{E_d}
      \]

      or alternatively,

      \[
      P \left(1 + \frac{1}{E_d}\right) = MC
      \]

      Substitute $-2$ for the elasticity and $40$ for price, and then solve for $MC = 20$.

   b. What is the firm’s percentage markup of price over marginal cost?

      \[
      \frac{(P - MC)}{P} = \frac{(40 - 20)}{40} = 0.5
      \]

      so the markup is 50% of the price.
c. Suppose that the average cost of the last unit produced is $15 and the firm’s fixed cost is $2000. Find the firm’s profit.

Total revenue is price times quantity, or $40(800) = $32,000. Total cost is equal to average cost times quantity, or $15(800) = $12,000. Profit is therefore $32,000 – 12,000 = $20,000. Fixed cost is already included in average cost, so we do not use the $2000 fixed cost figure separately.

8. A firm has two factories for which costs are given by:

\[
\text{Factory #1: } C_1(Q_1) = 10Q_1^2 \\
\text{Factory #2: } C_2(Q_2) = 20Q_2^2
\]

The firm faces the following demand curve:

\[P = 700 - 5Q\]

where \(Q\) is total output – i.e., \(Q = Q_1 + Q_2\).

a. On a diagram, draw the marginal cost curves for the two factories, the average and marginal revenue curves, and the total marginal cost curve (i.e., the marginal cost of producing \(Q = Q_1 + Q_2\)). Indicate the profit-maximizing output for each factory, total output, and price.

The average revenue curve is the demand curve,

\[P = 700 - 5Q.\]

For a linear demand curve, the marginal revenue curve has the same intercept as the demand curve and a slope that is twice as steep:

\[MR = 700 - 10Q.\]

Next, determine the marginal cost of producing \(Q\). To find the marginal cost of production in Factory 1, take the derivative of the cost function with respect to \(Q_1\):

\[MC_1 = \frac{dC_1(Q_1)}{dQ_1} = 20Q_1.\]

Similarly, the marginal cost in Factory 2 is

\[MC_2 = \frac{dC_2(Q_2)}{dQ_2} = 40Q_2.\]

We know that total output should be divided between the two factories so that the marginal cost is the same in each factory. Let \(MC_T\) be this common marginal cost value. Then, rearranging the marginal cost equations in inverse form and horizontally summing them, we obtain total marginal cost, \(MC_T\):

\[Q = Q_1 + Q_2 = \frac{MC_1}{20} + \frac{MC_2}{40} = \frac{3MC_T}{40}, \quad \text{or}\]

\[MC_T = \frac{40Q}{3}.\]
Profit maximization occurs where $MC_T = MR$. See the figure below for the profit-maximizing output for each factory, total output $Q_T$, and price $P_M$.

![Figure 10.8.a](image)

**b. Calculate the values of $Q_1$, $Q_2$, $Q$, and $P$ that maximize profit.**

To calculate the total output $Q$ that maximizes profit, set $MC_T = MR$:

$$\frac{40Q}{3} = 700 - 10Q, \text{ or } Q = 30.$$

When $Q = 30$, marginal revenue is $MR = 700 - (10)(30) = 400$. At the profit-maximizing point, $MR = MC_1 = MC_2$. Therefore,

$$MC_1 = 400 = 20Q_1, \text{ or } Q_1 = 20 \text{ and } MC_2 = 400 = 40Q_2, \text{ or } Q_2 = 10.$$

To find the monopoly price, $P$, substitute for $Q$ in the demand equation:

$$P = 700 - 5(30), \text{ or } P_M = $550.$$

**c. Suppose that labor costs increase in Factory 1 but not in Factory 2. How should the firm adjust (i.e., raise, lower, or leave unchanged) the following: Output in Factory 1? Output in Factory 2? Total output? Price?**

An increase in labor costs will lead to a horizontal shift to the left in $MC_1$, causing $MC_T$ to shift to the left as well (since it is the horizontal sum of $MC_1$ and $MC_2$). The new $MC_T$ curve will intersect the $MR$ curve at a lower total quantity and higher marginal revenue. You can see this in Figure 10.8.a above. At a higher level of marginal revenue, $Q_2$ is greater than at the original level of $MR$. Since $Q_T$ falls and $Q_2$ rises, $Q_1$ must fall. Since $Q_T$ falls, price must rise.
9. A drug company has a monopoly on a new patented medicine. The product can be made in either of two plants. The costs of production for the two plants are \( MC_1 = 20 + 2Q_1 \) and \( MC_2 = 10 + 5Q_2 \). The firm’s estimate of demand for the product is \( P = 20 - 3(Q_1 + Q_2) \). How much should the firm plan to produce in each plant? At what price should it plan to sell the product?

First, notice that only \( MC_2 \) is relevant because the marginal cost curve of the first plant lies above the demand curve.

![Graph showing marginal cost and demand curves]

This means that the demand curve becomes \( P = 20 - 3Q_2 \). With an inverse linear demand curve, we know that the marginal revenue curve has the same vertical intercept but twice the slope, or \( MR = 20 - 6Q_2 \). To determine the profit-maximizing level of output, equate \( MR \) and \( MC_2 \):

\[
20 - 6Q_2 = 10 + 5Q_2, \text{ or } Q_2 = 0.91.
\]

Also, \( Q_1 = 0 \), and therefore total output is \( Q = 0.91 \). Price is determined by substituting the profit-maximizing quantity into the demand equation:

\[
P = 20 - 3(0.91) = $17.27.
\]

10. One of the more important antitrust cases of the 20th century involved the Aluminum Company of America (Alcoa) in 1945. At that time, Alcoa controlled about 90% of primary aluminum production in the United States, and the company had been accused of monopolizing the aluminum market. In its defense, Alcoa argued that although it indeed controlled a large fraction of the primary market, secondary aluminum (i.e., aluminum produced from the recycling of scrap) accounted for roughly 30% of the total supply of aluminum and that many competitive firms were engaged in recycling. Therefore, Alcoa argued, it did not have much monopoly power.

a. Provide a clear argument in favor of Alcoa’s position.

Although Alcoa controlled about 90% of primary aluminum production, secondary production by recyclers accounted for 30% of the total aluminum supply. Therefore, Alcoa actually controlled about 63% (90% of the 70% that did not come from recyclers) of the aluminum supply. Alcoa’s ability to raise prices was constrained by the recyclers because with a higher price, a much larger proportion of aluminum supply could come from these secondary sources, as there was a large stock of potential scrap supply in the economy. Therefore, the price elasticity of demand for
Alcoa’s primary aluminum was much higher (in absolute value) than might be expected, given Alcoa’s dominant position in primary aluminum production. In addition, other metals such as copper and steel are feasible substitutes for aluminum in some applications. Again, the demand elasticity Alcoa faced might be higher than would otherwise be expected.

b. Provide a clear argument against Alcoa’s position.

While Alcoa could not raise its price by very much at any one time, the stock of potential aluminum supply is limited. Therefore, by keeping a stable high price, Alcoa could reap monopoly profits. Also, since Alcoa had originally produced most of the metal reappearing as recycled scrap, it would have considered the effect of scrap reclamation on future prices. Therefore, it exerted effective monopolistic control over the secondary metal supply.

c. The 1945 decision by Judge Learned Hand has been called “one of the most celebrated judicial opinions of our time.” Do you know what Judge Hand’s ruling was?

Judge Hand ruled against Alcoa but did not order it to divest itself of any of its United States production facilities. The two remedies imposed by the court were (1) that Alcoa was barred from bidding for two primary aluminum plants constructed by the government during World War II (they were sold to Reynolds and Kaiser), and (2) that it divest itself of its Canadian subsidiary, which became Alcan.

11. A monopolist faces the demand curve \( P = 11 - Q \), where \( P \) is measured in dollars per unit and \( Q \) in thousands of units. The monopolist has a constant average cost of $6 per unit.

a. Draw the average and marginal revenue curves and the average and marginal cost curves. What are the monopolist’s profit-maximizing price and quantity? What is the resulting profit? Calculate the firm’s degree of monopoly power using the Lerner index.

Because demand (average revenue) is \( P = 11 - Q \), the marginal revenue function is \( MR = 11 - 2Q \). Also, because average cost is constant, marginal cost is constant and equal to average cost, so \( MC = 6 \).

To find the profit-maximizing level of output, set marginal revenue equal to marginal cost:

\[
11 - 2Q = 6, \text{ or } Q = 2.5.
\]

That is, the profit-maximizing quantity equals 2500 units. Substitute the profit-maximizing quantity into the demand equation to determine the price:

\[
P = 11 - 2.5 = $8.50.
\]

Profits are equal to total revenue minus total cost,

\[
\pi = TR - TC = PQ - (AC)(Q), \text{ or }
\]

\[
\pi = (8.50)(2.5) - (6)(2.5) = 6.25, \text{ or } $6250.
\]

The diagram below shows the demand, \( MR \), \( AC \), and \( MC \) curves along with the optimal price and quantity and the firm’s profits.
The degree of monopoly power according to the Lerner Index is:

\[
\frac{P - MC}{P} = \frac{8.5 - 6}{8.5} = 0.294.
\]

b. A government regulatory agency sets a price ceiling of $7 per unit. What quantity will be produced, and what will the firm’s profit be? What happens to the degree of monopoly power?

To determine the effect of the price ceiling on the quantity produced, substitute the ceiling price into the demand equation.

\[7 = 11 - Q, \text{ or } Q = 4.\]

Therefore, the firm will choose to produce 4000 units rather than the 2500 units without the price ceiling. Also, the monopolist will choose to sell its product at the $7 price ceiling because $7 is the highest price that it can charge, and this price is still greater than the constant marginal cost of $6, resulting in positive monopoly profit.

Profits are equal to total revenue minus total cost:

\[\pi = 7(4000) - 6(4000) = 4000.\]

The degree of monopoly power falls to

\[\frac{P - MC}{P} = \frac{7 - 6}{7} = 0.143.\]

c. What price ceiling yields the largest level of output? What is that level of output? What is the firm’s degree of monopoly power at this price?

If the regulatory authority sets a price below $6, the monopolist would prefer to go out of business because it cannot cover its average variable costs. At any price above $6, the monopolist would produce less than the 5000 units that would be produced in a competitive industry.

Therefore, the regulatory agency should set a price ceiling of $6, thus making the monopolist face a horizontal effective demand curve up to \(Q = 5\) (i.e., 5000 units). To ensure a positive output (so that the monopolist is not indifferent between producing 5000 units and shutting down), the price ceiling should be set at \(6 + \delta\), where \(\delta\) is small.

Thus, 5000 is the maximum output that the regulatory agency can extract from the monopolist by using a price ceiling. The degree of monopoly power is
12. Michelle’s Monopoly Mutant Turtles (MMMT) has the exclusive right to sell Mutant Turtle t-shirts in the United States. The demand for these t-shirts is \( Q = \frac{10,000}{P^2} \). The firm’s short-run cost is \( SRTC = 2000 + 5Q \), and its long-run cost is \( LRTC = 6Q \).

   a. What price should MMMT charge to maximize profit in the short run? What quantity does it sell, and how much profit does it make? Would it be better off shutting down in the short run?

   MMMT should offer enough t-shirts so that \( MR = MC \). In the short run, marginal cost is the change in \( SRTC \) as the result of the production of another t-shirt, i.e., \( SRMC = 5 \), the slope of the \( SRTC \) curve. Demand is:

   \[ Q = \frac{10,000}{P^2}, \]
   
   or, in inverse form,

   \[ P = 100Q^{-1/2}. \]

   Total revenue is \( TR = PQ = 100Q^{1/2} \). Taking the derivative of \( TR \) with respect to \( Q \), \( MR = 50Q^{-1/2} \). Equating \( MR \) and \( MC \) to determine the profit-maximizing quantity:

   \[ 5 = 50Q^{-1/2}, \] or \( Q = 100. \)

   Substituting \( Q = 100 \) into the demand function to determine price:

   \[ P = (100)(100^{-1/2}) = 10. \]

   The profit at this price and quantity is equal to total revenue minus total cost:

   \[ \pi = 10(100) - [2000 + 5(100)] = -1500. \]

   Although profit is negative, price is above the average variable cost of 5, and therefore the firm should not shut down in the short run. Since most of the firm’s costs are fixed, the firm loses $2000 if nothing is produced. If the profit-maximizing (i.e., loss-minimizing) quantity is produced, the firm loses only $1500.

   b. What price should MMMT charge in the long run? What quantity does it sell and how much profit does it make? Would it be better off shutting down in the long run?

   In the long run, marginal cost is equal to the slope of the \( LRTC \) curve, which is 6.

   Equating marginal revenue and long run marginal cost to determine the profit-maximizing quantity:

   \[ 50Q^{-1/2} = 6, \] or \( Q = 69.444 \)

   Substituting \( Q = 69.444 \) into the demand equation to determine price:

   \[ P = (100)(69.444)^{-1/2} = (100)(1/8.333) = 12. \]

   Total revenue is \( TR = 12(69.444) = 833.33 \) and total cost is \( LRTC = 6(69.444) = 416.67. \) Profit is therefore \( 833.33 - 416.67 = 416.66. \) The firm should remain in business in the long run.

   c. Can we expect MMMT to have lower marginal cost in the short run than in the long run? Explain why.
In the long run, \( MMT \) can change all its inputs when it changes output level. Therefore, \( LRMC \) includes the costs of all inputs that are fixed in the short run but variable in the long run. These costs do not appear in \( SRMC \). As a result we can expect \( SRMC \) to be lower than \( LRMC \) in many cases.

13. You produce widgets for sale in a perfectly competitive market at a market price of $10 per widget. Your widgets are manufactured in two plants, one in Massachusetts and the other in Connecticut. Because of labor problems in Connecticut, you are forced to raise wages there so that marginal costs in that plant increase. In response to this, should you shift production and produce more in your Massachusetts plant?

No, production should not shift to the Massachusetts plant, although production in the Connecticut plant should be reduced. To maximize profits, a multiplant firm will schedule production so that the following two conditions are met:

- Marginal costs of production at each plant are equal.
- Marginal revenue of the last unit sold is equal to the marginal cost at each plant.

These two rules can be summarized as \( MR = MC_1 = MC_2 \), where the subscripts indicate plants. The firm in this example has two plants and sells in a perfectly competitive market. In a perfectly competitive market \( P = MR \). Therefore, production among the plants should be allocated such that:

\[
P = MC_c(Q_c) = MC_m(Q_m),
\]

where the subscripts denote plant locations (\( c \) for Connecticut, etc.). The marginal costs of production have increased in Connecticut but have not changed in Massachusetts. \( MC \) shifts up and to the left in Connecticut, so production in the Connecticut plant should drop from \( Q_c \) to \( Q'_c \) in the diagram below. Since costs have not changed in Massachusetts, the level of \( Q_m \) that sets \( MC_m(Q_m) = P \), should not change.

14. The employment of teaching assistants (TAs) by major universities can be characterized as a monopsony. Suppose the demand for TAs is \( W = 30,000 - 125n \), where \( W \) is the wage (as an annual salary), and \( n \) is the number of TAs hired. The supply of TAs is given by \( W = 1000 + 75n \).

a. If the university takes advantage of its monopsonist position, how many TAs will it hire? What wage will it pay?

The supply curve is equivalent to the average expenditure curve. With a supply curve of \( W = 1000 + 75n \), the total expenditure is \( Wn = 1000n + 75n^2 \). Taking the derivative of the total expenditure function with respect to the number of TAs, the marginal expenditure curve is \( ME = 1000 + 150n \). As a monopsonist, the university would equate marginal value (demand) with marginal expenditure to determine the number of TAs to hire:

\[
30,000 - 125n = 1000 + 150n,
\]

or \( n = 105.5 \).

Substituting \( n = 105.5 \) into the supply curve to determine the wage:
b. If, instead, the university faced an infinite supply of TAs at the annual wage level of $10,000, how many TAs would it hire?

With an infinite number of TAs at $10,000, the supply curve is horizontal at $10,000. Total expenditure is 10,000\(n\), and marginal expenditure is 10,000. Equating marginal value and marginal expenditure:

\[30,000 - 125n = 10,000,\text{ or } n = 160.\]

15. Dayna’s Doorstops, Inc. (DD) is a monopolist in the doorstop industry. Its cost is \(C = 100 - 5Q + Q^2\), and demand is \(P = 55 - 2Q\).

a. What price should DD set to maximize profit? What output does the firm produce? How much profit and consumer surplus does DD generate?

To maximize profit, DD should equate marginal revenue and marginal cost. Given a demand of \(P = 55 - 2Q\), we know that total revenue, \(PQ\), is \(55Q - 2Q^2\). Marginal revenue is found by taking the first derivative of total revenue with respect to \(Q\) or:

\[\frac{dTR}{dQ} = MR = 55 - 4Q.\]

Similarly, marginal cost is determined by taking the first derivative of the total cost function with respect to \(Q\) or:

\[\frac{dTC}{dQ} = MC = 2Q - 5.\]

Equating \(MC\) and \(MR\) to determine the profit-maximizing quantity,

\[55 - 4Q = 2Q - 5,\text{ or } Q = 10.\]

Substitute \(Q = 10\) into the demand equation to find the profit-maximizing price:

\[P = 55 - 2(10) = 35.\]

Profits are equal to total revenue minus total cost:

\[\pi = (35)(10) - [100 - 5(10) + 10^2] = 200.\]

Consumer surplus is equal to one-half times the profit-maximizing quantity, 10, times the difference between the demand intercept (55) and the monopoly price (35):

\[CS = (0.5)(10)(55 - 35) = 100.\]

b. What would output be if DD acted like a perfect competitor and set \(MC = P\)? What profit and consumer surplus would then be generated?

In competition, profits are maximized at the point where price equals marginal cost. So set price (as given by the demand curve) equal to \(MC\):

\[55 - 2Q = 2Q - 5,\text{ or } Q = 15.\]

Substituting \(Q = 15\) into the demand equation to determine the price:

\[P = 55 - 2(15) = 25.\]

Profits are total revenue minus total cost or:
\[ \pi = (25)(15) - [100 - 5(15) + 15^2] = $125. \]

Consumer surplus is

\[ CS = (0.5)(15)(55 - 25) = $225. \]

So consumer surplus increases by $125 and producer surplus decreases by $75.

c. **What is the deadweight loss from monopoly power in part a?**

The deadweight loss is equal to the area below the demand curve, above the marginal cost curve, and between the quantities of 10 and 15, or numerically

\[ DWL = (0.5)(35 - 15)(15 - 10) = $50. \]

d. **Suppose the government, concerned about the high price of doorstops, sets a maximum price at $27. How does this affect price, quantity, consumer surplus, and DD’s profit? What is the resulting deadweight loss?**

With the price ceiling, the maximum price that DD may charge is $27.00. Note that when a ceiling price is set below the monopoly price the ceiling price is the firm’s marginal revenue for each unit sold up to the quantity demanded at the ceiling price.

Substitute the ceiling price of $27.00 into the demand equation to determine the effect on the equilibrium quantity sold:

\[ 27 = 55 - 2Q, \text{ or } Q = 14. \]

Compared to part a, price drops from $35 to $27 and output increases from 10 to 14.

Consumer surplus is

\[ CS = (0.5)(14)(55 - 27) = $196. \]

Profits are

\[ \pi = (27)(14) - [100 - 5(14) + 14^2] = $152. \]

So CS increases from $100 to $196 and profit falls from $200 to $152.

The deadweight loss is \[ DWL = (0.5)(15 - 14)(27 - 23) = $2 \]

e. **Now suppose the government sets the maximum price at $23. How does this decision affect price, quantity, consumer surplus, DD’s profit, and deadweight loss?**

With a ceiling price set below the competitive price, DD’s output will be less than the competitive output of 15. Equate marginal revenue (the ceiling price) and marginal cost to determine the profit-maximizing level of output:

\[ 23 = 2Q - 5, \text{ or } Q = 14. \]

With the government-imposed maximum price of $23, profits are

\[ \pi = (23)(14) - [100 - 5(14) + 14^2] = $96. \]
Consumer surplus is realized on 14 doorsteps. Therefore, it is equal to the consumer surplus in part d, which was $196, plus an additional area due to the fact that price is now $23 instead of $27. The additional amount is \((27 - 23)(14) = 56\). Therefore, consumer surplus is 
\(196 + 56 = 252\).

Compared to part d, price is $4 less and quantity is the same. Consumer surplus increases by $56 and \(DD\)'s profit decreases by $56, which is also the drop in producer surplus. Since the increase in consumer surplus equals the drop in producer surplus, deadweight loss is the same as before at $2.00.

**f. Finally, consider a maximum price of $12. What will this do to quantity, consumer surplus, profit, and deadweight loss?**

With a maximum price of only $12, output decreases considerably:
\[ 12 = 2Q - 5, \text{ or } Q = 8.5. \]

Profits are
\[ \pi = (12)(8.5) - [100 - 5(8.5) + 8.5^2] = -27.75. \]

Even though the firm is making losses, it will continue to produce in the short run because revenue ($102) is greater than total variable cost ($29.75).

Consumer surplus is realized on only 8.5 units. Note that the consumer buying the last unit would have been willing to pay a price of $38 (\(38 = 55 - 2(8.5)\)). Therefore,
\[ CS = (0.5)(8.5)(55 - 38) + (8.5)(38 - 12) = 293.25. \]
\[ DWL = (0.5)(15 - 8.5)(38 - 12) = 84.50. \]

The result of this low price is that output falls, consumer surplus increases, profit drops, and deadweight loss increases. In the long run, the firm will shut down, and then output, consumer surplus, and profit will all drop to zero.

16. **There are 10 households in Lake Wobegon, Minnesota, each with a demand for electricity of \(Q = 50 - P\). Lake Wobegon Electric’s (\(LWE\)) cost of producing electricity is \(TC = 500 + Q\).**

   **a. If the regulators of \(LWE\) want to make sure that there is no deadweight loss in this market, what price will they force \(LWE\) to charge? What will output be in that case? Calculate consumer surplus and \(LWE\)’s profit with that price.**

The first step in solving the regulator’s problem is to determine the market demand for electricity in Lake Wobegon. The quantity demanded in the market is the sum of the quantity demanded by each individual at any given price. Graphically, we horizontally sum each household’s demand for electricity to arrive at market demand, and mathematically
\[ Q_M = \sum_{i=1}^{10} Q_i = 10(50 - P) = 500 - 10P \Rightarrow P = 50 - 0.1Q. \]

To avoid deadweight loss, the regulators will set price equal to marginal cost. Given \(TC = 500 + Q\), \(MC = 1\) (the slope of the total cost curve). Setting price equal to marginal cost, and solving for quantity:
\[ 50 - 0.1Q = 1, \text{ or } Q = 490. \]
Profits are equal to total revenue minus total costs:

\[ \pi = (1)(490) - (500 + 490) = -500 \text{ (a loss)} \]

Total consumer surplus is:

\[ CS = (0.5)(490)(50 - 1) = 12,005 \text{, or } 1200.50 \text{ per household.} \]

b. If regulators want to ensure that LWE doesn’t lose money, what is the lowest price they can impose? Calculate output, consumer surplus, and profit. Is there any deadweight loss?

To guarantee that LWE does not lose money, regulators will allow LWE to charge the average cost of production, where

\[ AC = \frac{TC}{Q} = \frac{500}{Q} + 1. \]

To determine the equilibrium price and quantity under average cost pricing, set price equal to average cost:

\[ 50 - 0.1Q = \frac{500}{Q} + 1. \]

Solving for \( Q \) yields the following quadratic equation:

\[ 0.1Q^2 - 49Q + 500 = 0. \]

Note: if \( aQ^2 + bQ + c = 0 \), then

\[ Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \]

Using this quadratic formula:

\[ Q = \frac{49 \pm \sqrt{49^2 - (4)(0.1)(500)}}{(2)(0.1)} \]

there are two solutions: 10.4 and 479.6. Note that at a quantity of 10.4, marginal revenue is greater than marginal cost, and the firm will gain by producing more output. Also, note that the larger quantity results in a lower price and hence a larger consumer surplus. Therefore, \( Q = 479.6 \) and \( P = 2.04. \) At this quantity and price, profit is zero (given some slight rounding error). Consumer surplus is

\[ CS = (0.5)(479.6)(50 - 2.04) = 11,500.81 \text{, or } 1150.08 \text{ per household}. \]

Deadweight loss is

\[ DWL = (0.5)(490 - 479.6)(2.04 - 1) = 5.41. \]

c. Kristina knows that deadweight loss is something that this small town can do without. She suggests that each household be required to pay a fixed amount just to receive any electricity at all, and then a per-unit charge for electricity. Then LWE can break even even while charging the price calculated in part a. What fixed amount would each household have to pay for Kristina’s plan to work? Why can you be sure that no household will choose instead to refuse the payment and go without electricity?

Fixed costs are $500. If each household pays $50, the fixed costs are covered and the utility can charge marginal cost for electricity. Because consumer surplus per household under marginal cost pricing is $1200.50, each would be willing to pay the $50.

17. A certain town in the Midwest obtains all of its electricity from one company, Northstar Electric. Although the company is a monopoly, it is owned by the citizens of the town, all of
whom split the profits equally at the end of each year. The CEO of the company claims that because all of the profits will be given back to the citizens, it makes economic sense to charge a monopoly price for electricity. True or false? Explain.

The CEO’s claim is false. If the company charges the monopoly price it will produce a smaller quantity than the competitive equilibrium. Therefore, even though all of the monopoly profits are given back to the citizens, there is still a deadweight loss associated with the fact that too little electricity is produced and consumed.

18. A monopolist faces the following demand curve: $Q = 144/P^2$, where $Q$ is the quantity demanded and $P$ is price. Its average variable cost is $AVC = Q^{0.5}$, and its fixed cost is 5.

a. What are its profit-maximizing price and quantity? What is the resulting profit?

To maximize profit the monopolist should set marginal revenue equal to marginal cost. To find marginal revenue, first rewrite the demand function as a function of $Q$ so that you can then express total revenue as a function of $Q$ and calculate marginal revenue:

$$Q = \frac{144}{P^2} \Rightarrow P^2 = \frac{144}{Q} \Rightarrow P = \sqrt{\frac{144}{Q}} = \frac{12}{\sqrt{Q}} \Rightarrow P = 12Q^{-0.5}$$

$$R = PQ = \frac{12}{\sqrt{Q}} Q = 12\sqrt{Q} = 12Q^{0.5}$$

$$MR = \frac{dR}{dQ} = 0.5(12Q^{-0.5}) = 6Q^{-0.5} = \frac{6}{\sqrt{Q}}.$$ 

To find marginal cost, first find total cost, which is equal to fixed cost plus variable cost. Fixed cost is 5, and variable cost is equal to average variable cost times $Q$. Therefore, total cost and marginal cost are:

$$TC = 5 + (Q^{0.5})Q = 5 + Q^{1.5}$$

$$MC = \frac{dTC}{dQ} = \frac{3}{2}Q^{-\frac{1}{2}} = \frac{3\sqrt{Q}}{2}.$$ 

To find the profit-maximizing level of output, set $MR = MC$:

$$\frac{6}{\sqrt{Q}} = \frac{3\sqrt{Q}}{2} \Rightarrow Q = 4.$$ 

Now find price and profit:

$$P = \frac{12}{\sqrt{Q}} = \frac{12}{\sqrt{4}} = $6$$

$$\pi = PQ - TC = (6)(4) - (5 + 4^{1.5}) = $11.$$
b. Suppose the government regulates the price to be no greater than $4 per unit. How much will the monopolist produce? What will its profit be?

The price ceiling truncates the demand curve that the monopolist faces at \( P = 4 \), so \( Q = \frac{144}{4^2} = 9 \). Therefore, if the monopolist produces 9 units or less, the price must be $4. Because of the regulation, the demand curve now has two parts:

\[
P = \begin{cases} 
4, & \text{if } Q \leq 9 \\
12Q^{-\frac{1}{2}}, & \text{if } Q > 9.
\end{cases}
\]

Thus, total revenue and marginal revenue also have two parts:

\[
TR = \begin{cases} 
4Q, & \text{if } Q \leq 9 \\
12Q^{\frac{1}{2}}, & \text{if } Q > 9
\end{cases}
\quad \text{and} \quad
MR = \begin{cases} 
4, & \text{if } Q \leq 9 \\
6Q^{-\frac{1}{2}}, & \text{if } Q > 9.
\end{cases}
\]

To find the profit-maximizing level of output, set marginal revenue equal to marginal cost, so that for \( P = 4 \),

\[
4 = \frac{3}{2} \sqrt{Q}, \text{ or } \sqrt{Q} = \frac{8}{3}, \text{ or } Q = 7.11.
\]

If the monopolist produces an integer number of units, the profit-maximizing production level is 7 units, price is $4, revenue is $28, total cost is $23.52, and profit is $4.48. There is a shortage of two units, since the quantity demanded at the price of $4 is 9 units.

c. Suppose the government wants to set a ceiling price that induces the monopolist to produce the largest possible output. What price will accomplish this goal?

To maximize output, the regulated price should be set so that demand equals marginal cost, which occurs where

\[
\frac{12}{\sqrt{Q}} = \frac{3\sqrt{Q}}{2} \Rightarrow \sqrt{Q} = 8 \quad \text{and} \quad Q = 8^2 = 64.
\]

The regulated price becomes the monopolist’s marginal revenue up to a quantity of 8. So \( MR \) is a horizontal line with an intercept equal to the regulated price of $4.24. To maximize profit, the firm produces where marginal cost is equal to marginal revenue, which results in a quantity of 8 units.

The monopolist’s profit is

\[
\pi = TR - TC = (4.24)(8) - (5 + 8^{\frac{1}{2}}) = 33.92 - 27.63 = 6.29.
\]

"