Chapter 15
Investment, Time, and Capital Markets

■ Review Questions

1. **A firm uses cloth and labor to produce shirts in a factory that it bought for $10 million. Which of its factor inputs are measured as flows and which as stocks? How would your answer change if the firm had leased a factory instead of buying one? Is its output measured as a flow or a stock? What about profit?**

   Inputs that are purchased and used up during a particular time period are flows. Flow variables can be measured over time periods such as hours, days, weeks, months, or years. Inputs measured at a particular point in time are stocks. All stock variables have an associated flow variable. At any particular time, a firm will have a stock of buildings and machines that it owns, which are stock variables. During a given time period, as the firm uses its capital stock, that stock will depreciate, and this depreciation is a flow. In this question, cloth and labor are flows, while the factory is a stock. If the firm instead leases the building, the factory is still a stock variable that is owned in this case by someone else. The firm would pay rent during a particular time period, which would be a flow just like depreciation when the firm owns the capital stock. Output is always a flow variable that is measured over some time period. Since profit is the difference between revenues and costs over some given time period, it is also a flow.

2. **How do investors calculate the net present value of a bond? If the interest rate is 5%, what is the present value of a perpetuity that pays $1000 per year forever?**

   The present value of a bond is the sum of discounted values of each payment to the bond holder over the life of the bond. This includes the payment of interest in each period and the repayment of the principal at the end of the bond's life. A perpetuity involves the payment of interest in every future period forever, with no repayment of the principal. The present discounted value of a perpetuity is $PDV = \frac{A}{R}$, where A is the annual payment and R is the annual interest rate. If A = \$1000 and R = 0.05, $PDV = \frac{\$1000}{0.05} = \$20{,}000.$

3. **What is the *effective yield* on a bond? How does one calculate it? Why do some corporate bonds have higher effective yields than others?**

   The effective yield is the interest rate that equates the present value of a bond's payment stream with the bond's current market price. The present discounted value of a payment made in the future is

   $$PDV = FV/(1 + R)^t,$$

   where t is the length of time before payment. The bond's selling price is its PDV. The payments it makes are the future values, FV, paid in time t. Thus the selling price P equals $P = \frac{A}{1+R} + \frac{A}{(1+R)^2} + \cdots + \frac{A}{(1+R)^N} + \frac{I}{(1+R)^N}$, where A is the annual interest payment, I is the principal repayment, and N is the number of years until maturity. We must solve for R, which is the bond's effective yield. The effective yield is determined by the interaction of buyers and sellers in the bond market. Some corporate bonds have higher effective yields because they are thought to be a more risky investment, and hence buyers must be rewarded with a higher rate of return so that they will be willing to hold the bonds. Higher rates of return imply a lower present discounted value. If bonds have the same payment stream, the bonds of riskier firms will sell for less than the bonds of less risky firms.

4. **What is the Net Present Value (*NPV*) criterion for investment decisions? How does one calculate the *NPV* of an investment project? If all cash flows for a project are certain, what discount rate should be used to calculate *NPV*?**

   The Net Present Value criterion for investment decisions says to invest if the present discounted value of the expected future cash flows from an investment is larger than the cost of the investment (Section 15.4). We calculate the *NPV* by (1) determining the present discounted value of all future cash flows and (2) subtracting the discounted value of all costs, present and future. To discount both income and cost, the firm should use a discount rate that reflects its opportunity cost of capital, the next highest return on an alternative investment of similar riskiness. Therefore, the risk-free interest rate should be used if the cash flows are certain.

5. **You are retiring from your job and are given two options. You can accept a lump sum payment from the company, or you can accept a smaller annual payment that will continue for as long as you live. How would you decide which option is best? What information do you need?**

   The best option is the one that has the highest present discounted value. The lump sum payment has a present discounted value equal to the amount of the lump sum payment. To calculate the present discounted value of the payment stream, you need to know approximately how many years you might live. If you made a guess of 25 years you could then discount each of the 25 future payments back to the current year and add them up to see how this sum compares to the lump sum payment. The discount factor would be the rate of return you expect to earn each year when you invest the lump sum payment. Finally, you must consider the time and risks involved in managing a lump sum on your own and decide if it is better or easier to just take the annual payments. The critical information you need to make this decision is how long you will live and the rate of return you will earn on your investment each year. Unfortunately, neither is knowable with certainty.

6. **You have noticed that bond prices have been rising over the past few months. All else equal, what does this suggest has been happening to interest rates? Explain.**

   This suggests that interest rates have been falling because bond prices and interest rates are inversely related. When the price of a bond (with a fixed stream of future payments) rises, then the effective yield on the bond falls. So if bond prices have been rising, the yields on bonds must be falling, and the only way people will be willing to hold bonds whose yields have fallen is if interest rates in general have also fallen.

7. **What is the difference between a real discount rate and a nominal discount rate? When should a real discount rate be used in an *NPV* calculation and when should a nominal rate be used?**

   The real discount rate is net of inflation, whereas the nominal discount rate includes inflationary expectations. The real discount rate is approximately equal to the nominal discount rate minus the expected rate of inflation. If cash flows are in real terms, the appropriate discount rate is the real rate, but if the cash flows are in nominal terms, a nominal discount rate should be used. For example, in applying the *NPV* criterion to a manufacturing decision, if future prices of inputs and outputs are projected using current dollars (and have not been increased to account for future inflation), a real discount rate should be used to determine whether the *NPV* is positive. On the other hand, if the future dollar amounts have been adjusted to account for inflation, then a nominal discount rate should be used. In sum, all numbers should either be expressed in real terms or nominal terms; not a mix of real and nominal.

8. **How is risk premium used to account for risk in *NPV* calculations? What is the difference between diversifiable and nondiversifiable risk? Why should only nondiversifiable risk enter into the risk premium?**

To determine the present discounted value of a cash flow, the discount rate should reflect the riskiness of the project generating the cash flow. The risk premium is the difference between a discount rate that reflects the riskiness of the cash flow and a discount rate for a risk-free flow, e.g., the discount rate associated with a short-term government bond. The higher the riskiness of a project, the higher the risk premium should be.

Diversifiable risk can be eliminated by investing in many projects. Hence, an efficient capital market will not compensate an investor for taking on risk that can be eliminated costlessly through diversification. Nondiversifiable risk is that part of a project's risk that cannot be eliminated by investing in a large number of other projects. It is that part of a project's risk which is correlated with the portfolio of all projects available in the market. Since investors can eliminate diversifiable risk, they cannot expect to earn a risk premium on diversifiable risk. Only nondiversifiable risk should enter into the risk premium.

9. **What is meant by the "market return" in the Capital Asset Pricing Model (*CAPM*)? Why is the market return greater than the risk-free interest rate? What does an asset's "beta" measure in the *CAPM*? Why should high-beta assets have a higher expected return than low-beta assets?**

In the Capital Asset Pricing Model (*CAPM*), the market return is the rate of return on the portfolio of all risky assets in the market. Thus the market return reflects only nondiversifiable risk.

Since the market portfolio has no diversifiable risk, the market return reflects the risk premium associated with holding one unit of nondiversifiable risk. The market rate of return is greater than the risk-free rate of return, because risk-averse investors must be compensated with higher average returns for holding a risky asset.

An asset's beta reflects the sensitivity (covariance) of the asset's return with the return on the market portfolio. An asset with a high beta will have a greater expected return than a low-beta asset, because the high-beta asset has greater nondiversifiable risk than the low-beta asset.

10. **Suppose you are deciding whether to invest $100 million in a steel mill. You know the expected cash flows for the project, but they are risky—steel prices could rise or fall in the future. How would the *CAPM* help you select a discount rate for an *NPV* calculation?**

To evaluate the net present value of a $100 million investment in a steel mill, you should use the stock market's current evaluation of firms that own steel mills as a guide to selecting the appropriate discount rate. For example, you would (1) identify nondiversified steel companies that are primarily involved in steel production, (2) determine the beta associated with stocks issued by those companies (this can be done statistically or by relying on a financial service that publishes stock betas, such as *Value Line*), and (3) take a weighted average of these betas, where the weights are equal to each firm's assets divided by the sum of all the nondiversified steel firms' assets. With an estimate of beta, plus estimates of the expected market and risk-free rates of return, you could infer the discount rate using Equation (15.7) in the text: Discount rate $= r_f + \beta(r_m - r_f)$.

11. How does a consumer trade off current and future costs when selecting an air conditioner or other major appliance? How could this selection be aided by an *NPV* calculation?

There are two major costs to be considered when purchasing a durable good: the initial purchase price and the cost of operating (and perhaps repairing) the appliance over its lifetime. Since these costs occur at different points in time, consumers should calculate the present discounted value of all the future costs and add that to the purchase price to determine the total cost of the appliance. This is what an *NPV* calculation does, so selecting the best appliance can be aided by doing an *NPV* calculation using the consumer's opportunity cost of money as the discount rate.

12. What is meant by the "user cost" of producing an exhaustible resource? Why does price minus extraction cost rise at the rate of interest in a competitive market for an exhaustible resource?

In addition to the marginal cost of extracting the resource and preparing it for sale, there is an additional opportunity cost arising from the depletion of the resource, because producing and selling a unit today makes that unit unavailable for production and sale in the future. User cost is the difference between price and the marginal cost of production. User cost rises over time because as reserves of the resource become depleted, the remaining reserves become more valuable.

Given constant demand over time, the price of the resource minus its marginal cost of extraction, $P - MC$, should rise over time at the rate of interest. If $P - MC$ rises faster than the rate of interest, no extraction should occur in the present period, because holding the resource for another year would earn a higher profit a year from now than selling the resource now and investing the proceeds for another year. If $P - MC$ rises more slowly than the rate of interest, current extraction should increase, thus increasing supply, lowering the equilibrium price, and decreasing the return on producing the resource. In equilibrium, the price of an exhaustible resource rises at the rate of interest.

13. What determines the supply of loanable funds? The demand for loanable funds? What might cause the supply or demand for loanable funds to shift? How would such a shift affect interest rates?

The supply of loanable funds is determined by the interest rate offered to savers. A higher interest rate induces households to consume less today (save) in favor of greater consumption in the future. The demand for loanable funds comes from consumers who wish to consume more today than tomorrow or from investors who wish to borrow money. Demand depends on the interest rate at which these two groups can borrow. Several factors can shift the demand and supply of loanable funds. For example, a recession decreases demand at all interest rates, shifting the demand curve inward and causing the equilibrium interest rate to fall. On the other hand, the supply of loanable funds will shift out if the Federal Reserve increases the money supply, again causing the interest rate to fall.

■ Exercises

1. Suppose the interest rate is 10%. If \$100 is invested at this rate today, how much will it be worth after one year? After two years? After five years? What is the value today of \$100 paid one year from now? Paid two years from now? Paid five years from now?

The future value, *FV*, of \$100 invested today at an interest rate of 10% is

$$FV = \$100 + (\$100)(0.10) = \$110.$$

Two years from now we will earn interest on the original \$100 (interest = \$10) *and* we will earn interest on the interest from the first year, i.e., (\$10)(0.10) = \$1. Thus our investment will be worth \$100 + \$10 (from the first year) + \$10 (from the second year) + \$1 (interest on the first year's interest) = \$121.

Algebraically, $FV = PDV(1 + R)^t$, where PDV is the present discounted value (or initial amount) of the investment, R is the interest rate, and t is the number of years. After two years,

$$FV = PDV(1 + R)^t = (\$100)(1.1)^2 = (\$100)(1.21) = \$121.00.$$

After five years

$$FV = PDV(1 + R)^t = (\$100)(1.1)^5 = (\$100)(1.61051) = \$161.05.$$

To find the present discounted value of \$100 paid one year from now, we must determine how much we would need to invest today at 10% to have \$100 one year from now. Using the formula for FV, solve for PDV as a function of FV:

$$PDV = (FV)(1 + R)^{-t}.$$

With $t = 1$, $R = 0.10$, and $FV = \$100$,

$$PDV = (100)(1.1)^{-1} = \$90.91.$$

With $t = 2$, $PDV = (100)(1.1)^{-2} = \$82.64$,

With $t = 5$, $PDV = (100)(1.1)^{-5} = \$62.09$.

2. You are offered the choice of two payment streams: (a) \$150 paid one year from now and \$150 paid two years from now; (b) \$130 paid one year from now and \$160 paid two years from now. Which payment stream would you prefer if the interest rate is 5%? If it is 15%?

To compare two income streams, calculate the present discounted value of each and choose the one with the higher value. Use the formula $PDV = FV(1 + R)^{-t}$ for each cash flow. First, use an interest rate of 5%. Stream (a) has two payments:

$$PDV_a = FV_1(1 + R)^{-1} + FV_2(1 + R)^{-2}$$

$$PDV_a = (\$150)(1.05)^{-1} + (\$150)(1.05)^{-2}, \text{ or}$$

$$PDV_a = \$142.86 + 136.05 = \$278.91.$$

Stream (b) has two payments:

$$PDV_b = (\$130)(1.05)^{-1} + (\$160)(1.05)^{-2}, \text{ or}$$

$$PDV_b = \$123.81 + \$145.12 = \$268.93.$$

At an interest rate of 5%, you should select (a) because it has the higher present discounted value.

If the interest rate is 15%, the present discounted values of the two income streams would be:

$$PDV_a = (\$150)(1.15)^{-1} + (\$150)(1.15)^{-2}, \text{ or}$$

$$PDV_a = \$130.43 + \$113.42 = \$243.85, \text{ and}$$

$$PDV_b = (\$130)(1.15)^{-1} + (\$160)(1.15)^{-2}, \text{ or}$$

$$PDV_b = \$113.04 + \$120.98 = \$234.02.$$

You should still select (a).

3. **Suppose the interest rate is 10%. What is the value of a coupon bond that pays $80 per year for each of the next five years and then makes a principal repayment of $1000 in the sixth year? Repeat for an interest rate of 15%.**

The value of the bond is the present discounted value, *PDV*, of the stream of payments made by the bond over the next six years. The *PDV* of each payment is:

$$PDV = \frac{FV}{(1+R)^t},$$

where *R* is the interest rate, equal to 10% (i.e., 0.10), and *t* is the number of years in the future. The value of all coupon payments over five years is therefore

$$PDV = \frac{80}{(1+R)} + \frac{8}{(1+R)^2} + \frac{80}{(1+R)^3} + \frac{80}{(1+R)^4} + \frac{80}{(1+R)^5}, \text{ or}$$

$$PDV = 80\left(\frac{1}{1.1} + \frac{1}{1.21} + \frac{1}{1.331} + \frac{1}{1.4641} + \frac{1}{1.61051}\right) = \$303.26.$$

The present value of the final payment of $1000 in the sixth year is

$$PDV = \frac{\$1000}{1.1^6} = \frac{\$1000}{1.771} = \$564.47.$$

Thus the present value of the bond is $303.26 + $564.47 = $867.73 when the interest rate is 5%.

With an interest rate of 15%, the value of the bond is

$$PDV = 80(0.870 + 0.756 + 0.658 + 0.572 + 0.497) + (1000)(0.432), \text{ or}$$

$$PDV = \$268.17 + \$432.33 = \$700.50.$$

As the interest rate increases, the value of the bond decreases.

4. **A bond has two years to mature. It makes a coupon payment of $100 after one year and both a coupon payment of $100 and a principal repayment of $1000 after two years. The bond is selling for $966. What is its effective yield?**

Determine the interest rate that will yield a present value of $966 for an income stream of $100 after one year and $1100 after two years. Find *R* such that

$$966 = (100)(1 + R)^{-1} + (1100)(1 + R)^{-2}.$$

Algebraic manipulation yields

$$966(1 + R)^2 = 100(1 + R) + 1100, \text{ or}$$

$$966 + 1932R + 966R^2 - 100 - 100R - 1100 = 0, \text{ or}$$

$$966R^2 + 1832R - 234 = 0.$$

Use the quadratic formula to solve for *R:*

$$R = 0.12 \text{ or } -2.017.$$

Since −2.017 does not make economic sense, the effective yield is 12%.

5. Equation (15.5) (page 572) shows the net present value of an investment in an electric motor factory. Half of the $10 million cost is paid initially and the other half after a year. The factory is expected to lose money during its first two years of operation. If the discount rate is 4%, what is the *NPV*? Is the investment worthwhile?

Using $R = 0.04$, Equation (15.5) becomes

$$NPV = -5 - \frac{5}{(1.04)} - \frac{1}{(1.04)^2} - \frac{0.5}{(1.04)^3} + \frac{0.96}{(1.04)^4} + \frac{0.96}{(1.04)^5} + \cdots \frac{0.96}{(1.04)^{20}} + \frac{1}{(1.04)^{20}}.$$

Calculating the *NPV* we find:

$NPV = -5 - 4.808 - 0.925 - 0.445 + 0.821 + 0.789 + 0.759 + 0.730 + 0.701 + 0.674 + 0.649 + 0.624 + 0.600 + 0.577 + 0.554 + 0.533 + 0.513 + 0.493 + 0.474 + 0.456 + 0.438 + 0.456 = -0.338.$

The investment loses $338,000 and is not worthwhile. However, were the discount rate 3%, the *NPV* = $866,000, and the investment would be worth undertaking.

6. The market interest rate is 5% and is expected to stay at that level. Consumers can borrow and lend all they want at this rate. Explain your choice in each of the following situations:

a. Would you prefer a $500 gift today or a $540 gift next year?

The present value of $500 today is $500. The present value of $540 next year is

$$\frac{\$540.00}{1.05} = \$514.29.$$

Therefore, you should prefer the $540 next year.

b. Would you prefer a $100 gift now or a $500 loan without interest for four years?

If you take the $500 loan, you can invest it for the four years and then pay back the $500. The future value of the $500 is

$$500(1.05)^4 = \$607.75.$$

After you pay back the $500 you will have $107.75 left to keep. The future value of the $100 gift is

$$100(1.05)^4 = \$121.55.$$

You should take the $100 gift because its future value is larger.

c. Would you prefer a $350 rebate on an $8000 car or one year of financing for the full price of the car at 0% interest?

The interest rate is 0%, which is 5% less than the current market rate. You save $400 = (0.05)($8000) one year from now. The present value of this $400 is

$$\frac{\$400}{1.05} = \$380.95.$$

This is greater than $350. Therefore, choose the financing.

d. You have just won a million-dollar lottery and will receive $50,000 a year for the next 20 years. How much is this worth to you today?

If you get the first year's payment immediately (as is usually the case), the *PDV* of the payments of $50,000 per year for 20 years is less than two-thirds of $1 million.

$$PDV = 50{,}000 + \frac{50{,}000}{1.05} + \frac{50{,}000}{(1.05)^2} + \frac{50{,}000}{(1.05)^3} + \cdots \frac{50{,}000}{(1.05)^{18}} + \frac{50{,}000}{(1.05)^{19}} = \$654{,}266.04.$$

e. You win the "honest million" jackpot. You can have $1 million today or $60,000 per year for eternity (a right that can be passed on to your heirs). Which do you prefer?

The present discounted value of the $60,000 perpetuity is $60,000/0.05 = $1,200,000, which makes it advisable take the $60,000 per year payment. Note that it takes 32 years before the *PDV* of the perpetuity exceeds $1,000,000, so if you don't expect to live that long, you should take the perpetuity only if you care about your heirs.

f. In the past, adult children had to pay taxes on gifts over $10,000 from their parents, but parents could make interest-free loans to their children. Why did some people call this policy unfair? To whom were the rules unfair?

Any gift of $N from parent to child could be made without taxation by lending the child $\frac{\$N(1+r)}{r}$. For example, to avoid taxes on a $20,000 gift, the parent would lend the child $420,000. With that money, the child could earn $21,000 in interest at the 5% interest rate during the next year. At the end of the year the child would repay the $420,000 loan (interest-free) and keep the $21,000 in interest. The present discounted value of the $21,000 received one year in the future is $21,000/1.05 = $20,000. So the child gets a "gift" with a present value of $20,000 without paying any taxes. People of more moderate incomes would find these rules unfair: they might be able to afford to give the child $20,000 directly (which would not be tax free) but would not have the resources to lend the child $420,000.

7. Ralph is trying to decide whether to go to graduate school. If he spends two years in graduate school, paying $15,000 tuition each year, he will get a job that will pay $60,000 per year for the rest of his working life. If he does not go to school, he will go into the work force immediately. He will then make $30,000 per year for the next three years, $45,000 for the following three years, and $60,000 per year every year after that. If the interest rate is 10%, is graduate school a good financial investment?

After the sixth year, Ralph's income will be the same with or without the graduate school education, so we can ignore all income after the first six years. With graduate school, the present value of income for the next six years (assuming all payments occur at the end of the year) is

$$-\frac{\$15{,}000}{(1.1)^1} - \frac{\$15{,}000}{(1.1)^2} + \frac{\$60{,}000}{(1.1)^3} + \frac{\$60{,}000}{(1.1)^4} + \frac{\$60{,}000}{(1.1)^5} + \frac{\$60{,}000}{(1.1)^6} = \$131{,}150.35.$$

Without graduate school, the present value of income for the next six years is

$$\frac{\$30{,}000}{(1.1)^1} + \frac{\$30{,}000}{(1.1)^2} + \frac{\$30{,}000}{(1.1)^3} + \frac{\$45{,}000}{(1.1)^4} + \frac{\$45{,}000}{(1.1)^5} + \frac{\$45{,}000}{(1.1)^6} = \$158{,}683.95.$$

The payoff from graduate school is not large enough to justify the foregone income and tuition expense while Ralph is in school; he should therefore not go to school.

8. Suppose your uncle gave you an oil well like the one described in Section 15.8. (Marginal production cost is constant at $50.) The price of oil is currently $80 but is controlled by a cartel that accounts for a large fraction of total production. Should you produce and sell all your oil now or wait to produce? Explain your answer.

If a cartel accounts for a large fraction of total production, today's price minus marginal cost, $P^t - MC$ will rise at a rate less than the rate of interest. This is because the cartel will choose output such that *marginal revenue* minus *MC* rises at the rate of interest. Since price exceeds marginal revenue, $P^t - MC$ will rise at a rate less than the rate of interest. So, to maximize net present value, all your oil should be sold today, and your profits should be invested at the rate of interest.

9. You are planning to invest in fine wine. Each case costs $100, and you know from experience that the value of a case of wine held for t years is $100t^{1/2}$. One hundred cases of wine are available for sale, and the interest rate is 10%.

a. How many cases should you buy, how long should you wait to sell them, and how much money will you receive at the time of their sale?

One way to get the answer is to compare holding a case of wine to putting your $100 in the bank. The bank pays interest of 10%, while the wine increases in value at the rate of

$$\frac{\frac{d(value)}{dt}}{value} = \frac{50t^{-0.5}}{100t^{0.5}} = \frac{1}{2t}.$$

As long as $t < 5$, the return on wine is greater than or equal to 10%. When $t > 5$, the return on wine drops below 10%. Therefore, $t = 5$ is the time to switch your wealth from wine to the bank. Since each case is a good investment, you should buy all 100 cases.

Another way to get the answer is to use an advanced formula for calculating the *PDV*. This formula assumes continuous compounding rather than annual compounding. If you buy a case and sell it after *t* years, you pay $100 now and receive $100t^{0.5}$ when it is sold. The *NPV* of this investment is

$$NPV = -100 + e^{-rt}(100\ t^{0.5}) = -100 + e^{-0.1t}(100\ t^{0.5})$$

If you do buy a case, you will want to choose *t* to maximize the *NPV*. This implies differentiating with respect to *t* to obtain the necessary condition that

$$\frac{dNPV}{dt} = (e^{-0.1t})(50\ t^{-0.5}) - (0.1e^{-0.1t})(100\ t^{0.5}) = 0.$$

Multiply both sides of the first order condition by $e^{0.1t}$ to obtain

$$50\ t^{-0.5} - 10\ t^{0.5} = 0, \text{ or } t = 5.$$

If you held the case for five years, the *NPV* would be

$$-100 + e^{(-0.1)(5)}(100)(5^{0.5}) = \$35.62.$$

[Note that you get a slightly different *NPV* if you use annual discounting: $NPV = -100 + 223.61/(1.1)^5 = 38.84$.]

Therefore, you should buy a case and hold it for five years. The value of the case at the time of sale is $(\$100)(5^{0.5}) = \$223.61$. Again, you should buy all 100 cases.

b. Suppose that at the time of purchase, someone offers you $130 per case immediately. Should you take the offer?

You just bought the wine and are offered $130 for resale, so you would make an immediate profit of $30. However, if you hold the wine for five years, the *NPV* of your profit is $35.62 as shown in part a. Therefore, the *NPV* if you sell immediately rather than hold for five years is $30 − 35.62 = −$5.62, so you should not sell. You are better off holding the wine for five years rather than selling it for a quick profit.

c. How would your answers change if the interest rate were only 5%?

If the interest rate changes from 10% to 5%, the *NPV* calculation is

$$NPV = -100 + (e^{-0.05t})(100t^{0.5}).$$

As before, we maximize this expression:

$$\frac{dNPV}{dt} = (e^{-0.05t})(50\ t^{-0.5}) - (0.05e^{-0.05t})(100\ t^{0.5}) = 0.$$

By multiplying both sides of the first order condition by $e^{0.05t}$, it becomes

$$50t^{-0.5} - 5t^{0.5} = 0,$$

or $t = 10$. If we hold the case 10 years, *NPV* is

$$-100 + e^{(-0.05)(10)}(100)(10^{0.5}) = \$91.80.$$

With a lower interest rate, it pays to hold onto the wine longer before selling it, because the value of the wine is increasing at the same rate as before. Again, you should buy all 100 cases.

Now if someone offers you $130 immediately, you should definitely not sell. By selling you make a quick $30 profit per case, but by holding for 10 years the present value of your profit is $91.80 per case.

10. Reexamine the capital investment decision in the disposable diaper industry (Example 15.4) from the point of view of an incumbent firm. If P&G or Kimberly-Clark were to expand capacity by building three new plants, they would not need to spend $60 million on R&D before startup. How does this advantage affect the *NPV* calculations in Table 15.5 (page 577)? Is the investment profitable at a discount rate of 12%?

If the only change in the cash flow for an incumbent firm is the absence of an initial pre-2015 $60 million R&D expenditure, then the *NPV* calculations in Table 15.5 simply increase by $60 million for each discount rate:

Discount Rate:	0.05	0.10	0.15
NPV:	140.5	43.1	−15.1

To determine whether the investment is profitable at a discount rate of 12%, we must recalculate the *NPV*. At 12%,

$$NPV = -60 - \frac{93.4}{1.12} - \frac{56.6}{(1.12)^2} + \frac{40}{(1.12)^3} + \frac{40}{(1.12)^4} + \frac{40}{(1.12)^5} + \frac{40}{(1.12)^6} + \frac{40}{(1.12)^7} + \frac{40}{(1.12)^8}$$
$$+ \frac{40}{(1.12)^9} + \frac{40}{(1.12)^{10}} + \frac{40}{(1.12)^{11}} + \frac{40}{(1.12)^{12}} + \frac{40}{(1.12)^{13}} + \frac{40}{(1.12)^{14}} + \frac{40}{(1.12)^{15}} = \$16.3 \text{ million.}$$

Thus, the incumbent would find it profitable to expand capacity.

11. Suppose you can buy a new Toyota Corolla for $20,000 and sell it for $12,000 after six years. Alternatively, you can lease the car for $300 per month for three years and return it at the end of the three years. For simplification, assume that lease payments are made yearly instead of monthly – i.e., that they are $3600 per year for each of three years.

a. If the interest rate, *r*, is 4%, is it better to lease or buy the car?

Compute the *NPV* of each option. The *NPV* of buying the car is:

$$-20{,}000+\frac{12{,}000}{1.04^6}=-10{,}516.23.$$

The *NPV* of leasing the car, assuming you have to pay at the beginning of each year, is:

$$-3600-\frac{3600}{1.04}-\frac{3600}{1.04^2}=-10{,}389.94.$$

In this case, you are better off leasing the car because the *NPV* is higher (i.e., less negative).

b. Which is better if the interest rate is 12%?

Again, compute the *NPV* of each option. The *NPV* of buying the car is:

$$-20{,}000+\frac{12{,}000}{1.12^6}=-13{,}920.43.$$

The *NPV* of leasing the car is:

$$-3600-\frac{3600}{1.12}-\frac{3600}{1.12^2}=-9684.18.$$

You are still better off leasing the car because the *NPV* is higher.

c. At what interest rate would you be indifferent between buying and leasing the car?

You are indifferent between buying and leasing if the two *NPV*'s are equal or:

$$-20{,}000+\frac{12{,}000}{(1+r)^6}=-3600-\frac{3600}{(1+r)}-\frac{3600}{(1+r)^2}.$$

In this case, you need to solve for *r*. The easiest way to do this is to use a spreadsheet and calculate the two *NPV*'s for different values of *r*. Observe first that the interest rate will be something less than 4% given that at 4% the best option was to lease, and as the interest rate rose to 12% leasing became even a better option. The interest rate will be in the neighborhood of 3.8% as shown in the table below.

r	*NPV* Buy	*NPV* Lease
0.03	−9,950.19	−10,488.49
0.037	−10,350.41	−10,419.24
0.038	−10,406.06	−10,409.45
0.039	−10,461.33	−10,399.68
0.04	−10,516.23	−10,389.94

11 (alternate) Suppose you can buy a new Toyota Corolla for $20,000 and sell it for $12,000 after six years. Alternatively, you can lease the car for $300 per month for three years and return it at the end of the three years. Then you can lease another Corolla for another three years for the same monthly payment. For simplification, assume that lease payments are made yearly instead of monthly—i.e., that they are $3600 per year for each of three years. *Compare leasing the car for two consecutive three-year periods against owning the car for six years.*

a. If the interest rate, *r*, is 4%, is it better to lease or buy the car?

The *NPV* of owning the car is

$$-20,000+\frac{12,000}{1.04^6}=-10,516.23.$$

The *NPV* of leasing the car for six years is

$$-3600-\frac{3600}{1.04}-\frac{3600}{(1.04)^2}-\frac{3600}{(1.04)^3}-\frac{3600}{(1.04)^4}-\frac{3600}{(1.04)^5}=-19,626.56.$$

You should buy the car because the *NPV* is much higher (less negative).

b. Which is better if the interest rate is 12%?

The *NPV* of owning the car is

$$-20,000+\frac{12,000}{1.12^6}=-13,920.43.$$

The *NPV* of leasing the car is now

$$-3600-\frac{3600}{1.12}-\frac{3600}{(1.12)^2}-\frac{3600}{(1.12)^3}-\frac{3600}{(1.12)^4}-\frac{3600}{(1.12)^5}=-16,577.19.$$

You are still better off buying the car than leasing it.

c. At what interest rate would you be indifferent between buying and leasing the car?

You are indifferent between buying and leasing if the two *NPV*'s are equal:

$$-20,000+\frac{12,000}{(1+r)^6}=-3600-\frac{3600}{1+r}-\frac{3600}{(1+r)^2}-\frac{3600}{(1+r)^3}-\frac{3600}{(1+r)^4}-\frac{3600}{(1+r)^5}$$

In this case, you need to solve for *r*. The easiest way to do this is to use a spreadsheet and calculate the two *NPV*'s for different values of *r*. Observe first that the interest rate will be something greater than 12% given that at 4% the best option was to buy, and as the interest rate rose to 12% buying was still better but by a smaller margin. The interest rate that equates these two *NPV*'s is almost exactly 16.6% as the spreadsheet results below reveal.

r	***NPV* Buy**	***NPV* Lease**
0.16	–15,074.69	–15,387.46
0.165	–15,200.17	–15,251.27
0.166	–15,224.82	–15,224.35
0.167	–15,249.32	–15,197.52
0.17	–15,321.94	–15,117.65

12. A consumer faces the following decision: She can buy a computer for $1000 and $10 per month for Internet access for three years, or she can receive a $400 rebate on the computer (so that its cost is $600) but agree to pay $25 per month for three years for Internet access. For simplification, assume that the consumer pays the access fees yearly (i.e., $10 per month = $120 per year).

a. What should the consumer do if the interest rate is 3%?

Calculate the *NPV* in each case. Assuming that the Internet fees are paid at the beginning of each year, the *NPV* of the first option is

$$-1000-120-\frac{120}{1.03}-\frac{120}{1.03^2}=-1349.62.$$

The *NPV* of the second option with the rebate is

$$-600-300-\frac{300}{1.03}-\frac{300}{1.03^2}=-1474.04.$$

In this case, the first option gives a higher (less negative) *NPV*, so the consumer should pay the $1000 now and pay $10 per month for Internet access.

b. What if the interest rate is 17%?

Repeat the calculations in part a using 17%. The *NPV* of the first option is

$$-1000-120-\frac{120}{1.17}-\frac{120}{1.17^2}=-1310.23.$$

The *NPV* of the second option with the rebate is

$$-600-300-\frac{300}{1.17}-\frac{300}{1.17^2}=-1375.56.$$

In this case, the first option still gives a higher *NPV*, so the consumer should pay the $1000 now and pay $10 per month for Internet access.

c. At what interest rate will the consumer be indifferent between the two options?

The consumer is indifferent between the two options if the *NPV* of each option is the same. Therefore, set the *NPV*'s equal to each other and solve for r:

$$-1000-120-\frac{120}{1+r}-\frac{120}{(1+r)^2}=-600-300-\frac{300}{1+r}-\frac{300}{(1+r)^2}$$

$$220=\frac{180}{1+r}+\frac{180}{(1+r)^2}$$

$$220(1+r)^2=180(1+r)+180$$

$$220r^2+260r-140=0.$$

Using the quadratic formula to solve for the interest rate r results in a value of approximately $r = 40.2\%$. So the consumer would have to have an extremely high interest rate to make the rebate worthwhile.