

# Chapter 16

## General Equilibrium and Economic Efficiency

### ■ Questions for Review

1. **Why can feedback effects make a general equilibrium analysis substantially different from a partial equilibrium analysis?**

A partial equilibrium analysis focuses on the interaction of supply and demand in *one* market. It ignores the effect that shifts in supply and demand in that market might have on the markets for complements, substitutes, and inputs. A general equilibrium analysis takes feedback effects into account, where a price or quantity adjustment in one market can cause a price or quantity adjustment in related markets. Ignoring these feedback effects can lead to inaccurate forecasts of the full effect of changes in one market. An initial shift in demand in one market, for example, can cause a shift in demand in a related market, which can then cause a second shift in demand in the first market. A partial equilibrium analysis will stop at the initial shift whereas a general equilibrium analysis will continue on and on, incorporating possible shifts in demand in related markets and the ensuing feedback effects on the first market.

2. **In the Edgeworth box diagram, explain how one point can simultaneously represent the market baskets owned by two consumers.**

The Edgeworth box diagram allows us to represent the distribution of two goods between two individuals. The box is formed by inverting the indifference curves of one individual and superimposing these on the indifference curves of another individual. The sides of the box represent the total amounts of the two goods available to the consumers. For example, the height of the vertical axis represents the total amount of, say, clothing that is available. For any point in the diagram, the vertical distance between the point and the bottom of the box is the amount of clothing that one consumer has, and the vertical distance between the point and the top of the box is the amount of clothing owned by the other consumer. Likewise, the horizontal distance between the point and the left side of the box represents the amount of the other good, say food, belonging to the first consumer and the distance to the right side of the box is the amount of food the other consumer has. Therefore, each point in the box represents a different allocation of the two goods between the two individuals.

3. **In the analysis of exchange using the Edgeworth box diagram, explain why both consumers' marginal rates of substitution are equal at every point on the contract curve.**

The contract curve in an Edgeworth box diagram is the set of points where the indifference curves of the two individuals are tangent. We know that the marginal rate of substitution is equal to the (negative) slope of the indifference curves. Also, when two curves are tangent at a point, their slopes are equal. Thus, by defining the contract curve as a set of indifference curve tangencies, the marginal rates of substitution between the two goods are equal for the two individuals at every point on the contract curve, assuming convex indifference curves.

**4. “Because all points on a contract curve are efficient, they are all equally desirable from a social point of view.” Do you agree with this statement? Explain.**

If society is only concerned with efficiency and not with equity, then all points on the contract curve are equally desirable. Since it is impossible to make comparisons of utility between individuals, economics focuses on efficiency. But most societies are also concerned with equity (i.e., whether the final allocation is fair, however that might be defined). In this case, all points on the contract curve are not equally desirable.

**5. How does the utility possibilities frontier relate to the contract curve?**

The utility possibilities frontier plots the utility levels of the two consumers for all points on the contract curve. One consumer's utility is plotted on the vertical axis and the other consumer's utility is shown on the horizontal axis. While points that lie between the origin and the utility possibilities frontier are feasible, they are not efficient because they are not on the frontier, and trades are possible that will make one or both individuals better off without making anyone worse off. Points outside the frontier are not feasible unless the individuals are given greater amounts of one or both goods.

**6. In the Edgeworth production box diagram, what conditions must hold for an allocation to be on the production contract curve? Why is a competitive equilibrium on the contract curve?**

When constructing an Edgeworth box for the production of two goods with two inputs, each point in the box represents an allocation of the two inputs between the two production processes. With production, each point can be ordered according to the total output. These points lie on isoquants instead of on indifference curves. Since each point simultaneously represents the allocation of inputs to two production processes, it lies on two isoquants, one for each production process. The production contract curve represents all combinations of inputs that are technically efficient. Thus there would be no way to increase the output of one good without decreasing the output of the other good.

A competitive equilibrium is one point on the production-contract curve. It is the intersection of the production-contract curve and a line passing through the initial allocation with a slope equal to the ratio of input prices. (The ratio of prices dictates the rates at which inputs can be traded in the market.) For a competitive equilibrium to hold, each producer must use inputs so that the slopes of the isoquants are equal to one another (assuming convex isoquants) and also equal to the ratio of the prices of the two inputs. Therefore, the competitive equilibrium is efficient in production.

**7. How is the production possibilities frontier related to the production contract curve?**

We can graph the quantities of each good produced (each point in the Edgeworth box) on a graph, where the vertical axis represents the output of one good and the horizontal axis represents the output of the other good. The production contract curve is represented in this graph as the production possibilities frontier. It is analogous to the utility possibilities frontier for consumers. Points inside the frontier are feasible but inefficient; points outside the frontier are infeasible and attainable only when more inputs become available or technological change increases productivity. Points on the production possibilities frontier are the same as those on the production contract curve. The difference is that the production contract curve measures inputs on the axes and the production possibilities frontier measures outputs on the axes.

**8. What is the marginal rate of transformation (*MRT*)? Explain why the *MRT* of one good for another is equal to the ratio of the marginal costs of producing the two goods.**

The marginal rate of transformation, *MRT*, is equal to the absolute value of the slope of the production possibilities frontier, *PPF*, and measures how much of one output must be given up to produce one more unit of the other output. The total cost of all inputs is the same at each point on the *PPF* because

the amount of each input is fixed. Therefore, when we move down along the frontier, the cost of

producing one output is reduced by the same amount as the cost of producing the other output is increased. Suppose the  $MRT$  is 4, then we must give up 4 units of the output on the vertical axis to get one more unit of the output on the horizontal axis.

This means that the total cost of producing the 4 units is the same as the total cost of producing the one unit, so the marginal cost of the good on the horizontal axis is 4 times the marginal cost of the good on the vertical axis.

**9. Explain why goods will not be distributed efficiently among consumers if the  $MRT$  is not equal to the consumers' marginal rate of substitution.**

If the marginal rate of transformation,  $MRT$ , is not equal to the marginal rate of substitution,  $MRS$ , we could reallocate inputs in producing output to leave the consumers and producers better off. If the  $MRS$  is greater than the  $MRT$ , then consumers are willing to pay more for another unit of a good than it will cost the producers to produce it. Both consumers and producers can therefore be made better off by arranging a trade somewhere between what consumers are willing to pay and what the producers have to pay to produce the extra unit. Note also that when  $MRT \neq MRS$ , the ratio of marginal costs will not be equal to the ratio of prices. This means that one good is being sold at a price below marginal cost and one good is being sold at a price above marginal cost. We should increase the output of the good whose price is above marginal cost and reduce the output of the other good whose price is below marginal cost.

**10. Why can free trade between two countries make consumers of both countries better off?**

Free trade between two countries expands each country's effective production possibilities frontier and allows each country to consume at a point above its original production possibilities frontier. Since each country has a comparative advantage in the production of some good or service, trade allows each country to specialize in the area where it has this advantage. It trades these outputs for those more cheaply produced in another country. Therefore, specialization benefits consumers in both countries.

**11. If Country  $A$  has an absolute advantage in the production of two goods compared to Country  $B$ , then it is not in Country  $A$ 's best interest to trade with Country  $B$ . True or false? Explain.**

This statement is false. A country can have an absolute advantage in the production of all goods, but it still will have a comparative advantage only in the production of some goods. Suppose Country  $A$  requires 4 units of labor to produce good 1 and 8 units of labor to produce good 2, whereas Country  $B$  requires 8 units and 12 units respectively. Country  $A$  can produce both goods more cheaply, so it has an absolute advantage in the production of both goods. However, trade is based on comparative advantage, which depends on how much of one good must be given up for one more unit of the other good. Here, Country  $A$  must give up 2 units of good 1 for another unit of good 2 whereas Country  $B$  must give up only 1.5 units of good 1 for another unit of good 2. Country  $B$  therefore has a comparative advantage in producing good 2. Likewise, Country  $A$  has a comparative advantage in producing good 1 and should trade good 1 to Country  $B$  for good 2.

**12. Do you agree or disagree with each of the following statements? Explain.**

- a. If it is possible to exchange 3 pounds of cheese for 2 bottles of wine, then the price of cheese is  $2/3$  the price of wine.**

This is a true statement. If 3 pounds of cheese can be exchanged for 2 bottles of wine, then cheese must have a cost that is  $2/3$  that of wine and the price of cheese will be  $2/3$  the price of wine.

- b. A country can only gain from trade if it can produce a good at a lower absolute cost than its trading partner.**

This statement is false. Trade is based on comparative advantage and not absolute advantage. A country can be absolutely worse at producing all goods, but will nonetheless be comparatively better at producing one or more goods and will gain by producing and trading these for other goods.

- c. If there are constant marginal and average costs of production, then it is in a country's best interest to specialize completely in the production of some goods but to import others.**

This is a true statement as long as the production costs differ across countries. If Country *A* has to always give up 2 units of good 1 for another unit of good 2 and Country *B* always has to give up 3 units of good 1 for another unit of good 2, then Country *A* should produce enough of good 2 to satisfy demand in both countries. Likewise, Country *B* should produce enough of good 1 to satisfy demand in both countries (its cost is 1/3 vs. 1/2 for Country *A*). Note that in reality, the marginal and average costs will tend to rise after a while as more resources are devoted to any given industry, so marginal and average costs are unlikely to remain constant regardless of output.

- d. Assuming that labor is the only input, if the opportunity cost of producing a yard of cloth is 3 bushels of wheat per yard, then wheat must require 3 times as much labor per unit produced as cloth.**

This statement is false. If the country must give up 3 bushels of wheat to produce another yard of cloth then the same labor resources that are producing the 3 bushels of wheat are required to produce the one yard of cloth. Therefore the yard of cloth requires three times as much labor as a bushel of wheat.

- 13. What are the four major sources of market failure? Explain briefly why each prevents the competitive market from operating efficiently.**

The four major sources of market failure are market power, incomplete information, externalities, and public goods. We know from the study of market structures that *market power* leads to situations where price does not equal marginal cost. In these situations, the producer is producing too little. Consumers would be made better off if the good were produced under a competitive market structure, thereby lowering price until price were equal to marginal cost. *Incomplete information* implies that prices do not reflect either the marginal cost of production or the change in utility from changes in consumption. Either too much or too little (at the extreme, none) is produced and consumed. *Externalities* occur when a consumption or production activity influences other consumption or production activities, and these effects are not reflected in market prices. *Public goods* are goods that can be consumed at prices below marginal cost (at the extreme, freely) because consumers cannot be excluded. In these four cases, prices do not send the proper signals to either producers or consumers to increase or decrease production or consumption. Thus the market mechanism cannot equate social marginal costs with social marginal benefits.

## ■ Exercises

- 1. Suppose gold (*G*) and silver (*S*) are substitutes for each other because both serve as hedges against inflation. Suppose also that the supplies of both are fixed in the short run ( $Q_G = 75$  and  $Q_S = 300$ ) and that the demands for gold and silver are given by the following equations:**

$$P_G = 975 - Q_G + 0.5P_S \quad \text{and} \quad P_S = 600 - Q_S + 0.5P_G.$$

**a. What are the equilibrium prices of gold and silver?**

In the short run, the quantity of gold,  $Q_G$ , is fixed at 75. Substitute  $Q_G$  into the demand equation for gold:

$$P_G = 975 - 75 + 0.5P_S = 900 + 0.5P_S.$$

In the short run, the quantity of silver,  $Q_S$ , is fixed at 300. Substituting  $Q_S$  into the demand equation for silver:

$$P_S = 600 - 300 + 0.5P_G = 300 + 0.5P_G.$$

Since we now have two equations and two unknowns, substitute the price of gold into the silver demand function and solve for the price of silver:

$$P_S = 300 + (0.5)(900 + 0.5P_S), \text{ or } P_S = \$1000.$$

Now substitute the price of silver into the demand function for gold:

$$P_G = 900 + (0.5)(1000) = \$1400.$$

**b. What if a new discovery of gold doubles the quantity supplied to 150. How will this discovery affect the prices of both gold and silver?**

When the quantity of gold increases by 75 units from 75 to 150, both prices fall. To see this, resolve the system of equations:

$$P_G = 975 - 150 + 0.5P_S, \text{ or } P_G = 825 + (0.5)(300 + 0.5P_G), \text{ or } P_G = \$1300.$$

The price of silver is equal to:  $P_S = 300 + (0.5)(1300) = \$950$ .

**2. Using general equilibrium analysis, and taking into account feedback effects, analyze the following:****a. The likely effects of outbreaks of disease on chicken farms on the markets for chicken and pork.**

If consumers are worried about the quality of the chicken, then they may choose to consume pork instead. This will shift the demand curve for chicken down and to the left and the demand curve for pork up and to the right. Feedback effects will partially offset the shift in the chicken demand curve, because some people may switch back to chicken when the price of pork rises. This will shift the demand curve for chicken back to the right by some amount, raising the price of chicken and shifting the demand curve for pork a bit further to the right. Overall, we would expect the price of chicken to drop, but not by as much as if there were no feedback effects. The price of pork will increase, perhaps by more than the increase in the absence of feedback effects.

The other possibility is that the outbreaks of disease cause a reduction in supply of chicken. This would increase the price of chicken, which would then lead to an increase in demand for pork. The price of pork would increase in response, which would boost the demand for chicken by some (probably small) amount, raising the price of chicken again. This would increase the demand for pork some more, and so on. Ultimately these effects will peter out, but the final increases in prices will be greater than if there had been no feedback effects.

**b. The effects of increased taxes on airline tickets on travel to major tourist destinations such as Florida and California and on the hotel rooms in those destinations.**

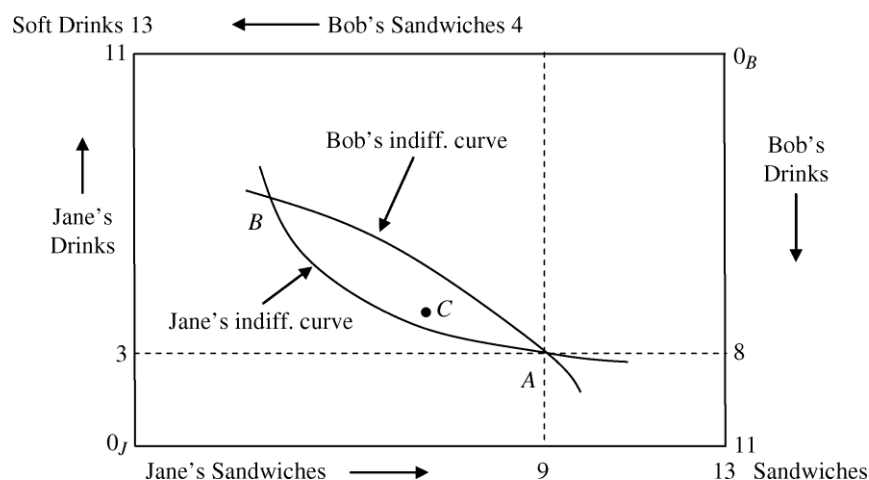
The increase in tax raises the price of airline tickets, making it more costly to fly. The resulting increase in ticket prices would reduce the quantity of airline tickets sold. This in turn would reduce the demand for hotel rooms by out-of-town visitors, causing the demand curve for hotel

rooms to shift down and to the left and reducing the price of hotel rooms. For the feedback effects, the lower price for hotel rooms may encourage some consumers to travel more, in which case the demand for airline tickets would shift back up and to the right, partially offsetting the initial decline in quantity demanded. The increase in airline demand would increase the demand for hotel rooms, causing hotel room prices to increase somewhat. In the end, we would still expect reduced quantities of both airline ticket sales and hotel rooms, higher airline ticket prices (due to the tax) and reduced prices for hotel rooms.

3. Jane has 3 liters of soft drinks and 9 sandwiches. Bob, on the other hand, has 8 liters of soft drinks and 4 sandwiches. With these endowments, Jane's marginal rate of substitution ( $MRS$ ) of soft drinks for sandwiches is 4 and Bob's  $MRS$  is equal to 2. Draw an Edgeworth box diagram to show whether this allocation of resources is efficient. If it is, explain why. If it is not, what exchanges will make both parties better off?

Given that  $MRS_{Bob} \neq MRS_{Jane}$ , the current allocation of resources is inefficient. Jane and Bob could trade to make one of them better off without making the other worse off. Although we do not know the exact shape of Jane and Bob's indifference curves, we do know the slope of both indifference curves *at the current allocation*, because we know that  $MRS_{Jane} = 4$  and  $MRS_{Bob} = 2$ . At the current allocation (point  $A$  in the diagram), Jane is willing to trade 4 sandwiches for 1 drink, or she will give up 1 drink in exchange for 4 sandwiches. Bob is willing to trade 2 sandwiches for 1 drink, or he will give up 1 drink in exchange for 2 sandwiches. Jane will give 4 sandwiches for 1 drink while Bob is willing to accept only 2 sandwiches in exchange for 1 drink.

Any exchange that leaves both parties inside the lens-shaped area between points  $A$  and  $B$  will make both better off. For example, if Jane gives Bob 3 sandwiches for 1 drink, they will be at point  $C$ . Jane is better off because she was willing to give up 4 sandwiches but only had to give up 3. Bob is better off because he was willing to accept 2 sandwiches and actually received 3. Jane ends up with 4 drinks and 6 sandwiches and Bob ends up with 7 drinks and 7 sandwiches, and both are better off than at point  $A$ .



4. Jennifer and Drew consume orange juice and coffee. Jennifer's  $MRS$  of orange juice for coffee is 1 and Drew's  $MRS$  of orange juice for coffee is 3. If the price of orange juice is \$2 and the price of coffee is \$3, which market is in excess demand? What do you expect to happen to the prices of the two goods?

Jennifer is willing to trade 1 coffee for 1 orange juice. Drew is willing to trade 3 coffees for 1 orange juice. In the market, it is possible to trade  $2/3$  of a coffee for an orange juice. Both will find it optimal

to trade coffee in exchange for orange juice since they are willing to give up more for orange juice than they have to. There is an excess demand for orange juice and an excess supply of coffee. The price of coffee will decrease and the price of orange juice will rise. Notice also that at the given rates of *MRS* and prices, both Jennifer and Drew have a higher marginal utility per dollar for orange juice as compared to coffee.

5. Fill in the missing information in the following tables. For each table, use the information provided to identify a possible trade. Then identify the final allocation and a possible value for the *MRS* at the efficient solution. (Note: There is more than one correct answer.) Illustrate your results using Edgeworth Box diagrams.

- a. Norman's *MRS* of food for clothing is 1 and Gina's *MRS* of food for clothing is 4:

Individual	Initial Allocation	Trade	Final Allocation
Norman	6F, 2C	1F for 3C	5F, 5C
Gina	1F, 8C	3C for 1F	2F, 5C

Gina will give 4 clothing for 1 food while Norman is willing to accept 1 clothing for 1 food. If they settle on 2 or 3 units of clothing for one unit of food they will both be better off. Let's say they settle on 3 units of clothing for 1 unit of food. Gina will give up 3 units of clothing and receive 1 unit of food so her final allocation is 2F and 5C. Norman will give up 1 food and gain 3 clothing so his final allocation is 5F and 5C. Gina's *MRS* will decrease and Norman's will increase, so given they must be equal in the end, it will be somewhere between 1 and 4, in absolute value terms. So a possible value for both individual's *MRS* at the efficient solution is 2.5.

- b. Michael's *MRS* of food for clothing is 1/2 and Kelly's *MRS* of food for clothing is 3.

Individual	Initial Allocation	Trade	Final Allocation
Michael	10F, 3C	3F for 3C	7F, 6C
Kelly	5F, 15C	3C for 3F	8F, 12C

Michael will give 1/2 clothing for 1 food (or 2 food for 1 clothing) while Kelly is willing to trade 3 clothing for 1 food (i.e., she will accept 1/3 food for 1 clothing). If they settle on a trading rate of 1 unit of food for 1 unit of clothing they will both be better off. Suppose Michael gives up 3 units of food and receives 3 units of clothing, so his final allocation is 7F and 6C. Kelly will thus give up 3 units of clothing and gain 3 units of food, so her final allocation is 8F and 12C. Kelly's *MRS* will decrease and Michael's will increase, so given they must be equal in equilibrium, their *MRS* value will be somewhere between 3 and 1/2. So a possible value for each person's *MRS* is 2 at the efficient solution.

6. In the analysis of an exchange between two people, suppose both people have identical preferences. Will the contract curve be a straight line? Explain. Can you think of a counterexample?

Given that the contract curve intersects the origin for each individual, a straight line contract curve would be a diagonal line running from one origin to the other. The slope of this line is  $\frac{Y}{X}$ , where  $Y$  is the total amount of the good on the vertical axis and  $X$  is the total amount of the good on the horizontal axis.  $(x_1, y_1)$  are the amounts of the two goods allocated to one individual and  $(x_2, y_2) = (X - x_1, Y - y_1)$  are the amounts of the two goods allocated to the other individual. A straight line contract curve would have the equation

$$y_1 = \left(\frac{Y}{X}\right)x_1.$$

We need to show that when the marginal rates of substitution for the two individuals are equal ( $MRS^1 = MRS^2$ ), the point lies on this linear contract curve.

For example, consider the utility function  $U = x_i^2 y_i$ . Then

$$MRS^i = \frac{MU_x^i}{MU_y^i} = \frac{2x_i y_i}{x_i^2} = \frac{2y_i}{x_i}.$$

If  $MRS^1$  equals  $MRS^2$ , then

$$\left(\frac{2y_1}{x_1}\right) = \left(\frac{2y_2}{x_2}\right).$$

Is this point on the contract curve? Yes, because

$$x_2 = X - x_1 \text{ and } y_2 = Y - y_1, \text{ so}$$

$$2\left(\frac{y_1}{x_1}\right) = 2\left(\frac{Y - y_1}{X - x_1}\right).$$

This means that

$$\begin{aligned} \frac{y_1(X - x_1)}{x_1} &= Y - y_1, \quad \text{or} \quad \frac{y_1 X - y_1 x_1}{x_1} = Y - y_1, \text{ and} \\ \frac{y_1 X}{x_1} - y_1 &= Y - y_1, \quad \text{or} \quad \frac{y_1 X}{x_1} = Y, \quad \text{or} \quad y_1 = \left(\frac{Y}{X}\right)x_1. \end{aligned}$$

With this utility function we find  $MRS^1 = MRS^2$ , and the contract curve is a straight line. This should be the case for utility functions with strictly convex indifference curves. However, if the consumers' preferences are such that the goods are perfect complements or perfect substitutes, the contract curve is not necessarily well defined. For example, with perfect substitutes the indifference curves are straight lines, so every point is a point of tangency between indifference curves, and thus there is no unique contract curve. With perfect complements, there may be a "thick" contract path, not a single line.

## 7. Give an example of conditions when the production possibilities frontier might not be concave.

The production possibilities frontier is concave if at least one of the production functions exhibits decreasing returns to scale. If both production functions exhibit constant returns to scale, then the production possibilities frontier is a straight line. If both production functions exhibit increasing returns to scale, then the production function is convex. The following numerical examples can be used to illustrate this concept. Assume that  $L = 5$  is the fixed labor input, and  $X$  and  $Y$  are the two goods. The first example is the decreasing returns to scale case, the second example is the constant returns to scale case, and the third example is the increasing returns to scale case. Note further that it is not necessary that both products have identical production functions.

Product X		Product Y		PPF	
L	X	L	Y	X	Y
0	0	0	0	0	30
1	10	1	10	10	28
2	18	2	18	18	24



3	24	3	24	24	18
4	28	4	28	28	10
5	30	5	30	30	0

Product X		Product Y		PPF	
L	X	L	Y	X	Y
0	0	0	0	0	50
1	10	1	10	10	40
2	20	2	20	20	30
3	30	3	30	30	20
4	40	4	40	40	10
5	50	5	50	50	0

Product X		Product Y		PPF	
L	X	L	Y	X	Y
0	0	0	0	0	80
1	10	1	10	10	58
2	22	2	22	22	38
3	38	3	38	38	22
4	58	4	58	58	10
5	80	5	80	80	0

8. A monopsonist buys labor for less than the competitive wage. What type of inefficiency will this use of monopsony power cause? How would your answer change if the monopsonist in the labor market were also a monopolist in the output market?

When market power exists, the market will not allocate resources efficiently. The wage paid by a monopsonist is below the competitive wage, so too little labor is used in the production process and hence too little output is produced. If the firm is also a monopolist in the output market, the firm's  $MRP_L$  curve, which is its demand curve for labor, lies below the  $MRP_L$  for a competitive firm. Thus, the monopolist's demand for labor is less than for a competitive firm. The result is that even less labor is hired and less output produced. So having monopoly power in the output market exacerbates the inefficiencies in both the labor market and the output market.

9. The Acme Corporation produces  $x$  and  $y$  units of goods Alpha and Beta, respectively.
- a. Use a production possibility frontier to explain how the willingness to produce more or less Alpha depends on the marginal rate of transformation of Alpha for Beta.

The production possibilities frontier shows all efficient combinations of Alpha and Beta. The marginal rate of transformation of Alpha for Beta is the absolute value of the slope of the production possibilities frontier. The slope measures the marginal cost of producing one good relative to the marginal cost of producing the other. To increase  $x$ , the units of Alpha, Acme must release inputs in the production of Beta and redirect them to producing Alpha. The rate at which it can efficiently substitute away from Beta to Alpha is given by the marginal rate of transformation.

- b. Consider two cases of production extremes: (i) Acme produces zero units of Alpha initially, or (ii) Acme produces zero units of Beta initially. If Acme always tries to stay on its production possibility frontier, describe the initial positions of cases (i) and (ii). What happens as the Acme Corporation begins to produce *both* goods?

In situation (i), Acme produces no units of Alpha and the maximum possible amount of Beta. In situation (ii), it produces the maximum possible amount of Alpha and no units of Beta. Consider what happens in case (i) as Acme starts producing some Alpha along with Beta. Assuming the usual concave shape for the production possibility frontier, Acme will initially enjoy large increases in Alpha production without giving up much Beta. As it produces more and more of Alpha, however, it will have to give up larger and larger increments of Beta because the *MRT* increases (assuming Alpha is plotted on the vertical axis). The same reasoning applies in case (ii) with the roles of Alpha and Beta reversed.

10. In the context of our analysis of the Edgeworth production box, suppose that a new invention changes a constant-returns-to-scale food production process into one that exhibits sharply increasing returns. How does this change affect the production contract curve?

In an Edgeworth production box, the production contract curve is made up of the points of tangency between the isoquants of the two production processes. A change from a constant-returns-to-scale production process to a sharply-increasing-returns-to-scale production process does not necessarily imply a change in the shape of the isoquants. One can simply redefine the quantities associated with each isoquant such that proportional increases in inputs yield greater-than-proportional increases in outputs. Under this scenario, the marginal rate of technical substitution would not change. Thus, there would be no change in the production contract curve.

However, if the change to a sharply-increasing-returns-to-scale technology involved a change in the trade-off between the two inputs (a change in the shape of the isoquants), then the production contract curve would change.

For example, suppose the original production functions for food and the other good (call it clothing) were both  $Q = LK$  with  $MRTS = \frac{K}{L}$ , then the shape of the isoquants would not change if the new production function for food were  $Q = L^2K^2$  with  $MRTS = \frac{K}{L}$ . In both cases the production contract curve would be linear. The shape of the production contract curve would become nonlinear, however, if the new production function for food were  $Q = L^2K$  with  $MRTS = 2\left(\frac{K}{L}\right)$ , assuming the production function for clothing was unchanged.

11. Suppose that Country A and Country B both produce wine and cheese. Country A has 800 units of available labor, while Country B has 600 units. Prior to trade, Country A consumes 40 pounds of cheese and 8 bottles of wine, and Country B consumes 30 pounds of cheese and 10 bottles of wine.

	Country A	Country B
Labor per pound cheese	10	10
Labor per bottle wine	50	30

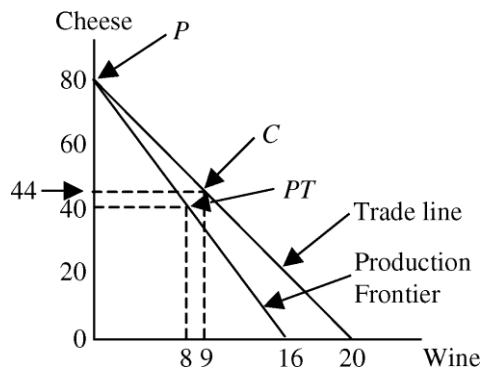
- a. Which country has a comparative advantage in the production of each good? Explain.

To produce another bottle of wine, Country A needs 50 units of labor, and must therefore produce 5 fewer pounds of cheese. The opportunity cost of a bottle of wine is therefore 5 pounds of

cheese. For Country *B* the opportunity cost of a bottle of wine is 3 pounds of cheese. Since Country *B* has a lower opportunity cost, it has the comparative advantage and should produce the wine and Country *A* should produce the cheese. Country *A* has the comparative advantage in producing cheese because the opportunity cost of cheese in *A* is 1/5 of a bottle of wine while in Country *B* it is 1/3 of a bottle of wine.

- b. Determine the production possibilities curve for each country, both graphically and algebraically. (Label the pre-trade production point *PT* and the post-trade production point *P*.)**

Country *A*'s production frontier is given by  $10C + 50W = 800$ , or  $C = 80 - 5W$ , and Country *B*'s production frontier is  $10C + 30W = 600$ , or  $C = 60 - 3W$ . The slope of the frontier for Country *A* is  $-5$ , which is the opportunity cost of wine in terms of cheese. Therefore, in Country *A* the price of wine is 5 and in Country *B* the price of wine is 3. Country *A*'s production possibilities curve is the steeper line in the diagram. If Country *A* uses all its labor to make cheese, it can produce 80 pounds of cheese; if it uses all its labor to make wine, it can make 16 bottles of wine.



Before trade, Country *A* produces the combination of cheese and wine at point *PT* (40C and 8W). After trade, the price of cheese will settle somewhere between 3 and 5. Country *A* will produce only cheese and Country *B* will produce only wine, and they will trade with each other for the other good. Country *A* will produce 80 pounds of cheese at point *P*. Each can consume at a point on the trade line that lies above and outside the production frontier. If the terms of trade result in a wine price of 4, Country *A* will be able to consume combinations of cheese and wine along the shifted-out line (the trade line) in the diagram.

- c. Given that 36 pounds of cheese and 9 bottles of wine are traded, label the post-trade consumption point *C*.**

See the graph for Country *A* above. Before trade the country consumed and produced at point *PT*, with 40 pounds of cheese and 8 bottles of wine. After trade, Country *A* will completely specialize in the production of cheese and will produce at point *P*. Given the quantities traded, Country *A* will consume  $80 - 36 = 44$  pounds of cheese and  $0 + 9 = 9$  bottles of wine. This is point *C* on the graph. The graph for Country *B* is similar except that Country *B* will produce only wine. Country *B*'s trade line will be the same as the one for Country *A*, and Country *B* will consume  $0 + 36 = 36$  pounds of cheese and  $20 - 9 = 11$  bottles of wine.

- d. Prove that both countries have gained from trade.**

Both countries have gained from trade because they can now both consume more of both goods than they could before trade. Graphically we can see this by noticing that the trade line lies to the right of the production frontier. After trade, each country can consume on the trade line and can consume more of both goods. Numerically, Country *A* consumes 4 more pounds of cheese and 1

more bottle of wine after trade as compared to pre-trade, and Country *B* consumes 6 more pounds of cheese and 1 more bottle of wine.

**e. What is the slope of the price line at which trade occurs?**

The slope is  $-4$  because 36 pounds of cheese were traded for 9 bottles of wine, so 4 pounds of cheese were given up for every bottle of wine. Therefore, the price of each bottle of wine is 4 (in terms of cheese).

**12. Suppose a bakery has 16 employees to be designated as bread bakers (*B*) and cake bakers (*C*), so that  $B + C = 16$ . Draw the production possibilities frontier for bread (*y*) and cakes (*x*) for the following production functions:**

**a.**  $y = 2B^{0.5}$  and  $x = C^{0.5}$

There are two ways to do this. First is to calculate a few points on the production possibilities frontier (*PPF*) and then draw the curve. Because there are 16 employees,  $B = 16 - C$ . So, for any value of *C* we can calculate the amount of cake produced by *C* employees and the amount of bread produced by  $16 - C$  employees using the production functions for *x* and *y*. The table below does this for selected values of *C*.

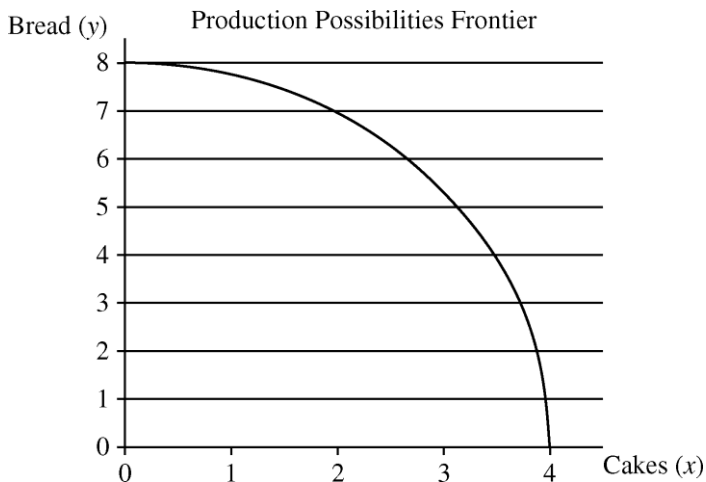
Cake Bakers ( <i>C</i> )	Cake ( <i>x</i> )	Bread ( <i>y</i> )
0	0	8.00
4	2.00	6.93
8	2.83	5.66
12	3.46	4.00
16	4.00	0

A more elegant method is to solve for the *PPF* function.

Since  $B = 16 - C$ , we can write  $y = 2(16 - C)^{0.5}$ . And since  $x = C^{0.5}$ , it follows that  $C = x^2$ .

Substituting for *C* in the expression for *y*, we get  $y = 2(16 - x^2)^{0.5}$ , which can also be written as  $y = 2\sqrt{16 - x^2}$ . This is the equation for the *PPF*. You can check that you get the same set of *x* and *y* values in the table above using this equation.

The plot of the *PPF* is given in the diagram below. It has the usual concave shape.



b.  $y = B$  and  $x = 2C^{.5}$

Since  $B = 16 - C$ , we can write the equation for  $y$  as  $y = 16 - C$ . Now solve  $x$ 's production function for  $C$ , which yields  $C = \left(\frac{x}{2}\right)^2$ . Substitute into the expression for  $y$  to get the function for the *PPF*:  $y = 16 - \left(\frac{x}{2}\right)^2$ . This *PPF* is plotted below. Note that even though the production function for bread ( $y$ ) is linear, the *PPF* is concave because the production function for cake ( $x$ ) has diminishing returns to scale.

