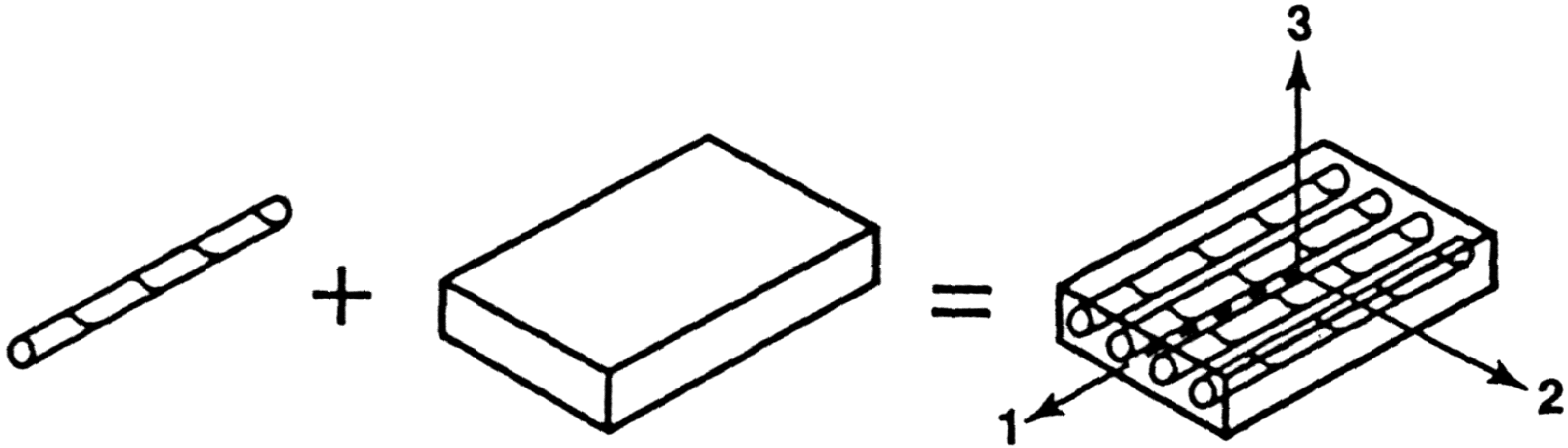
COSTRUZIONI NAVALI II

(materiali compositi: analisi micromeccanica)

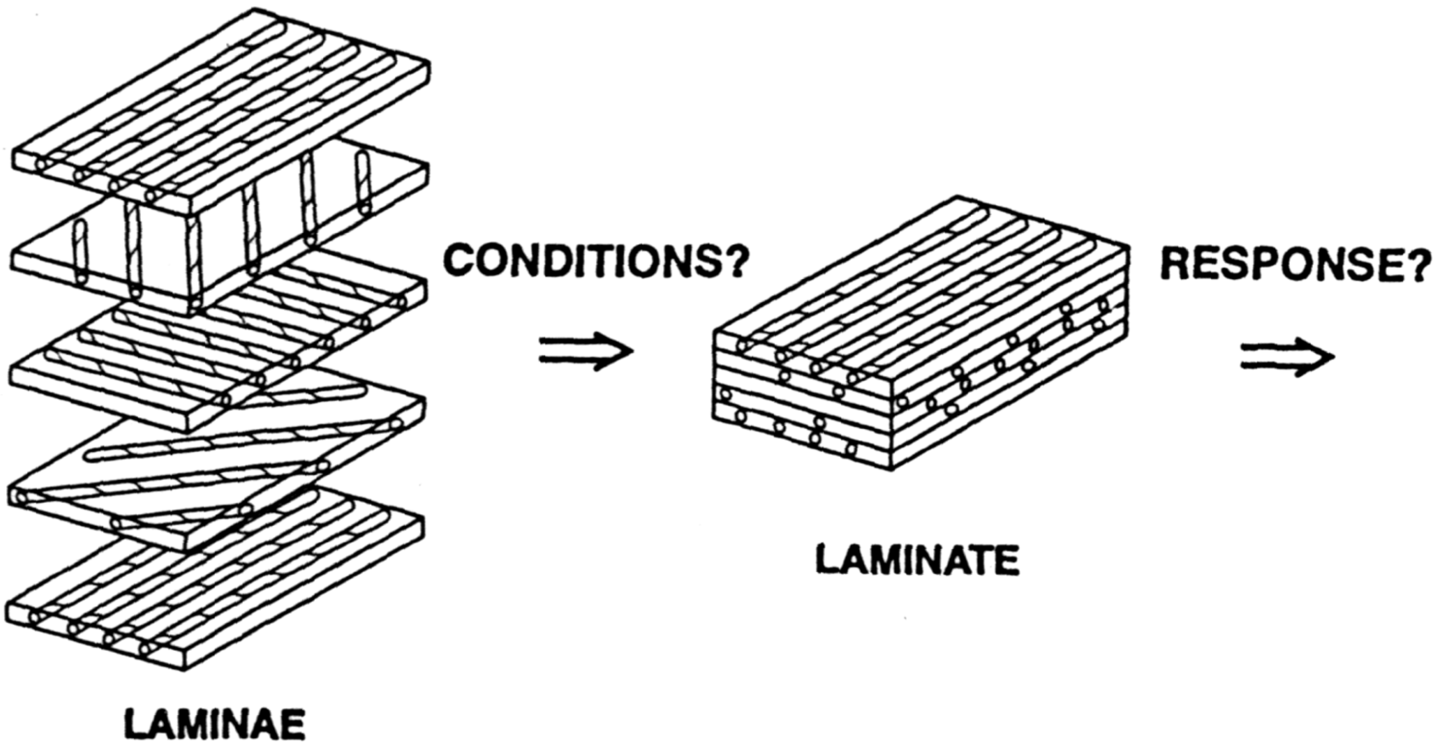
MICROMECHANICAL BEHAVIOR OF A LAMINA



FIBER	MATRIX	COMPOSITE MATERIAL
E_f	E_m	$E_{c_i}=?$
X_f	X_m	$X_{c_i}=?$

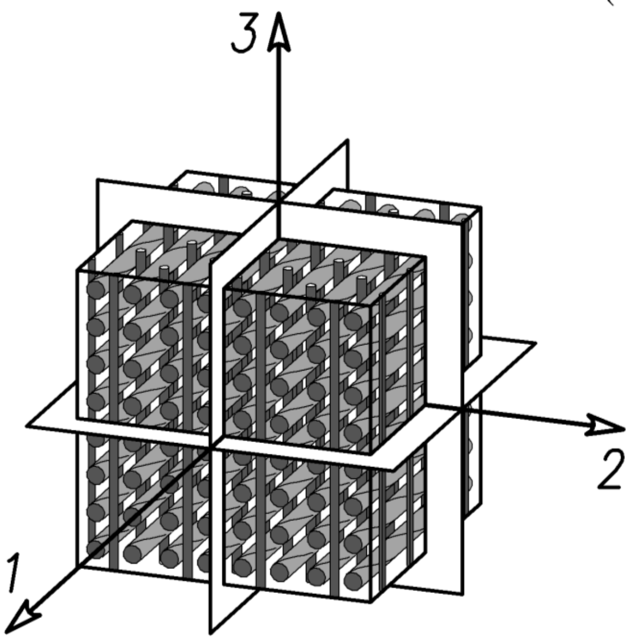
Basic Question of Micromechanics

MACROMECHANICAL BEHAVIOR OF A LAMINATE



The Basic Questions of Laminate Analysis

Materiale ORTOTROPO
(9 costanti elastiche indipendenti)

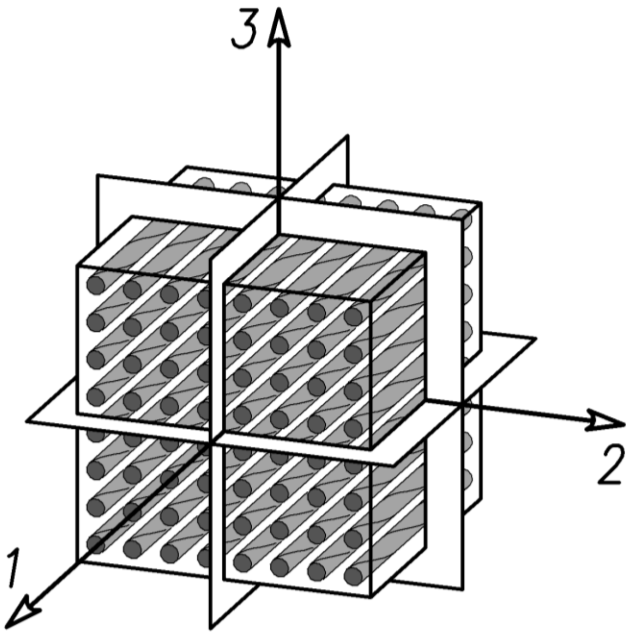


E_1	E_2	E_3
G_{12}	G_{23}	G_{31}
ν_{12}	ν_{23}	ν_{31}
ν_{21}	ν_{32}	ν_{13}

$$\begin{cases} \nu_{21} \implies \dfrac{\nu_{21}}{E_2} = \dfrac{\nu_{12}}{E_1} \\ \nu_{32} \implies \dfrac{\nu_{32}}{E_3} = \dfrac{\nu_{23}}{E_2} \\ \nu_{13} \implies \dfrac{\nu_{13}}{E_1} = \dfrac{\nu_{31}}{E_3} \end{cases}$$

Materiale ORTOTROPO TRASVERSALMENTE ISOTROPO

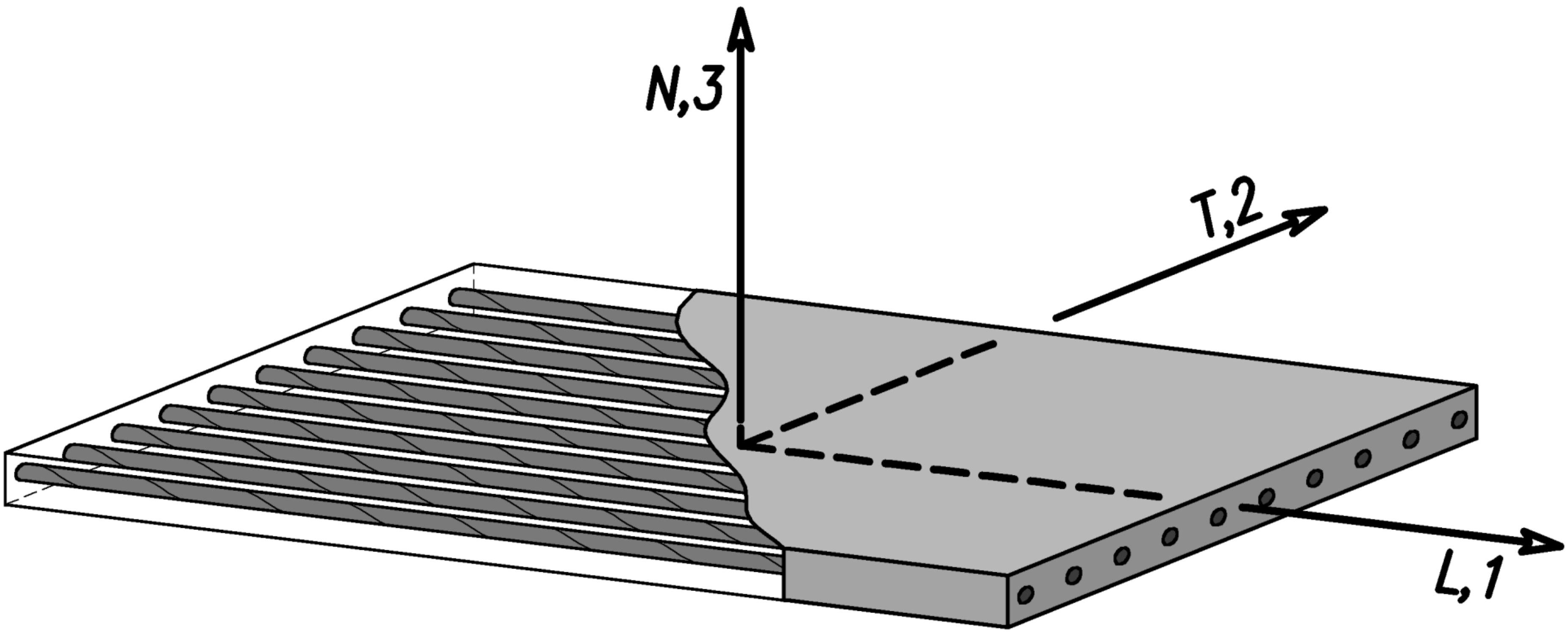
(5 costanti elastiche indipendenti)



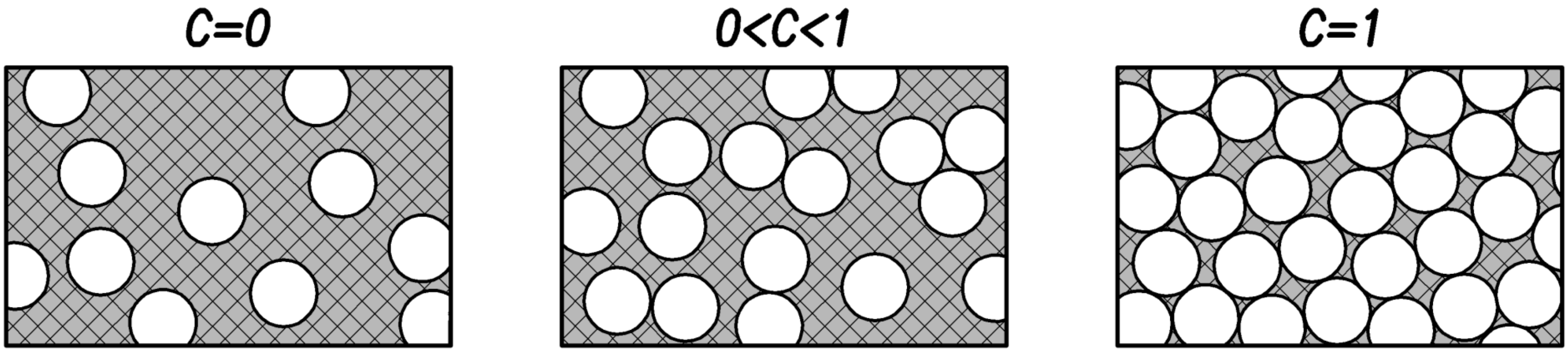
$\boxed{E_1}$	$\boxed{E_2}$	E_3
$\boxed{G_{12}}$	$\boxed{G_{23}}$	G_{31}
$\boxed{\nu_{12}}$	ν_{23}	ν_{31}
ν_{21}	ν_{32}	ν_{13}

$$\begin{cases} E_3 \implies E_3 = E_2 \\ G_{31} \implies G_{31} = G_{12} \\ \nu_{21} \implies \dfrac{\nu_{21}}{E_2} = \dfrac{\nu_{12}}{E_1} \\ \nu_{23} \implies G_{23} = \dfrac{E_2}{2(1+\nu_{23})} \\ \nu_{32} \implies \dfrac{\nu_{32}}{E_3} = \dfrac{\nu_{23}}{E_2} \\ \nu_{31} \implies \nu_{31} = \nu_{21} \\ \nu_{13} \implies \dfrac{\nu_{13}}{E_1} = \dfrac{\nu_{31}}{E_3} \end{cases}$$

Assi principali del materiale in una lamina



Grado di contiguità con disposizione casuale delle fibre



Frazioni ponderali e volumetriche

$$W_f = \frac{w_f}{w_c} \qquad V_f = \frac{v_f}{v_c}$$

$$W_m = \frac{w_m}{w_c} \qquad V_m = \frac{v_m}{v_c}$$

$$W_f + W_m = 1 \qquad V_f + V_m = 1$$

$$\boxed{\frac{1}{\rho_c} = \frac{1}{\rho_f} W_f + \frac{1}{\rho_m} W_m}$$

$$\boxed{\rho_c = \rho_f V_f + \rho_m V_m}$$

$$\rho_c = \sum_{i=1}^{n} \rho_i V_i$$

$$\frac{1}{\rho_c} = \sum_{i=1}^{n} \frac{1}{\rho_i} W_i$$

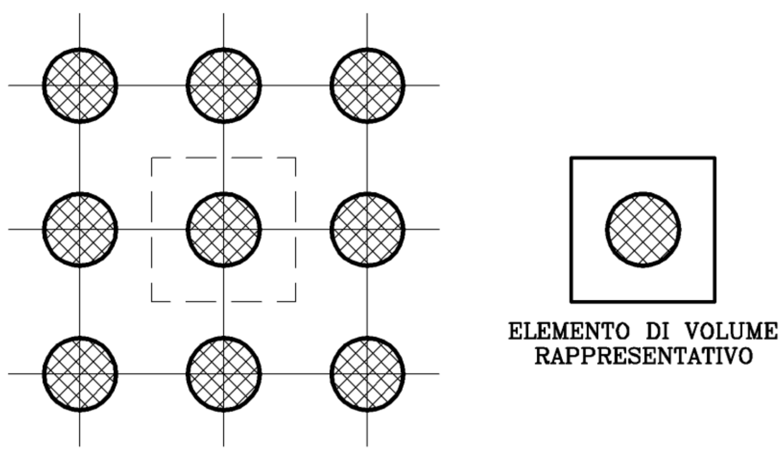
$$W_i = \frac{\rho_i}{\rho_c} V_i$$

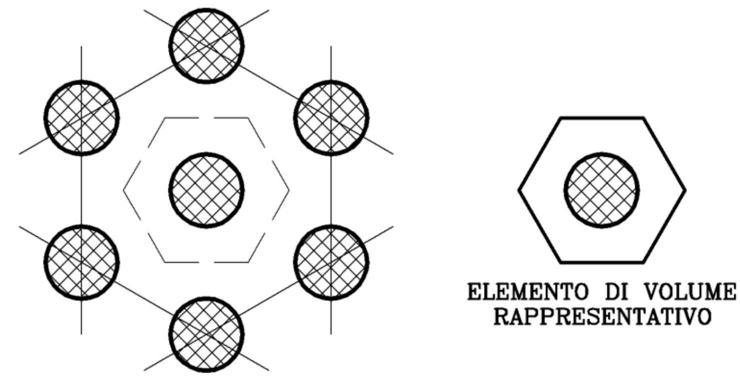
$$\boxed{W_{mf} = \frac{w_m}{w_f} = \frac{W_m}{W_f} = \frac{\rho_m V_m}{\rho_f V_f}}$$

Frazioni ponderali e volumetriche

$$
\begin{aligned}
V_m &= 1 - V_f = \left[1 + \frac{\rho_m/\rho_f}{W_m/W_f}\right]^{-1} = \left[1 + \frac{\rho_m/\rho_f}{W_{mf}}\right]^{-1} \\
V_f &= 1 - V_m = \left[1 + \frac{W_m/W_f}{\rho_m/\rho_f}\right]^{-1} = \left[1 + \frac{W_{mf}}{\rho_m/\rho_f}\right]^{-1} \\
W_m &= \left[1 + \frac{1}{(\rho_m/\rho_f)(V_m/V_f)}\right]^{-1} = 1 - W_f = \left[1 + \frac{1}{W_{mf}}\right]^{-1} \\
W_f &= \left[1 + (\rho_m/\rho_f)(V_m/V_f)\right]^{-1} = 1 - W_m = \left[1 + W_{mf}\right]^{-1} \\
W_{mf} &= (\rho_m/\rho_f)(V_m/V_f) = W_m/W_f
\end{aligned}
$$

Elementi di volume rappresentativo di materiale fibro-rinforzato





Formule di TSAI

$$E_L = k\,E_f\,V_f + E_m\,V_m$$

$$\begin{aligned} E_T \;=\; & 2\left[1-\nu_f+(\nu_f-\nu_m)\,V_m\right]\left\{(1-C)\left[\frac{K_f\,(2\,K_m+G_m)-G_m\,(K_f-K_m)\,V_m}{(2\,K_m+G_m)+2\,(K_f-K_m)\,V_m}\right]+\right. \\ & \left. + C\left[\frac{K_f\,(2\,K_m+G_f)-G_f\,(K_f-K_m)\,V_m}{(2\,K_m+G_f)+2\,(K_f-K_m)\,V_m}\right]\right\} \end{aligned}$$

$$\begin{aligned} \nu_{LT} \;=\; & (1-C)\left[\frac{K_f\,\nu_f\,(2\,K_m+G_m)\,V_f+K_m\,\nu_m\,(2\,K_f+G_m)\,V_m}{K_f\,(2\,K_m+G_m)-G_m\,(K_f-K_m)\,V_m}\right]+ \\ & + C\left[\frac{K_m\,\nu_m\,(2\,K_f+G_f)\,V_m+K_f\,\nu_f\,(2\,K_m+G_f)\,V_f}{K_f\,(2\,K_m+G_f)-G_f\,(K_f-K_m)\,V_m}\right] \end{aligned}$$

$$\frac{\nu_{LT}}{E_L} = \frac{\nu_{TL}}{E_T}$$

$$\begin{aligned} G_{LT} \;=\; & (1-C)\,G_m\left[\frac{2\,G_f-(G_f-G_m)\,V_m}{2\,G_m+(G_f-G_m)\,V_m}\right]+ \\ & + C\,G_f\left[\frac{(G_f+G_m)-(G_f-G_m)\,V_m}{(G_f+G_m)+(G_f-G_m)\,V_m}\right] \end{aligned}$$

Formule di WHITNEY e RILEY

$$L_f = 1 - \nu_f - 2\,\nu_f^2$$

$$L_m = 1 - \nu_m - 2\,\nu_m^2$$

$$k_f = E_f/(2\,L_f)$$

$$k_m = E_m/(2\,L_m)$$

$$\overline{k} = \frac{(k_f + G_m)\,k_m - G_m\,(k_f - k_m)\,V_f}{(k_f + G_m) - (k_f - k_m)\,V_f}$$

$$\overline{\nu} = \nu_f\,V_f + \nu_m\,V_m$$

$$\boxed{E_L = E_f\,V_f + E_m\,V_m}$$

$$\boxed{\nu_{LT} = \nu_m - \frac{2\,E_f\,(\nu_m - \nu_f)\,(1 - \nu_m^2)\,V_f}{E_m\,L_f\,V_m + [L_m\,V_f + (1 + \nu_m)]\,E_f}}$$

$$\boxed{E_T = \frac{2\,\overline{k}\,(1 - \overline{\nu})\,E_L}{E_L + 4\,\overline{k}\,\nu_{LT}^2}}$$

$$\boxed{\begin{aligned} G_{LT} = {} & (1 - C)\,G_m \left[\frac{2\,G_f - (G_f - G_m)\,V_m}{2\,G_m + (G_f - G_m)\,V_m}\right] + \\ & + C\,G_f \left[\frac{(G_f + G_m) - (G_f - G_m)\,V_m}{(G_f + G_m) + (G_f - G_m)\,V_m}\right] \end{aligned}}$$

Formule di HALPIN e TSAI

$$E_L = E_f V_f + E_m V_m$$

$$\nu_{LT} = \nu_f V_f + \nu_m V_m$$

$$\frac{\nu_{LT}}{E_L} = \frac{\nu_{TL}}{E_T}$$

$$\frac{E_T}{E_m} = \frac{1 + \xi_E \eta_E V_f}{1 - \eta_E V_f}$$

$$\frac{G_{LT}}{G_m} = \frac{1 + \xi_G \eta_G V_f}{1 - \eta_G V_f}$$

$$\frac{G_T}{G_m} = \frac{1 + \xi_T \eta_T V_f}{1 - \eta_T V_f}$$

$$\eta_E = \frac{(E_f/E_m) - 1}{(E_f/E_m) + \xi_E}$$

$$\eta_G = \frac{(G_f/G_m) - 1}{(G_f/G_m) + \xi_G}$$

$$\eta_T = \frac{(G_f/G_m) - 1}{(G_f/G_m) + \xi_T}$$

$$\xi_E = 2$$

$$\xi_G = 1$$

$$\xi_T = \frac{K_m/G_m}{(K_m/G_m) + 2}$$

$$K_m = \frac{E_m}{2\,(1 - \nu_m - 2\,\nu_m^2)}$$

Formule di HAHN

$$P = \frac{P_f\, V_f + \eta\, P_m\, V_m}{V_f + \eta\, V_m}$$

COSTANTE ELASTICA	P	P_f	P_m	η
E_L	E_L	E_{Lf}	E_m	1
ν_{LT}	ν_{LT}	ν_{LTf}	ν_m	1
G_{LT}	$1/G_{LT}$	$1/G_{LTf}$	$1/G_m$	$\eta_{G_{LT}}$
G_T	$1/G_T$	$1/G_{Tf}$	$1/G_m$	η_{G_T}
K_T	$1/K_T$	$1/K_f$	$1/K_m$	η_K

$$E_T = \frac{4\, K_T\, G_T}{K_T + \psi\, G_T}$$

$$\psi = 1 + \frac{4\, K_T\, \nu_{LT}^2}{E_L}$$

$$K_f = \frac{E_{Lf}}{2\,(1 - \nu_{LTf})}$$

$$K_m = \frac{E_m}{2\,(1 - \nu_m)}$$

$$G_m = \frac{E_m}{2\,(1 + \nu_m)}$$

$$\eta_{G_{LT}} = \frac{1 + G_m/G_{LTf}}{2}$$

$$\eta_{G_T} = \frac{3 - 4\,\nu_m + G_m/G_{Tf}}{4\,(1 - \nu_m)}$$

$$\eta_K = \frac{1 + G_m/K_f}{2\,(1 - \nu_m)}$$

Moduli elastici in compositi a fibre casualmente orientate

$$
\begin{aligned}
E &= \frac{3}{8}\,E_L + \frac{5}{8}\,E_T \\
G &= \frac{1}{8}\,E_L + \frac{1}{4}\,E_T
\end{aligned}
$$

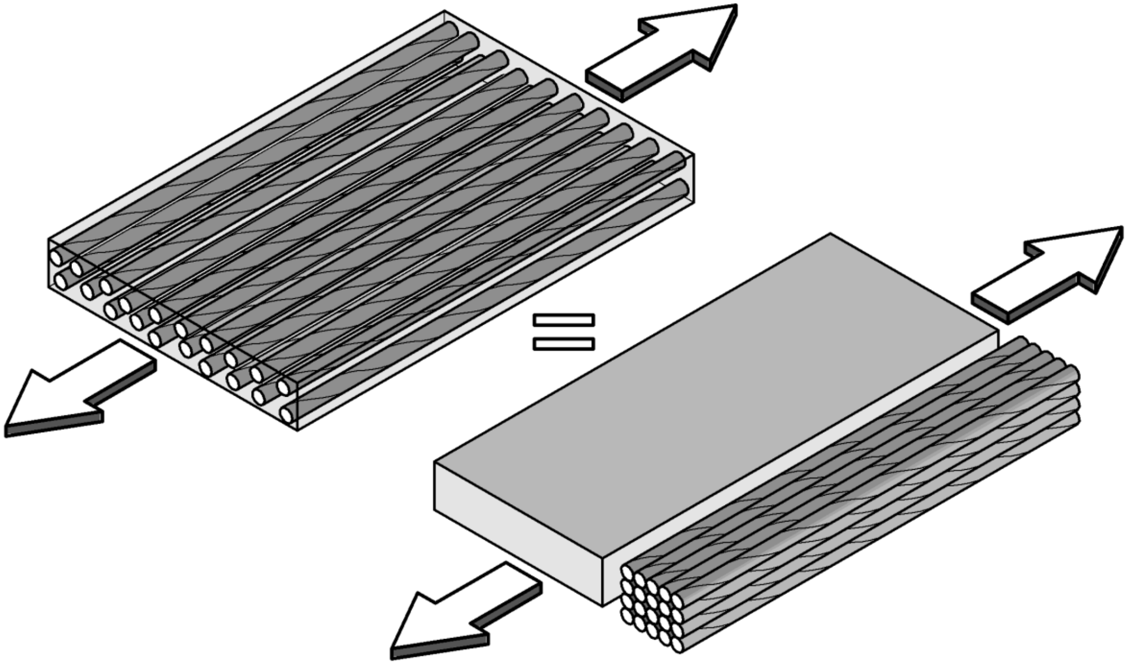
$$\nu = \frac{E}{2\,G} - 1$$

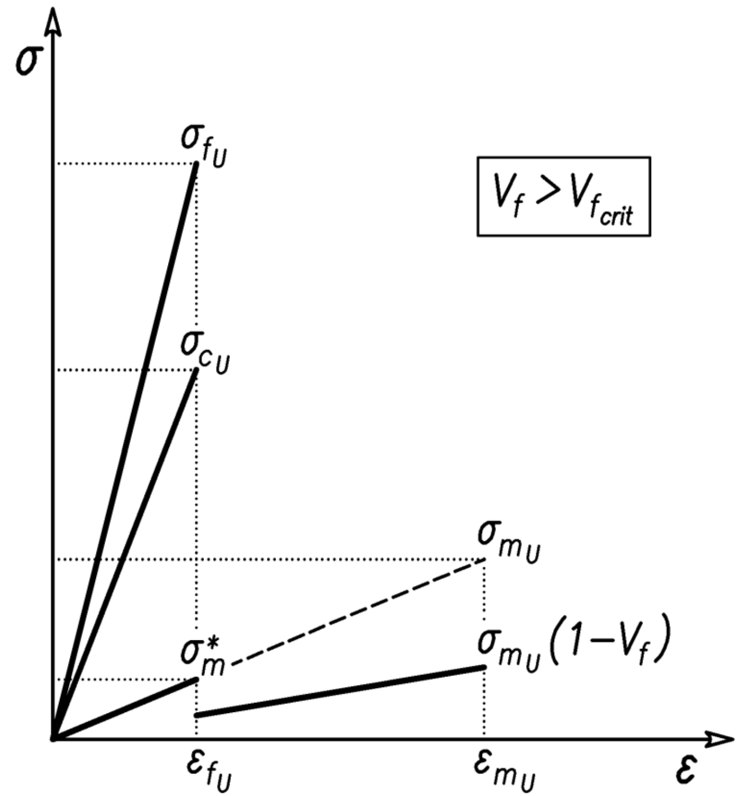
$$
\begin{aligned}
E &= (U_1 + U_4)\,(U_1 - U_4)\,/\,U_1 \\
G &= (U_1 - U_4)\,/\,2 \\
\nu &= U_4\,/\,U_1
\end{aligned}
$$

$$
\begin{aligned}
U_1 &= (3\,Q_{11} + 3\,Q_{22} + 2\,Q_{12} + 4\,Q_{66})\,/\,8 \\
U_4 &= (Q_{11} + Q_{22} + 6\,Q_{12} - 4\,Q_{66})\,/\,8
\end{aligned}
$$

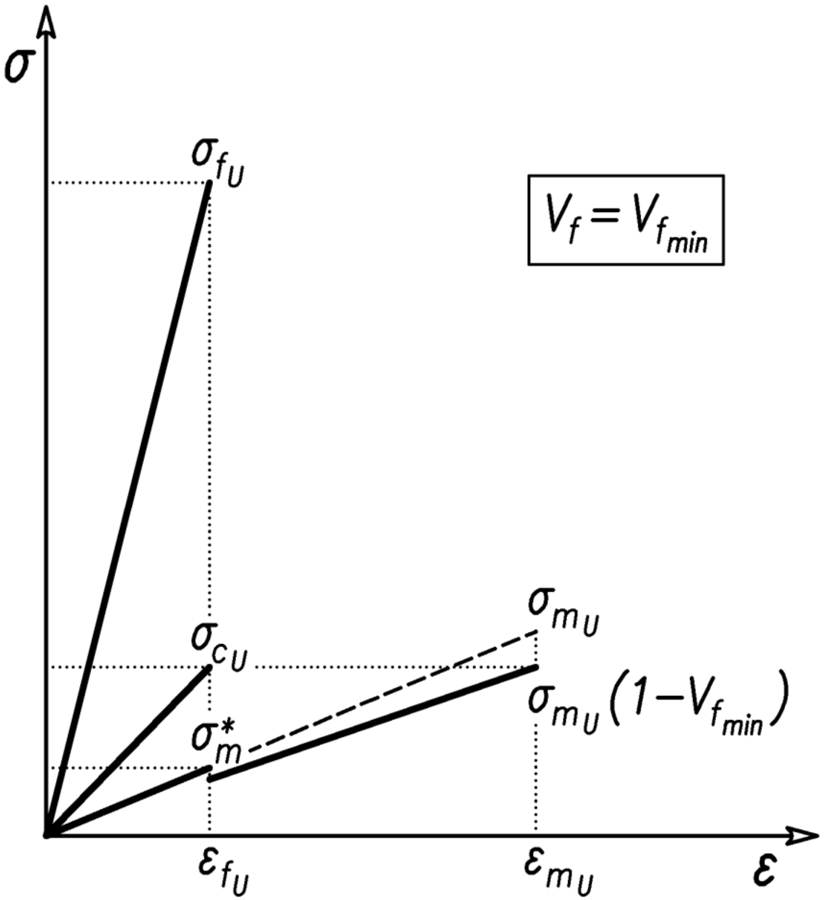
$$
\begin{aligned}
Q_{11} &= E_L\,/\,(1 - \nu_{LT}\,\nu_{TL}) \\
Q_{22} &= E_T\,/\,(1 - \nu_{LT}\,\nu_{TL}) \\
Q_{12} &= \nu_{LT}\,E_T\,/\,(1 - \nu_{LT}\,\nu_{TL}) \\
Q_{66} &= G_{LT}
\end{aligned}
$$

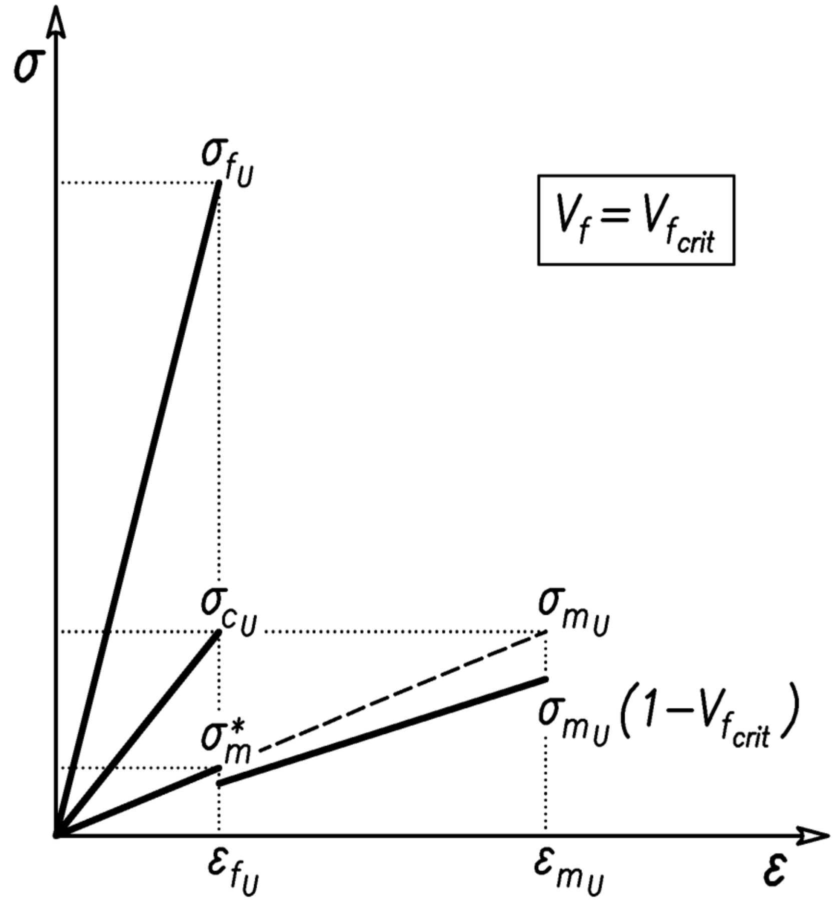
Modello per la previsione teorica della resistenza di una lamina



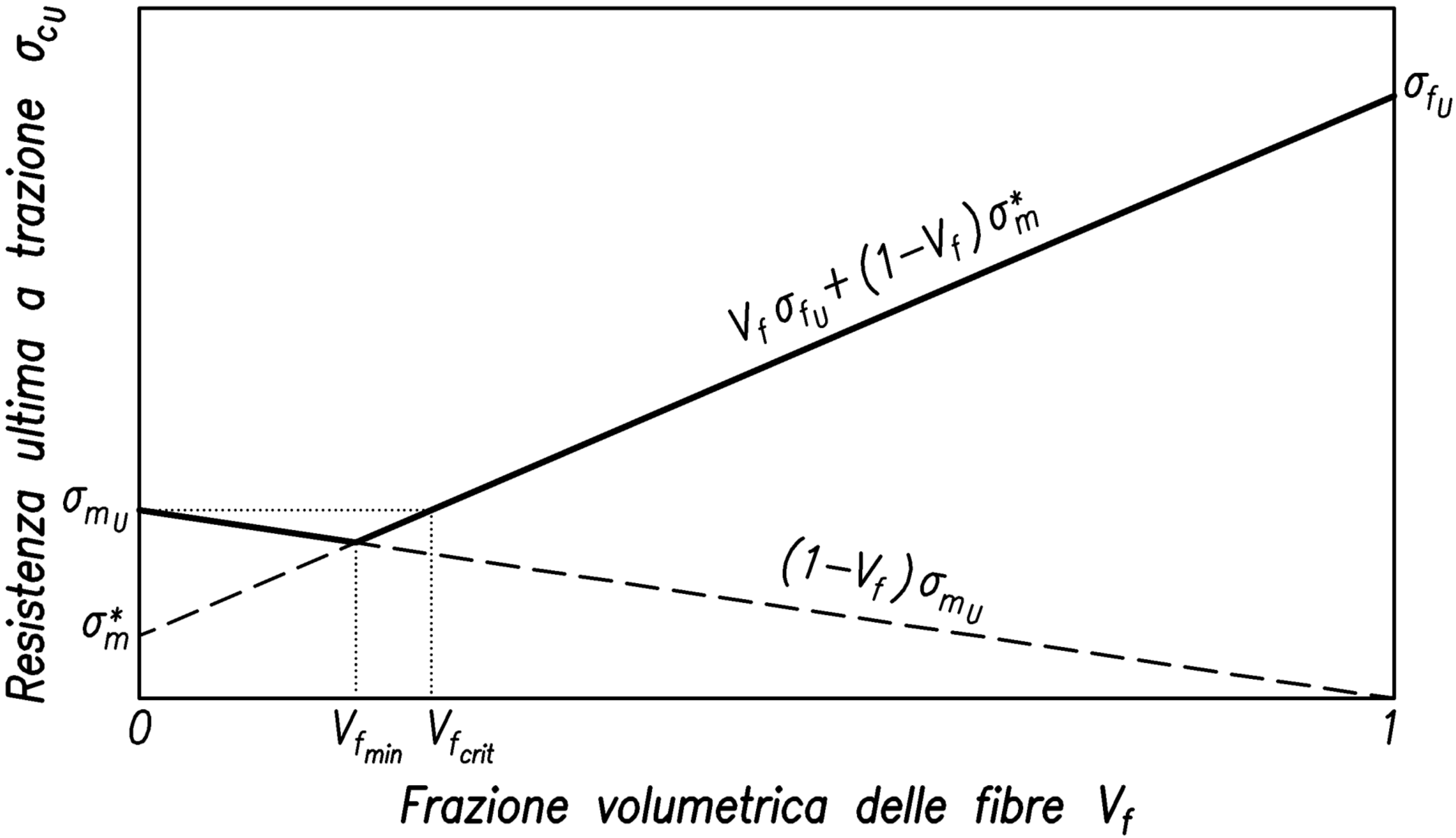


Leggi costitutive di una lamina per frazione volumetrica minima e critica

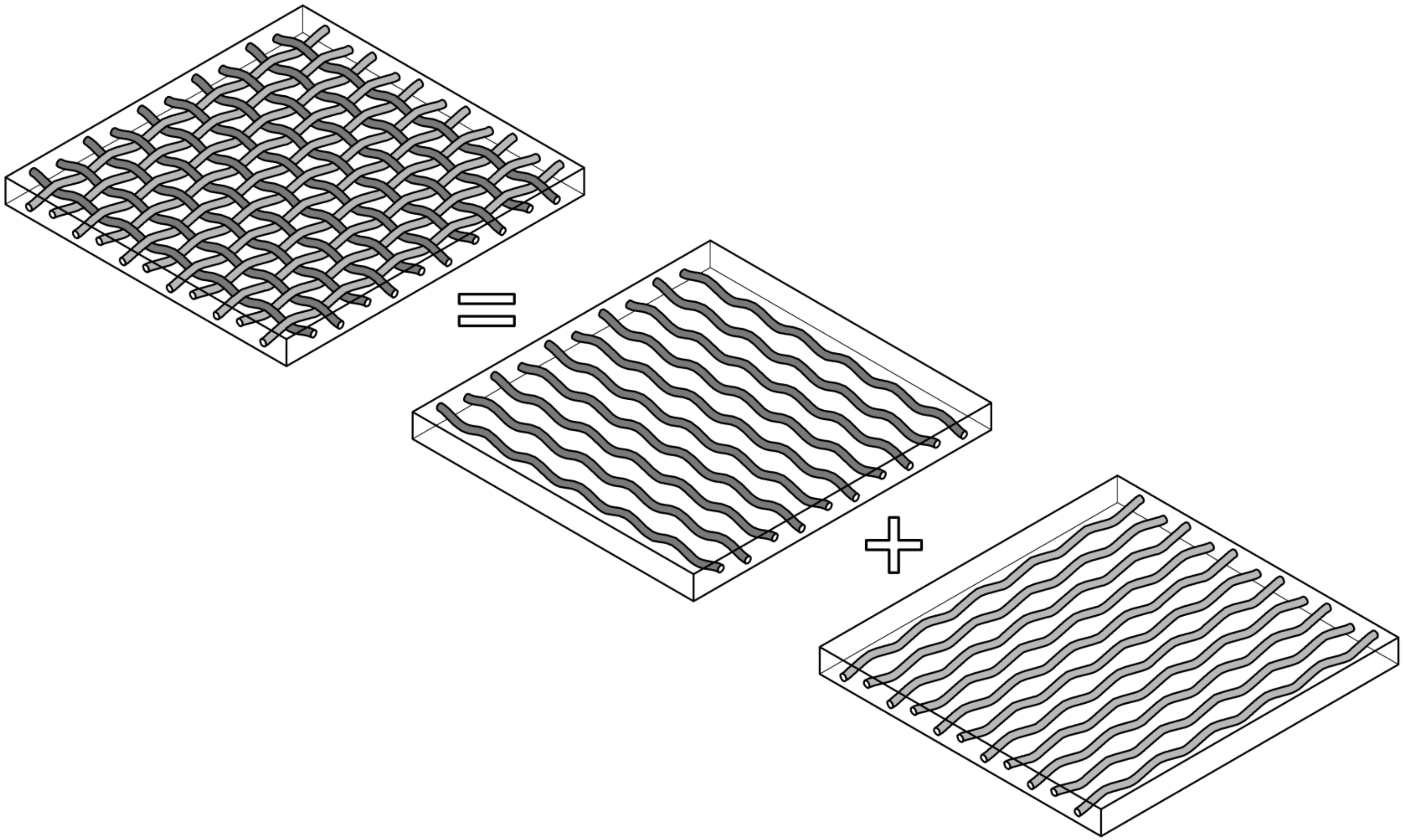




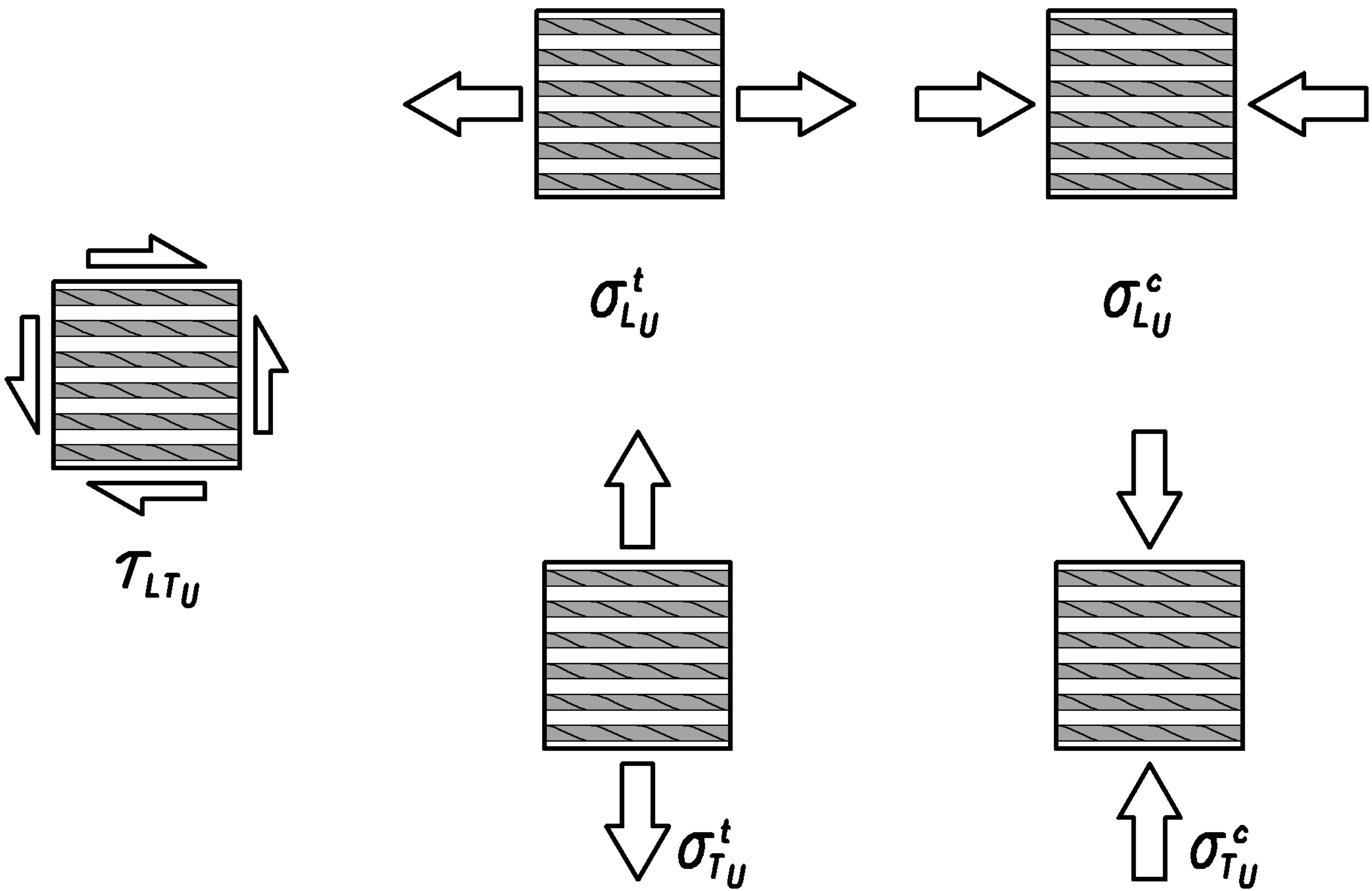
Resistenza ultima a trazione del composito in funzione della frazione volumetrica



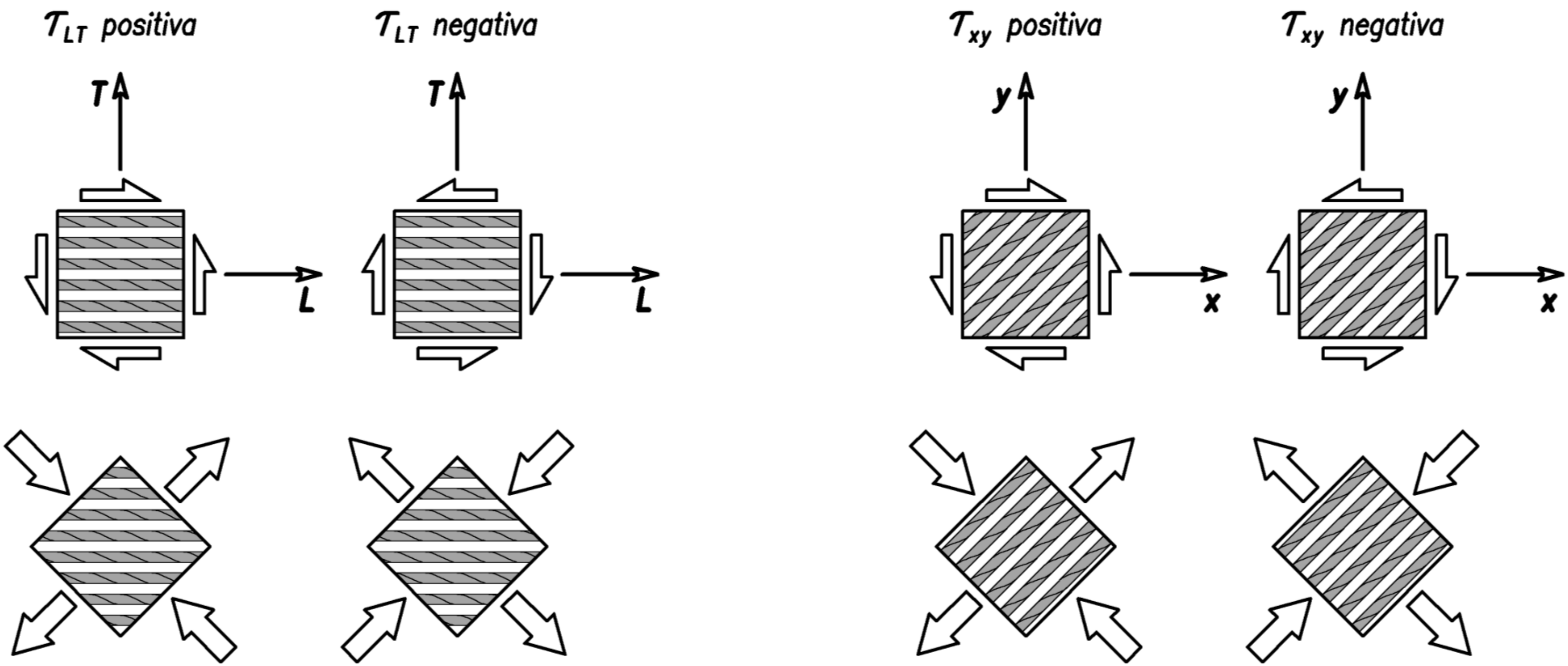
Scomposizione di una lamina rinforzata con stuoia



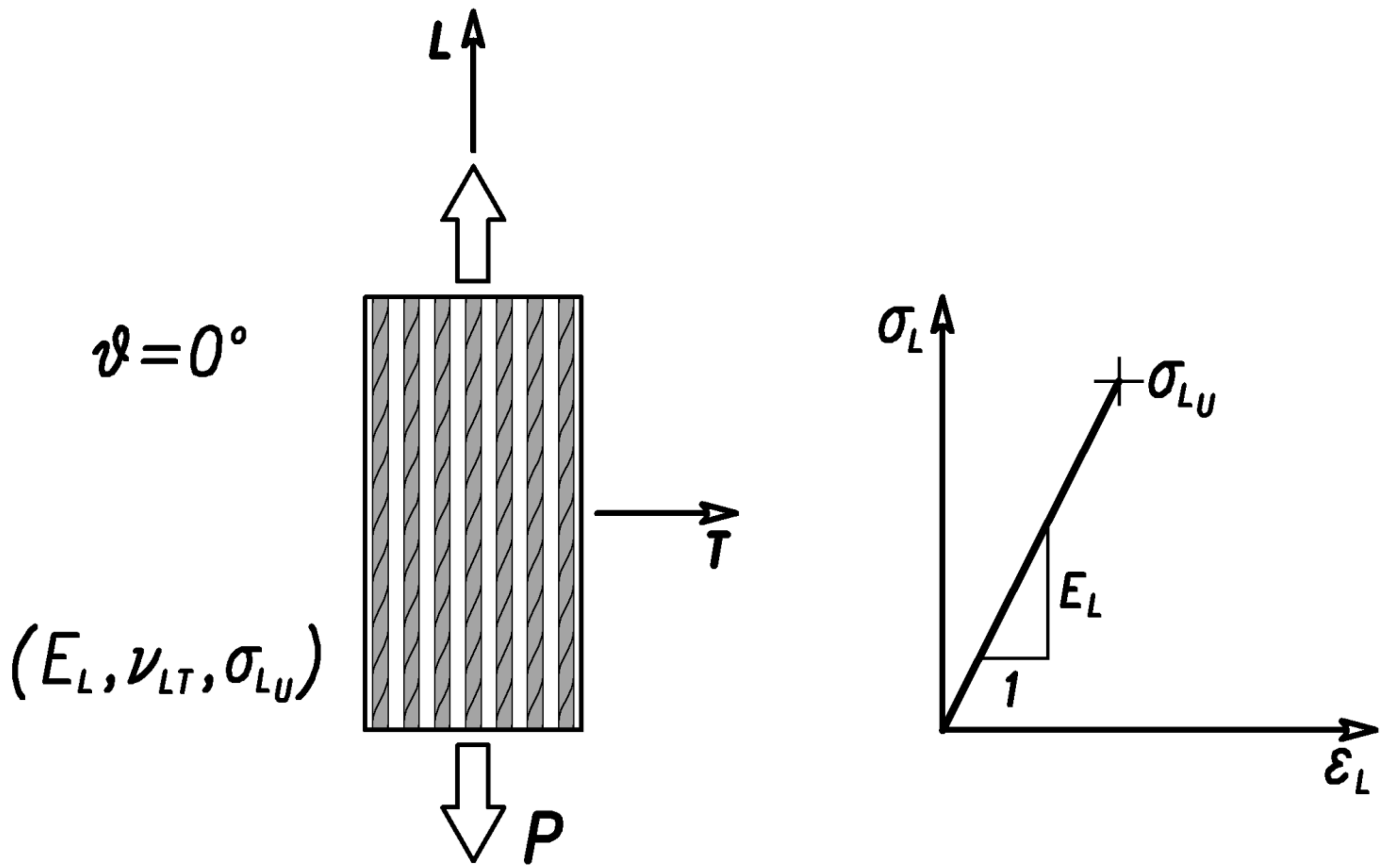
Resistenze fondamentali in una lamina unidirezionale



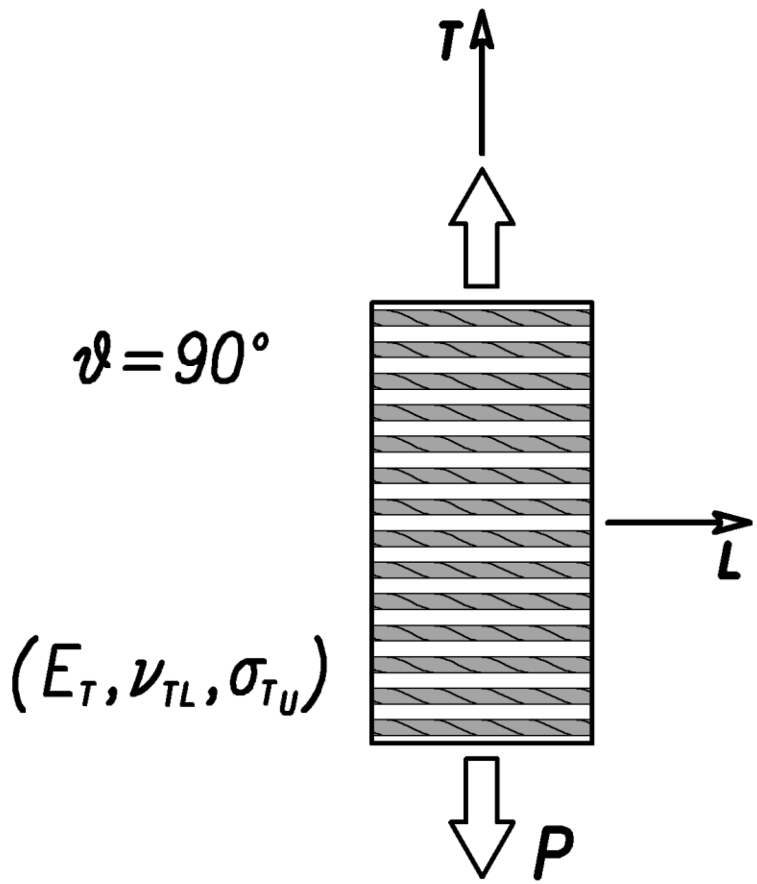
Effetto del segno delle tensioni tangenziali sulle resistenze a taglio

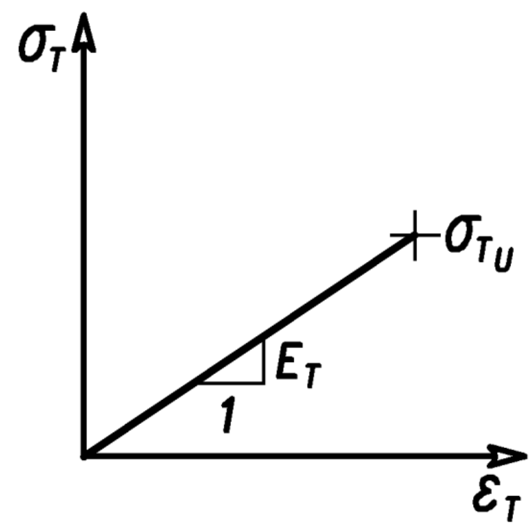


Prova di trazione in direzione parallela alle fibre

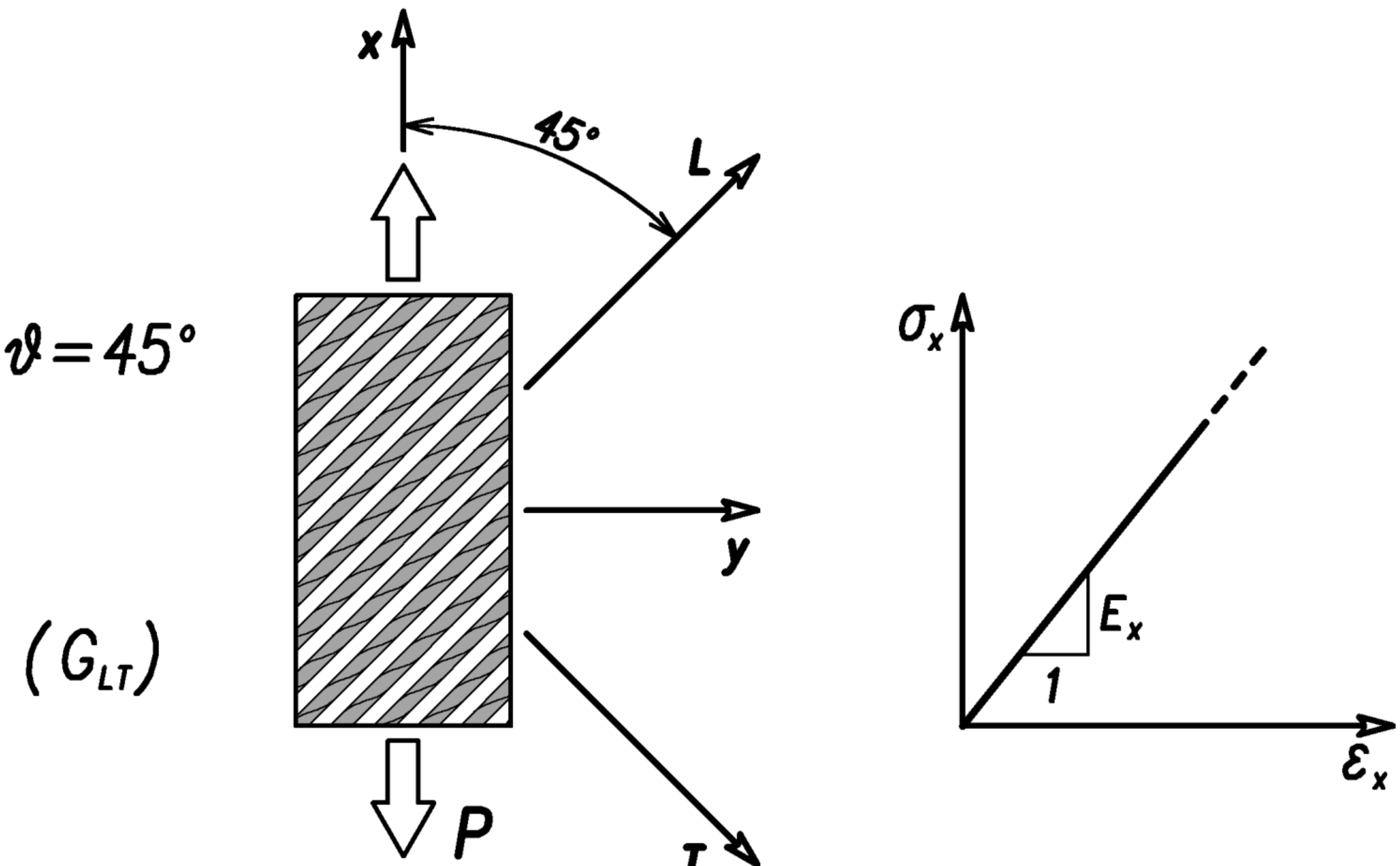


Prova di trazione in direzione trasversale alle fibre

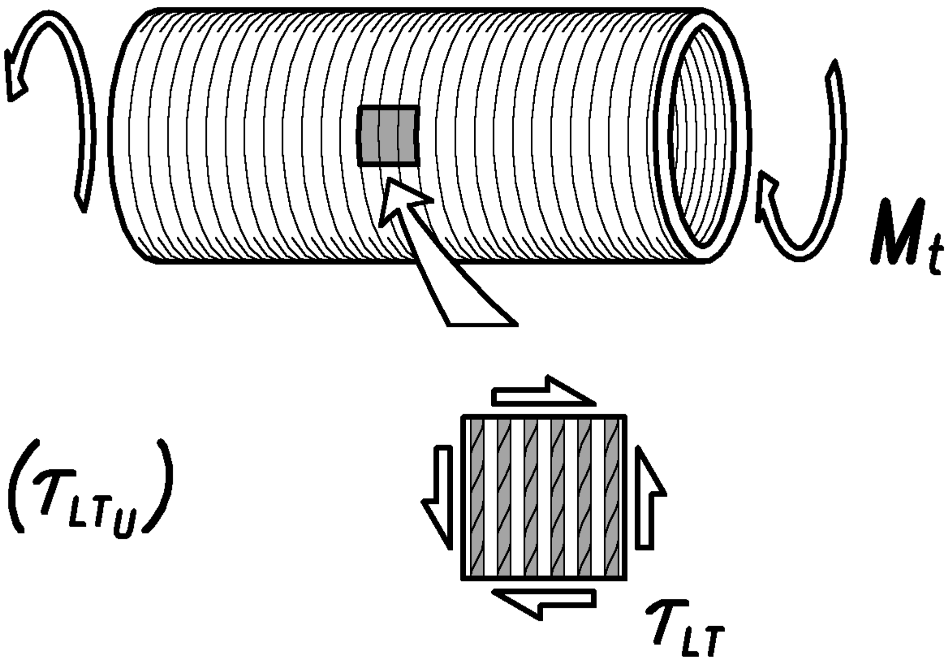


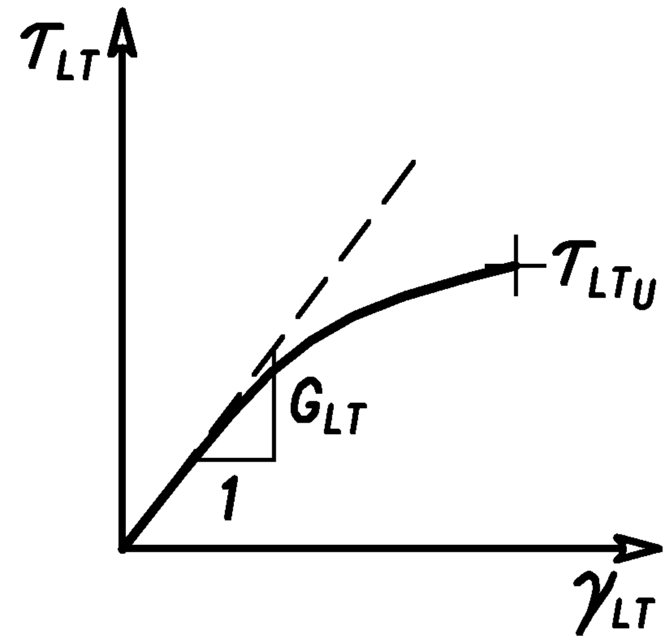


Prova di trazione in direzione a 45° rispetto alle fibre

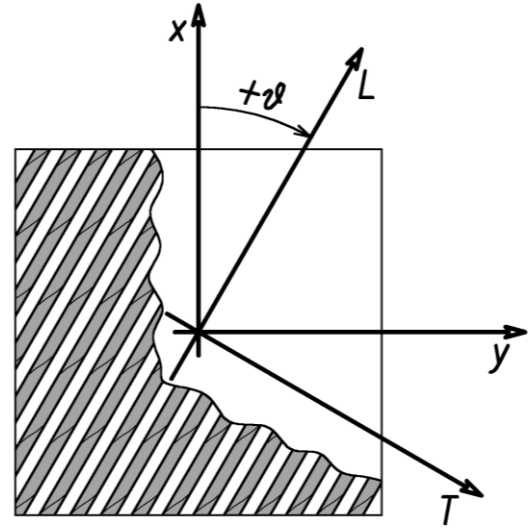


Prova di torsione





Costanti elastiche riferite ad assi arbitrari



$$\frac{1}{E_x} = \frac{\cos^4\theta}{E_L} + \left(\frac{1}{G_{LT}} - \frac{2\,\nu_{LT}}{E_L}\right)\sin^2\theta\,\cos^2\theta + \frac{\sin^4\theta}{E_T}$$

$$\frac{1}{E_y} = \frac{\sin^4\theta}{E_L} + \left(\frac{1}{G_{LT}} - \frac{2\,\nu_{LT}}{E_L}\right)\sin^2\theta\,\cos^2\theta + \frac{\cos^4\theta}{E_T}$$

$$\frac{1}{G_{xy}} = \frac{\cos^2 2\,\theta}{G_{LT}} + \left(\frac{1+\nu_{LT}}{E_L} + \frac{1+\nu_{TL}}{E_T}\right)\sin^2 2\,\theta$$

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} = \frac{\nu_{LT}}{E_L} - \frac{1}{4}\left(\frac{1+\nu_{LT}}{E_L} + \frac{1+\nu_{TL}}{E_T} - \frac{1}{G_{LT}}\right)\sin^2 2\,\theta$$

ESERCIZIO

Calcolo delle costanti elastiche di una lamina

Sia data una lamina formata con UDR di vetro E (grammatura $\overline{w} = 300\,\text{g/m}^2$) impregnata con resina poliestere isoftalica secondo un rapporto in peso matrice/fibre $W_{mf} = 0.85$ e grado di contiguità $C = 90\%$. Le caratteristiche meccaniche delle due fasi costituenti sono:

fibre	matrice
ρ_f = 2.54 g/cm³	ρ_m = 1.2 g/cm³
E_f = 72 GPa	E_m = 3.2 GPa
G_f = 30 GPa	G_m = 1.2 GPa
ν_f = 0.20	ν_m = 0.32

Valutare dapprima lo spessore nominale della lamina:

$t =$ mm

Successivamente confrontare i valori che si ottengono per le costanti elastiche che caratterizzano il composito mediante le formule di Tsai, Whitney-Riley, Halpin-Tsai e Hahn:

Formule	E_L [GPa]	ν_{LT} [−]	E_T [GPa]	G_{LT} [GPa]
Tsai				
Whitney-Riley				
Halpin-Tsai				
Hahn				