Electrons in crystals – I written test –academic year 2006/2007, November 6, 2006(revised)

(*Time: 3 hours*)

Solve all the exercises. Each one corresponds to a maximum score of 18 (36 total for both). If the score is between 33 and 36 it is considered equal to 30/30 *cum laude*, if it is between 30 and 32 it is considered equal to 30/30.

Exercise 1: *Free electron model*

First part

Sodium (Na) in normal conditions of temperature and pressure is a metal with BCC structure, with density of about 2.7 g cm^{-3}, mass number of about 23 and Fermi energy E_F of about 3 eV.

1. Calculate the Fermi temperature T_F.
2. Calculate the Fermi velocity v_F and the average kinetic energy at 0 K.
3. Which is the variation of the average kinetic energy at room temperature?
4. If you would consider the electron gas as a classical gas, which would be the average kinetic energy at 0 K? And at room temperature?
5. From the given Fermi energy, calculate the density n of free electrons present in the metal.
6. Calculate the numerical density of Na atoms and then the average number of free electrons per atoms. Is it what you expected? Comment.

Second part

1. Derive the expression of the density of the electron states for a two-dimensional electron gas.
2. Considering the electron density n as a parameter, study the behaviour of E_F as a function of the temperature.
3. Discuss the high and low temperature limits.
4. Derive the expression of the density of the electron states for a four-dimensional electron gas.

Exercise 2: *Diffraction of neutron on Cesium Chloride*

Consider Cesium Chloride. Let α and β be the atomic form factor of Cl and Cs respectively. In the following we will not consider the dependence of α and β on the transferred wave vector. This structure is evidently a Bravais lattice with basis.

1. Specify which is the Bravais lattice and the basis.
2. Write the expression of the intensity of the diffracted wave in terms of the geometrical structure factor $S(\mathbf{k})$.
3. Write the expression of $S(\mathbf{k})$ for a generic $\mathbf{k}$ of the reciprocal *space.*
4. Calculate explicitly $S(\mathbf{K})$ for $\alpha = \beta$ for $\mathbf{K}$ of the reciprocal *lattice.*
5. As before, but with $\alpha = -\beta$
6. What is the relationship between the two lattices found in (d) and (e)?

NOTE:

- Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.
- When required, numerical evaluations should be given exactly with 3 significant figures, if not otherwise indicated.