Electrons in crystals – final written test
Academic year 2006/2007 – January 12, 2007

(*Time: 3 hours*)

- Solve all the exercises, corresponding to a total maximum score of 36. If the score is between 33 and 36 it is considered equal to 30/30 *cum laude*, if it is between 30 and 32 it is considered equal to 30/30.
- Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.
- When required, numerical evaluations should be given exactly with 3 significant figures, if not otherwise indicated.

Exercise 1: *Free-electron model*

Consider Al at room temperature. Its electron density is $n = 18.1 \cdot 10^{22}/cm^3$ and its electrical resistivity is $\rho = 2.45\ \mu\Omega \cdot cm$.

1. Find its electron relaxation time τ and electron mean free path ℓ (consider the Sommerfeld model).
2. Consider AC conductivity. At which frequency w the conductivity $\sigma(\omega)$ will be 1/10 of its zero-frequency value?
3. Give an estimate of the thermal conductivity of Al.

Exercise 2: *Electrons in periodic solids*

1. Consider *free electrons* in a solid with FCC crystal structure. Write the explicit expressions and plot the energy-wave-vector curves $E(k)$ in a *reduced zone scheme* along the high symmetry direction $\Gamma - X$ (where $\Gamma = (0, 0, 0)$ and $X = \frac{2\pi}{a}(1, 0, 0)$ and a is the lattice parameter), considering all the bands with energy up to $4E(X)$.
2. Use the *tight-binding method* and consider the BCC crystal structure with s-type wavefunctions and nearest-neighbor interactions only and negligible overlap. With reference to the texbook for the definition of $\gamma(\mathbf{R})$, consider: $\gamma(\mathbf{R}_{NN})$=t where $\mathbf{R}_{NN}$ is for nearest neighbors, Write the explicit expression for $E(\mathbf{k})$.
3. Calculate the effective mass tensor in the points corresponding to the maxima and minima of the band calculated in (2). Comment your results.

Exercise 3: *Semiclassical model of electron dynamics*

Consider a 3D solid with band dispersion $E(\mathbf{K}) = E_0 - 2t[cos(k_x a) + cos(k_y a) + cos(k_z a)]$, with a the lattice parameter, under a uniform static magnetic field $\mathbf{H} = H\hat{x}$.

1. Write the Bloch electron velocity in the plane (y,z).
2. Write the equation of an orbit (in $\mathbf{k}$ space) for $k_x = 0$ on the constant energy surface $E(\mathbf{k}) = E^*$. (this is valid up to the end of the exercise)
3. What is the orbit when $E^* = E_0 - 6t + t\delta^2$ and $\delta \ll 1$?
4. Write explicitely the period of that orbit. What should be t in order to have the same period as for free electrons?
5. What is the orbit when $E^* = E_0 - 2t$? Write explicitely the equation, specifying the relation between k_y and k_z, and draw the orbit.
6. Indicate the direction of the motion along the orbit.