Condensed Matter Physics I
II partial written test
academic year 2014/2015
January 13, 2015

(*Time: 2.5 hours*)

NOTE: Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.

Exercise 1: *Metals and insulators*

1. Explain (shortly!) why these three facts are compatible:
   (i) sodium, which has a BCC structure and has 2 atoms in a conventional cubic unit cell, is a metal;
   (ii) calcium, which has a FCC structure and has 4 atoms in a conventional cubic unit cell, is a metal;
   (iii) diamond, which has a FCC Bravais lattice and has 8 atoms in a conventional cubic unit cell, is an insulator.

Exercise 2: *Band structure of a 1D solid*

1. Find the form of the van Hove singularity for a minimum or maximum in one dimension.
2. Consider a 1D band whose energy is given by $E(k) = E_0 - t\ \cos(ka)$. Calculate the density of states explicitly and check whether it has the expected van Hove singularities. as a function of k in the above model.
3. Suppose the band is 1/3 occupied. What is v_F, the group velocity at the Fermi level?
4. Show that a uniform electric field does not accelerate the electrons but lets them oscillate around some fixed position.

Exercise 3: *2D solids*

Consider a two-dimensional material whose crystalline structure is a square lattice with spacing a. Suppose that the dispersion relations for electron energies in the conduction and valence bands are given by:

$$E_c(\mathbf{k}) = 6 - 2(\cos(ak_x) + \cos(ak_y))$$

$$E_v(\mathbf{k}) = -2 + (\cos(ak_x) + \cos(ak_y))$$

where energies are given here in units of eV.

1. Using tight-binding theory, explain where the above dispersion relations come from.
2. Sketch E_c and E_v for the direction $k_x = k_y$. Indicate the value and position of the minimum band gap.
3. Show that close to the conduction and valence band edges, contours of constant energy are circles in $\mathbf{k}$-space, and determine the effective masses of both the electrons and the holes.
4. Sketch the density of states as a function of energy for the whole of both the conduction and the valence band.
5. Indicate the Fermi surface when the valence band is half filled.

Consider now a two-dimensional material having a "relativistic" dispersion relation $E(\mathbf{k}) = \alpha|\mathbf{k}| = \alpha k$.

6. Calculate the velocity. *[since the velocity is a vector, give modulus and direction!]*
7. Calculate the density of states, showing all steps. *[not just a sketch, but write the precise energy dependence and proper constant factors involved].*