Condensed Matter Physics I
I test - 18 November 2014
(2.5 hours)

- Solve all the exercises.
- Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.

Exercise 1: *Free electrons - Sommerfeld model*

1. Derive the expression of the density of states $g(E)$ for free electron gas in 1D and 2D.
2. Consider in both cases the variation of the chemical potential $\mu(T)$ for small non-zero temperatures: starting from the expression of n in terms of an energy integral (using the density of states $g(E)$ and the Fermi function $f(E, \mu)$ and *not* making use of Sommerfeld expansion) discuss qualitatively whether $\mu(T)$ increases, decreases or remains constant if T increases from 0K, for fixed n.
3. Using now the Sommerfeld expansion, give the explicit expression for the leading term of the variation of $\mu(T)$ in 1D as a function of T (n should not compare in such expressions). It the result consistent with the previous point?
4. Consider now Zn, described as a free electron gas in 3D, whose Fermi energy is E_F=9.47 eV. Neglect the variation of the chemical potential with the temperature and calculate the probability of an energy level at 9.7 eV being occupied by an electron at 300 K and 1000 K.
5. Consider now correctly the variation of the chemical potential with T, using the Sommerfeld expansion, at 300 K and at 1000 K. Discuss the validity of the assumption made in the previous point in the calculation of the occupation probability of the electronic level at 9.7 eV.
6. Estimate the fraction of electrons excited above the Fermi level at room temperature for Zn.

Exercise 2: *Crystalline structures*

The figure below shows a 2D periodic array of atoms of two different types.

1. Identify the type of Bravais lattice, write the primitive vectors $\{\mathbf{a}_i\}$, sketch them in the figure, together with the corresponding primitive unit cell.
2. Is this a *simple* Bravais lattice or is it a Bravais lattice *with a basis*? If it is a Bravais lattice *with a basis*: (i) how many points are in the basis? (ii) sketch them in the figure and write the corresponding vectors.
3. Write the primitive vectors $\{\mathbf{b}_i\}$ of the reciprocal lattice.
4. Write the geometrical structure factors $S(\mathbf{K})$ on a generic reciprocal lattice vector $\mathbf{K}$.
5. Specify the expression of $S(\mathbf{K})$ if all the atoms have the same atomic form factor.
6. Show that in the latter case there are some reciprocal lattice vectors $\mathbf{K}$ where $S(\mathbf{K})$ vanishes. Show that the removal of these vectors of zero structure factor from the reciprocal lattice reduces it to a new lattice. Specify which one and why.