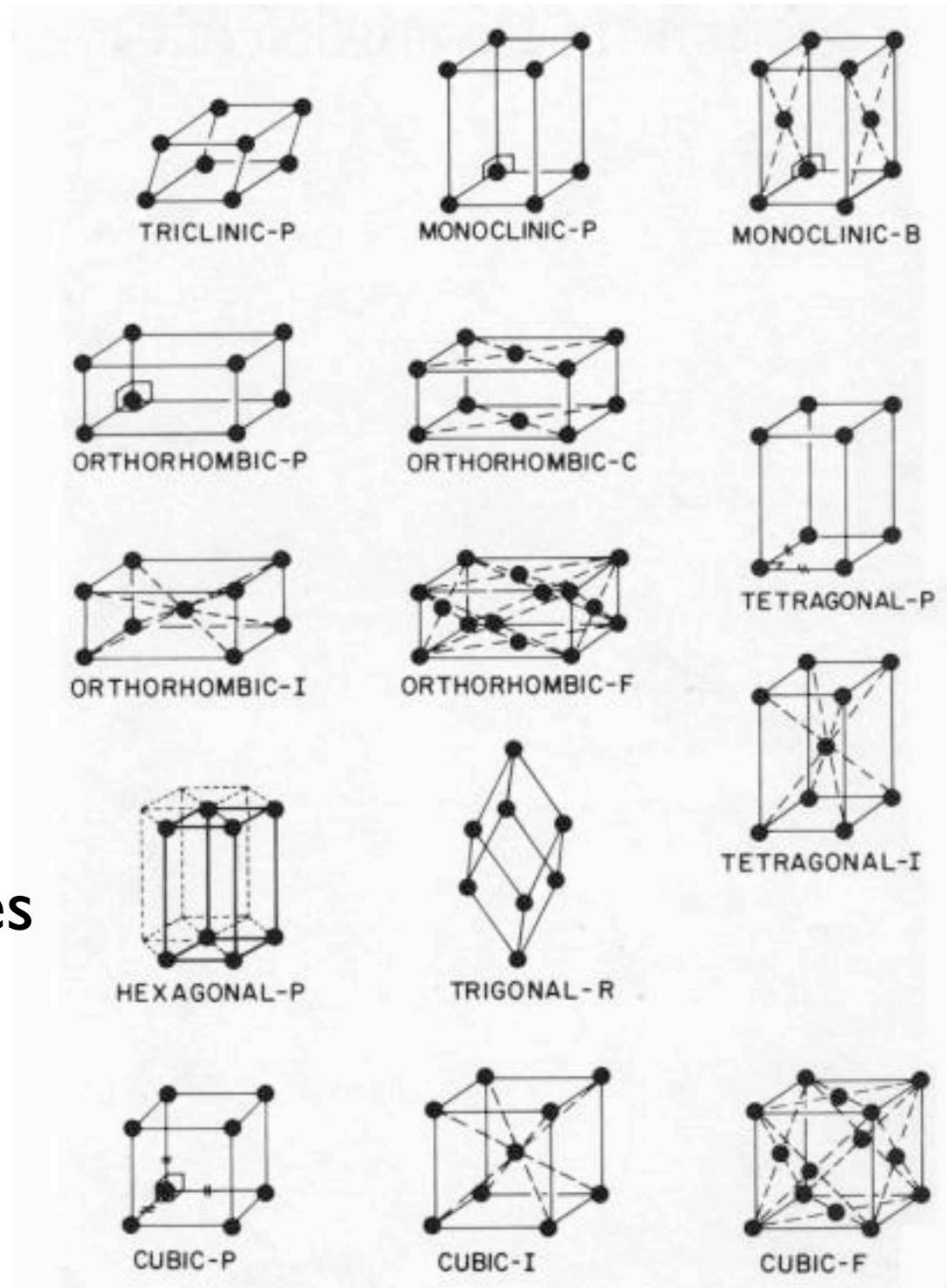# Bravais lattices

(few notes/pictures complementary to the A&M book)

The 14 Bravais lattices in 3D
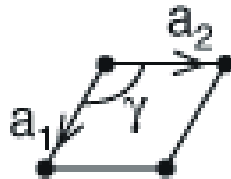
M. Peressi
Cond Matt Phys 1, UniTS, 2017/18

TRICLINIC-P   MONOCLINIC-P   MONOCLINIC-B

ORTHORHOMBIC-P   ORTHORHOMBIC-C   TETRAGONAL-P

ORTHORHOMBIC-I   ORTHORHOMBIC-F   TETRAGONAL-I

HEXAGONAL-P   TRIGONAL-R

CUBIC-P   CUBIC-I   CUBIC-F

The 5 Bravais lattices in 2D
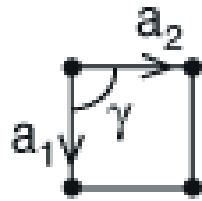
square $a_1 = a_2$ $\gamma = 90°$

hexagonal $a_1 = a_2$ $\gamma = 120°$
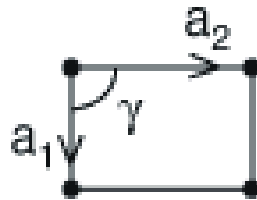
rectangular $a_1 \neq a_2$ $\gamma = 90°$
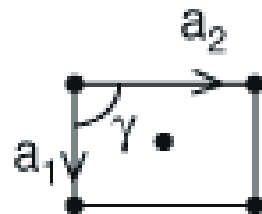
centered
rectangular $a_1 \neq a_2$ $\gamma = 90°$

oblique $a_1 \neq a_2$ $\gamma \neq 90°, 120°$
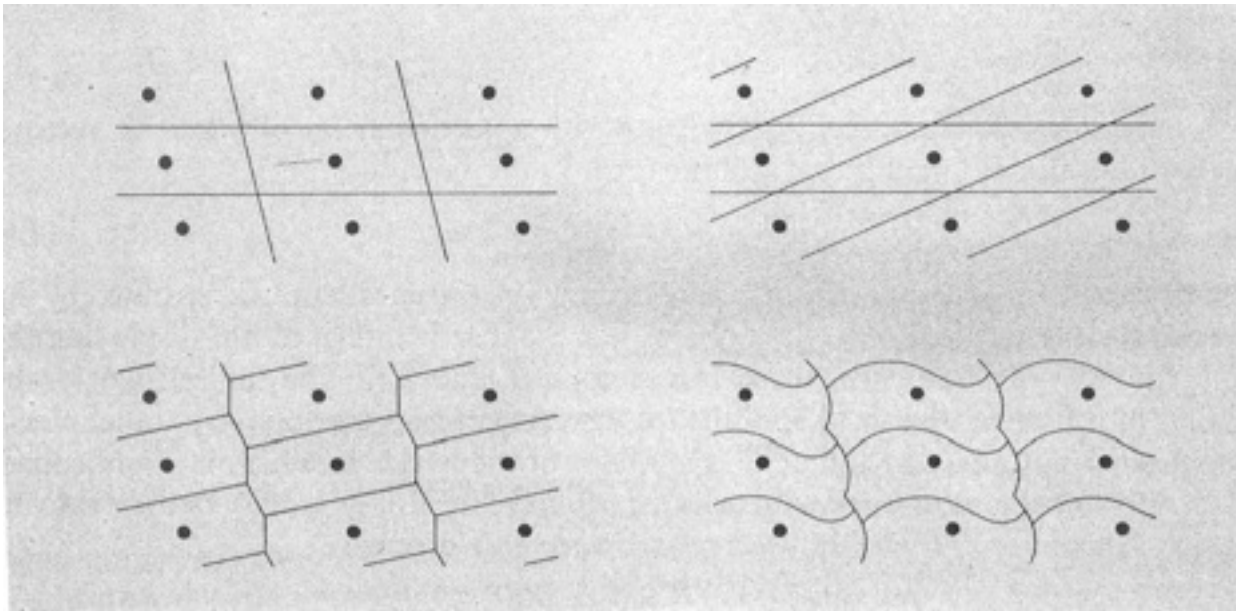
Wigner-Seitz cell
around a lattice point

- region of space that is closer to that point than to any other lattice point (topological def.)

- each point pertains to 1 WS cell

- translation => covers the whole space

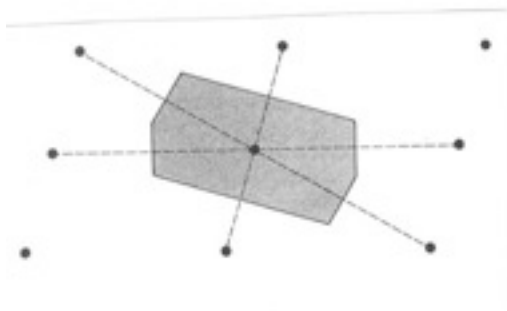- no reference to a particular choice of the primitive vectors: same symmetry of the lattice!
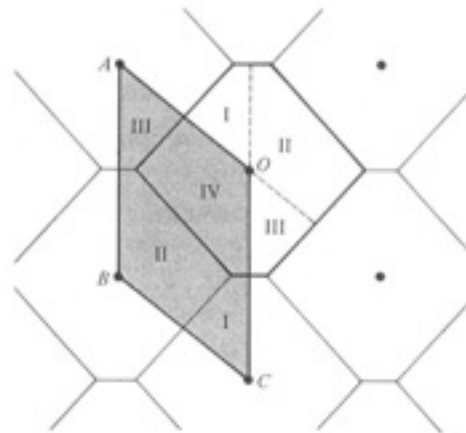
a Wigner-Seitz cell: construction and properties
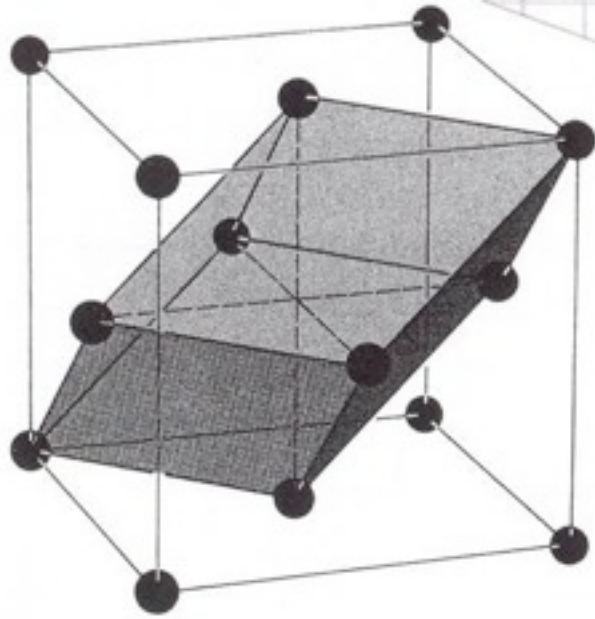

Wigner-Seitz
Elementary cell
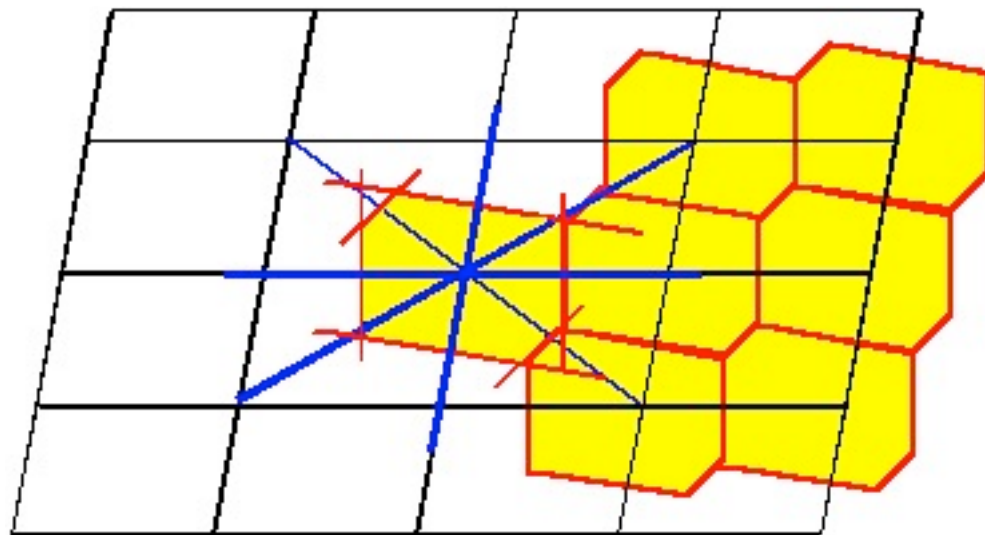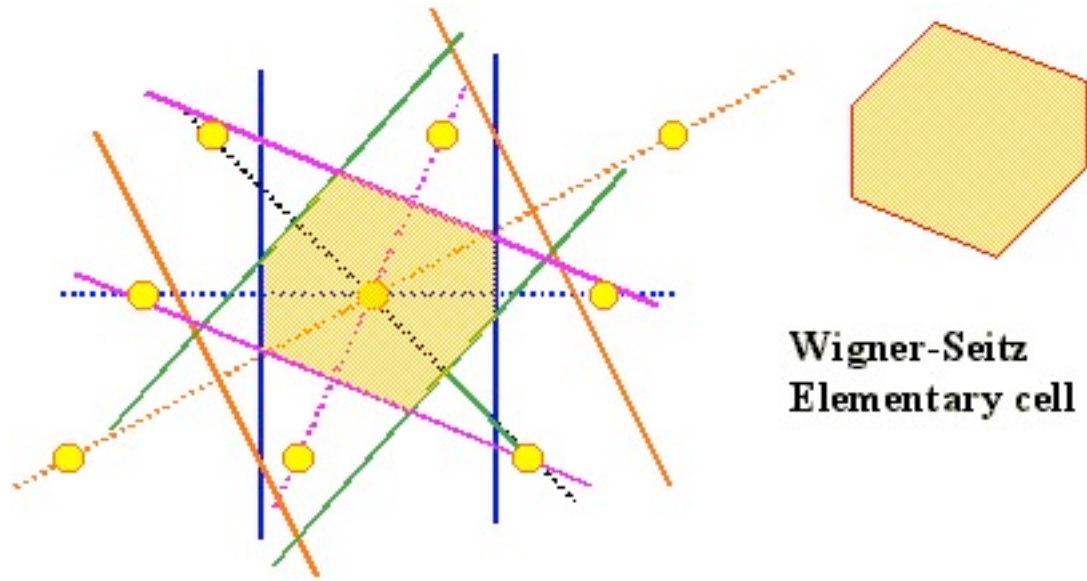
2D examples

some possible choices of **primitive unit cells** for oblique lattice
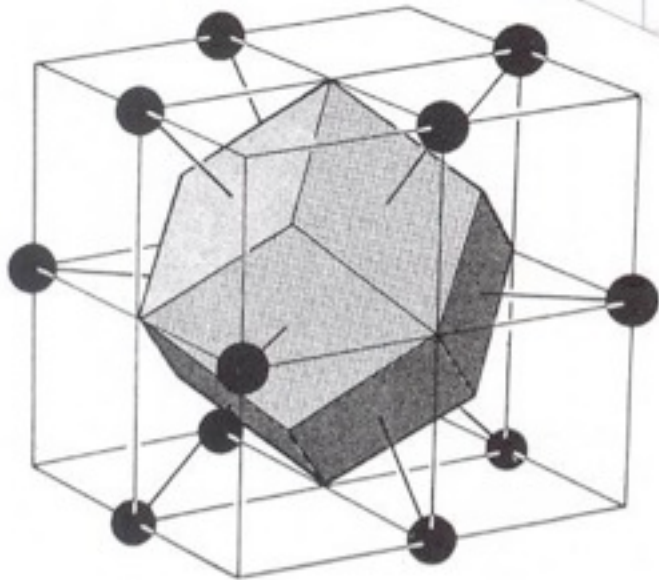
Wigner-Seitz cell for oblique lattice

3D - cubic lattices : example of FCC

Primitive and conventional unit cells for the face-centered cubic Bravais lattice. The conventional cell is the large cube. The primitive cell is the figure with six parallelogram faces. It has one quarter the volume of the cube, and rather less symmetry.
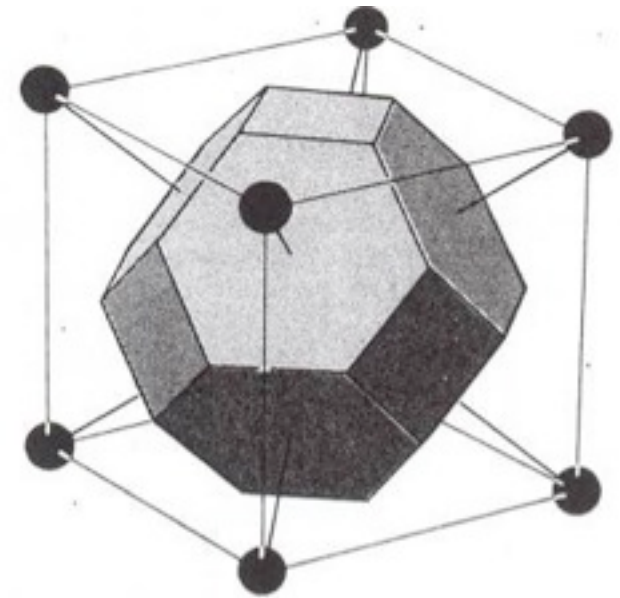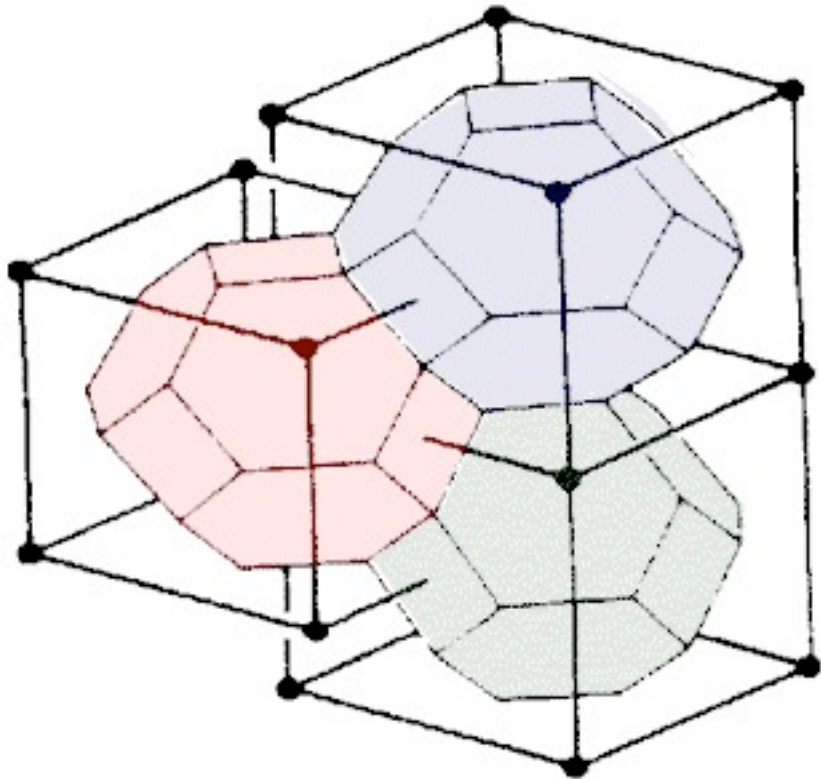
unit primitive ; conventional

Wigner-Seitz cell for the face-centered cubic Bravais lattice (a "rhombic dodecahedron"). The surrounding cube is *not* the conventional cubic cell of Figure 4.12, but one in which lattice points are at the center of the cube and at the center of the 12 edges. Each of the 12 (congruent) faces is perpendicular to a line joining the central point to a point on the center of an edge.
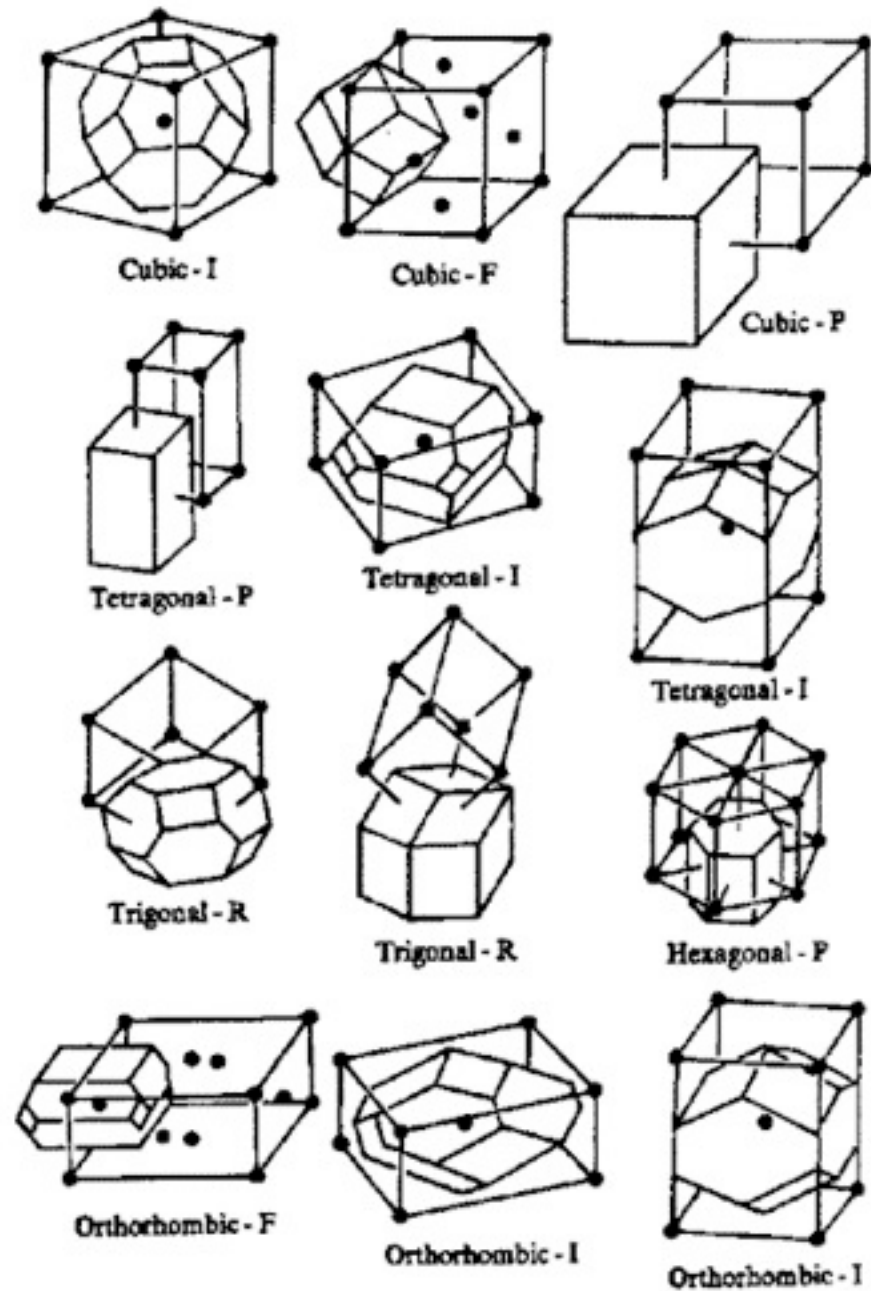
Wigner-Seitz

Wigner - Seitz for BCC

The Wigner-Seitz cell for the body-centered cubic Bravais lattice (a "truncated octahedron"). The surrounding cube is a conventional body-centered cubic cell with a lattice point at its center and on each vertex. The hexagonal faces bisect the lines joining the central point to the points on the vertices (drawn as solid lines). The square faces bisect the lines joining the central point to the central points in each of the six neighboring cubic cells (not drawn). The hexagons are regular (see
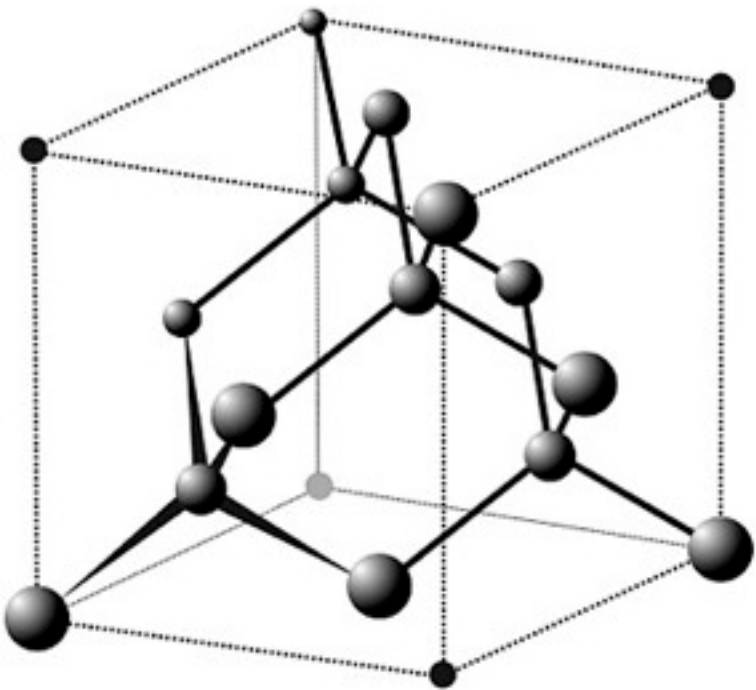
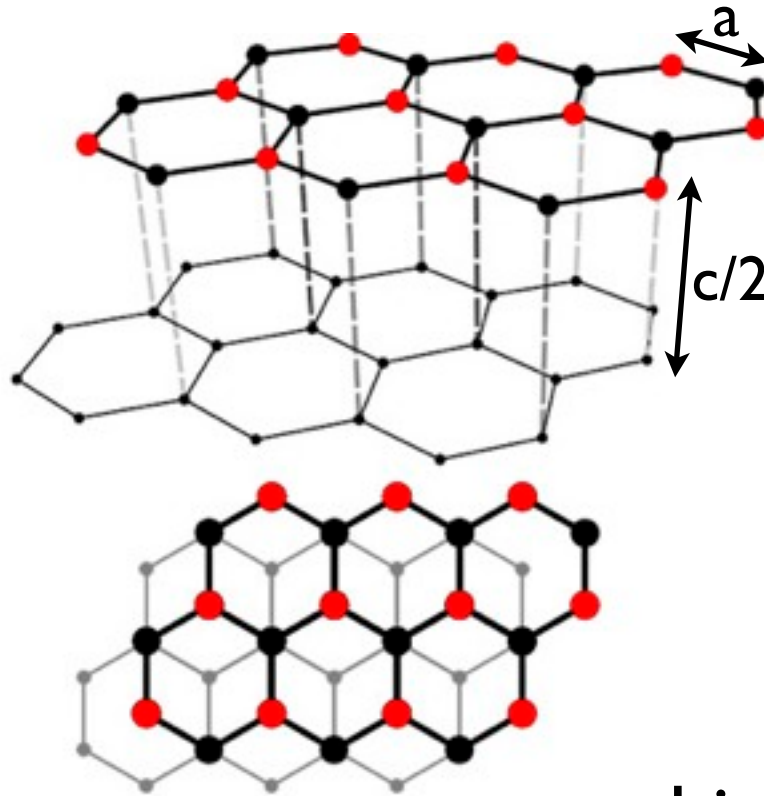# Wigner - Seitz for all Bravais Lattices

Cubic - I

Cubic - F

Cubic - P

Tetragonal - P

Tetragonal - I

Tetragonal - I

Trigonal - R

Trigonal - R

Hexagonal - P

Orthorhombic - F

Orthorhombic - I

Orthorhombic - I

Bravais lattices with basis
example: two allotropic forms of Carbon

diamond

graphite

primitive vectors
and
vectors of the basis:

$$a_1 = a\left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$$

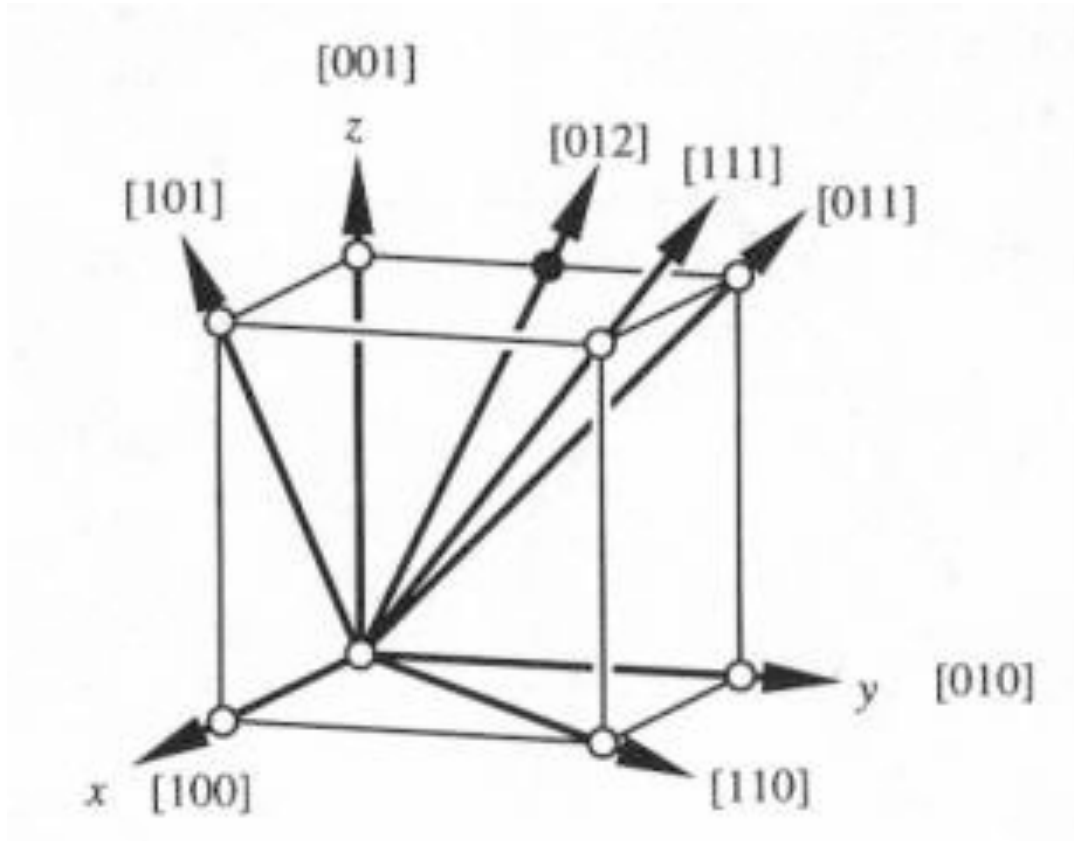$$a_2 = a\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$$

$$a_3 = c\,(0, 0, 1)$$

$$d_1 = (0, 0, 0)$$

$$d_2 = \left(0, \frac{a}{\sqrt{3}}, 0\right)$$

$$d_3 = \left(0, 0, \frac{c}{2}\right)$$

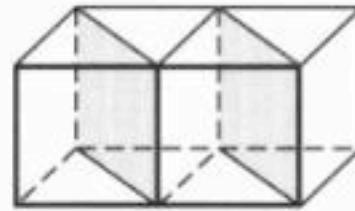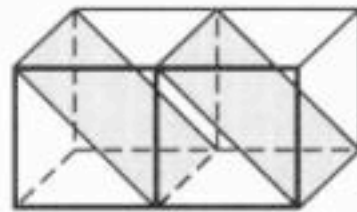$$d_4 = \left(0, \frac{2a}{\sqrt{3}}, \frac{c}{2}\right)$$
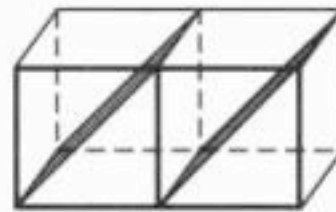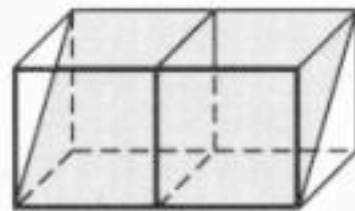
Directions

Miller indexes