Course of Geothermics

Dr. Magdala Tesauro

Course Outline:

1. Thermal conditions of the early Earth and present-day Earth's structure
2. Thermal parameters of the rocks
3. Thermal structure of the lithospheric continental areas (steady state)
4. Thermal structure of the lithospheric oceanic areas
5. **Thermal structure of the lithosphere for transient conditions in various tectonic settings**
6. Heat balance of the Earth
7. Thermal structure of the sedimentary basins
8. Thermal maturity of sediments
9. Mantle convection and hot spots
10. Magmatic processes and volcanoes
11. Heat transfer in hydrogeological settings

Effects of Surface Temperature Changes

The average surface temperature, T_0, can be estimated from:

$$T_0 = 3 + (T_{av.min} + T_{av.max})/2$$

$T_{av.min}$ = average annual minimum temperature
$T_{av.max}$ = average annual maximum temperature

- **Natural daily and seasonal temperature fluctuation propagate into the crust, but the effect decays exponentially with depth**

The effect of periodic surface heating is defined by: $T_\theta = T_0 \times \exp(-\varepsilon z)\sin(\omega t - \varepsilon z)$

The equation describes the departure (T_θ) from a mean value of T at a specific depth, z, and time t, resulting from a surface heating cycle with amplitude T_0 and frequency ω ($\omega = 2\pi/P$, P=period).

$\varepsilon = (\pi/P\kappa)^{1/2}$ ε=Thermal property of the medium

T_0=amplitude of the surface temperature cycle

$\sin(\omega t - \varepsilon z)$ =time lag between the temperature perturbation at the surface and at depth

$\exp(-\varepsilon z)$ = decay in the amplitude of the temperature perturbation with depth

$z = 2\pi/\varepsilon = 2\pi/(\pi/P\kappa)^{1/2} = (4\pi P\kappa)^{1/2}$ Depth at which the T fluctuation is in phase with the surface cycle (**$\varepsilon z = 2\pi$**), defining z_{wl}

z_{wl}=Effective wavelength for a temperature cycle near the surface of the Earth

The magnitude of the temperature perturbation at a depth of one effective wavelength is given by:

$$exp(-\varepsilon z_{wl}) = \exp(-2\pi) = 0.0019$$

Effects of Surface Temperature Changes

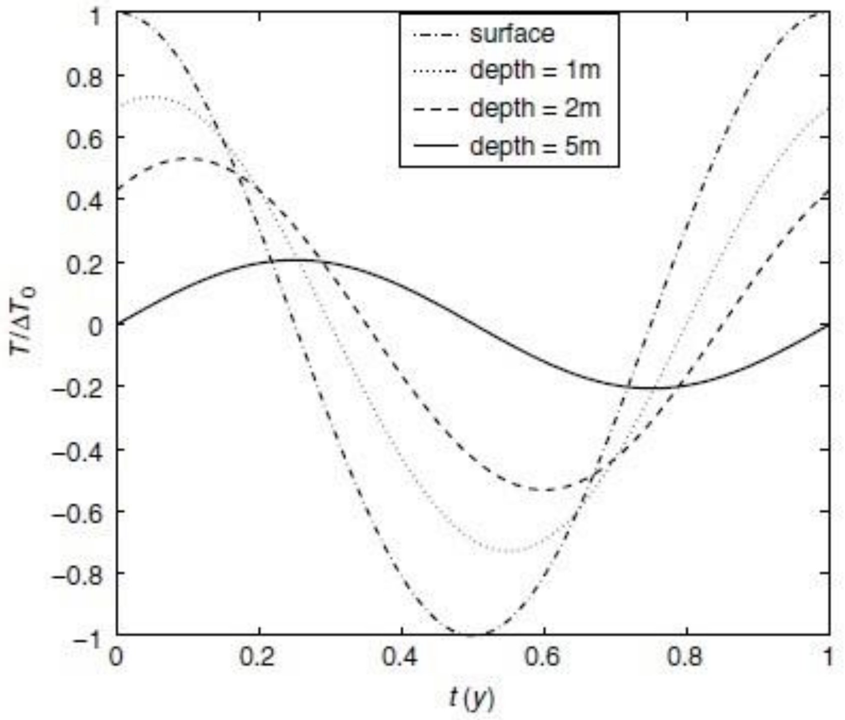
$$(\partial T/\partial z)_a = (\partial T/\partial z) + (\partial T_\theta/\partial z)$$ $(\partial T/\partial z)_a$ **= apparent thermal gradient**

$$(\partial T_\theta/\partial z) = T_0 \times (-\varepsilon)\exp(-\varepsilon z) \times [\sin(\omega t - \varepsilon z) + cos(\omega t - \varepsilon z)]$$ $(\partial T_\theta/\partial z)$ **= magnitude of perturbation**

Maximum disturbance is attained when: $\sin(\omega t - \varepsilon z) = \cos(\omega t - \varepsilon z)$ for $t_{\max} = (\pi n + \pi/4 + \varepsilon z)/\omega$

$$|(\partial T_\theta/\partial z)|_{\max} = T_0 \times (-\varepsilon) \times \exp(-\varepsilon z) \times [\sin(\pi n + \pi/4) + \cos(\pi n + \pi/4)] = T_0(\varepsilon)\exp(-\varepsilon z) \times [\sqrt{2}]$$

The threshold depth, z_{min}, at which the maximum departure from mean gradient is no longer significant: $z_{\min} = -\frac{1}{\varepsilon}\ln\left|\frac{0.01}{T_0\varepsilon\sqrt{2}}\frac{\partial T}{\partial z}\right|$



Effects of Surface Temperature Changes

- The effect in depth of surface temperature changes depends on the magnitude of the temperature step, the time since the event and the thermal diffusivity of the ground.

Climatic changes can be modelled as discrete events, each with an associated step function in surface temperature:

$$T_\theta = T_0 \times \mathrm{erfc}[z/(2\sqrt{\kappa t})]$$

T_θ = departure from original equilibrium temperature at depth *z* and time *t* after an instantaneous change in surface temperature of T_0

The effect of more than one temperature step (climatic event) is found by: $T_\theta = \Sigma T_{\theta i}$

The change in thermal gradient β due to a change in surface temperature, T_0, is: $\Delta\beta = -T_0 \times [(\pi\kappa t)^{-1/2} \times \exp(-z^2/(4\kappa t))]$

The effect of more than one event can be found: $\Delta\beta = \Sigma\Delta\beta_i$

We can define a skin depth *L* at which the amplitude of the *T* variations is *1/e* of that at the surface of the Earth:

L=1/e $\quad L = \sqrt{\dfrac{2\kappa}{\omega}} \quad \kappa=10^{-6}m^2s^{-1}$

skin depth (*L*) for the daily *T* variation ($\omega=7.27\times10^{-5}s^{-1}$) is less than 20 cm
skin depth (*L*) for the yearly *T* variation ($\omega= 2\times10^{-7}s^{-1}$) is 3.3 m
skin depth (*L*) for an ice age (10^5 yr) *T* variation ($\omega=1.99\times10^{-12}s^{-1}$) is >1km

Steady *vs* Transient Geotherms

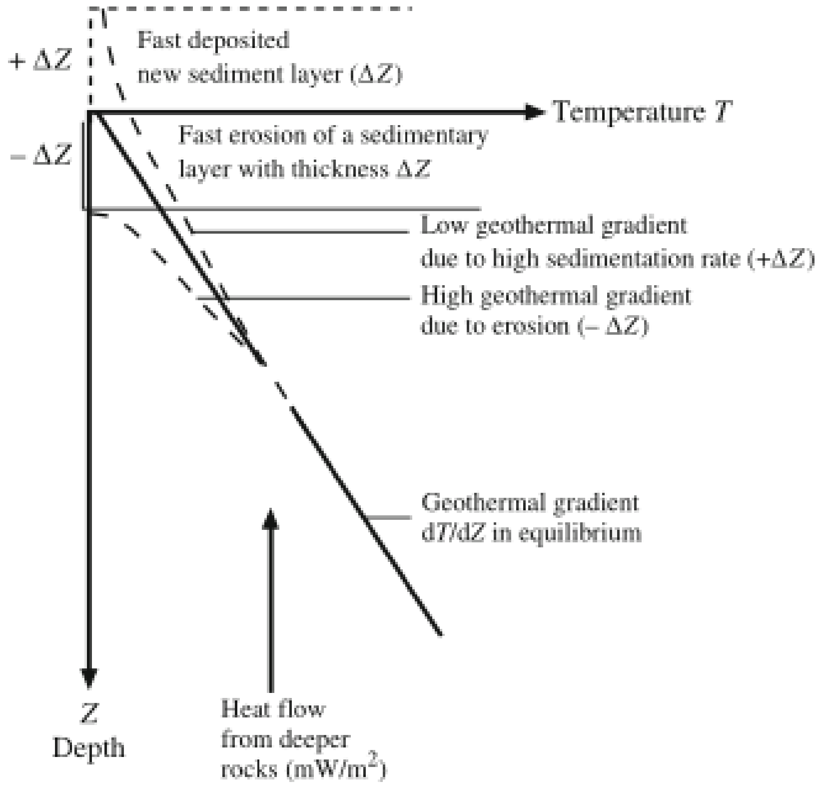
Heat flow > 90 mWm^2 imply melting in the crust or a weak lithospheric mantle
(other heat transport mechanisms are effective in tectonic active areas)

Crustal thickness variations imply changes of crustal heat production and deformation
(change of temperature distribution)

- Erosion or crustal extension initially cause steeper geotherms and enhanced heat flux and later the reduced crustal thickness and possible injection of basaltic melts (depleted in radioelements) leads to a lower heat flux than initial.
- Crustal thickening causes the geothermal gradient and the heat flux to decrease at first and then to increase due to higher crustal heat production (e.g., Tibet and Alps).
- Heat flux may record shallow processes such as the cooling of recently emplaced plutons. The anomalously high heat flux in the Basin and Range Province (about 110 mWm^2) and the high elevation (about 1750 m) is consistent with an extension of 100% and presence of shallow magma intrusions.

Steady *vs* Transient Geotherms

- Erosion or crustal extension cause initially steeper geotherms and enhance heat flux and when these transient effects decay the reduced crustal thickness leads to a lower heat flux.
- Sedimentation or crustal thickening causes the geothermal gradient and heat flux to decrease at first and then to increase due to higher crustal heat production.
- Other transient conditions are produced by crustal melting in the upper crust modify the vertical distribution of radioelements.



Crustal temperature return to equilibrium with local heat sources in < 100 Myr, while thick lithosphere last ~ 500 Myr

Cooling of rocks in proximity of intrusions (Step-shaped temperature distributions)

- **For the thermal modeling of intrusions it is possible to assume that their emplacement is infinitely rapid, compared to the time of the subsequent thermal equilibration (*instantaneous heating model*).**

T_i = temperatures of the intrusion
T_b= temperatures of the host rock

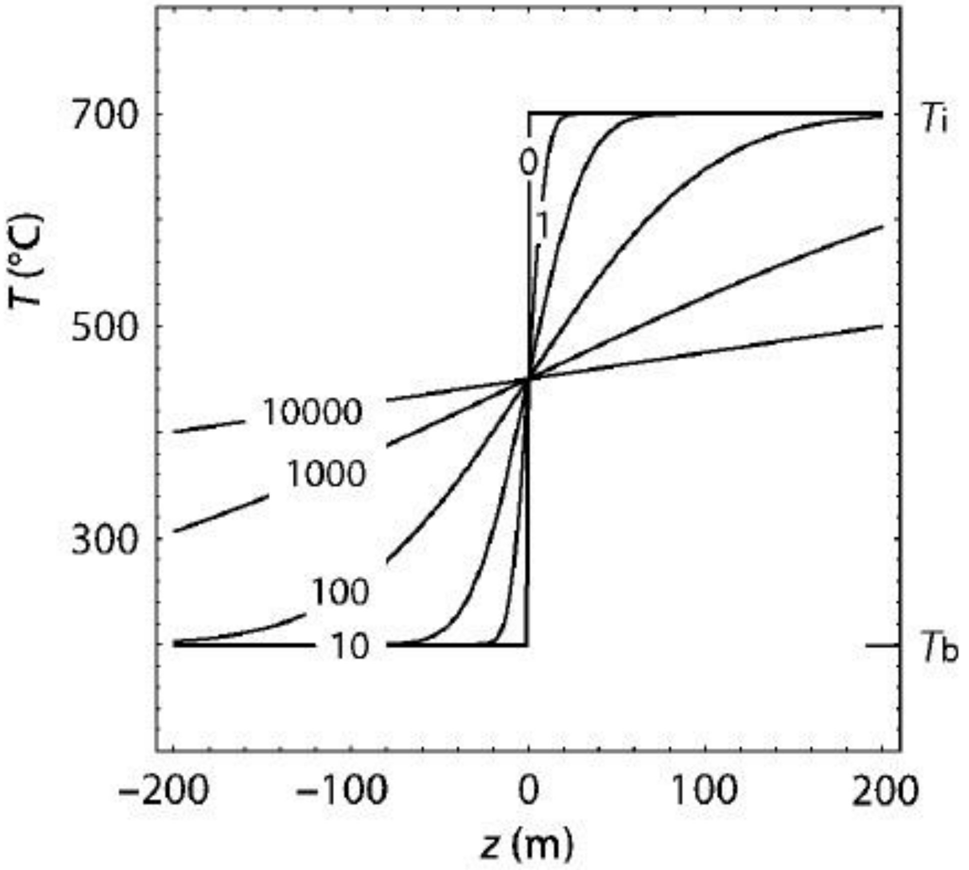
If we choose a one dimensional coordinate system in which the origin *z* = 0 is exactly at the contact of the model intrusion, then the initial and boundary conditions can be:
Initial Conditions: *T = Ti* for all *z> 0* and *T=Tb* for all *z<0* at *t=0*
Boundary Conditions: *T = Ti* for all *z=* $+\infty$ and *T=Tb* for all *z=* $-\infty$ at *t>0*

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2} \qquad T = T_\mathrm{b} + \frac{(T_\mathrm{i} - T_\mathrm{b})}{2}\left(1 + \mathrm{erf}\left(\frac{z}{\sqrt{4\kappa t}}\right)\right)$$

In another coordinate system in which the coordinate origin is located at a distance *l* from the temperature step:
Initial Conditions: *T = Ti* for all *z> l* and *T=Tb* for all *z<l* at *t=0*

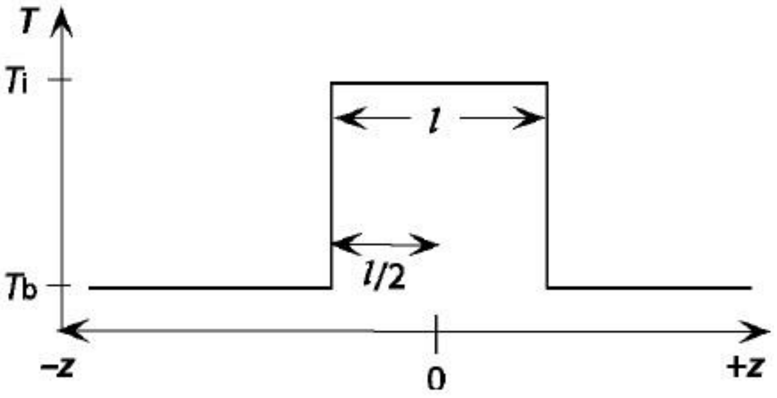
$$T = T_\mathrm{b} + \frac{(T_\mathrm{i} - T_\mathrm{b})}{2}\left(1 + \mathrm{erf}\left(\frac{z - l}{\sqrt{4\kappa t}}\right)\right)$$

Temperature profiles at different times (in years) after the intrusion event



- Thermal evolution on both sides of the mean temperature between T_i and T_b develops symmetrically.

Cooling of rocks in proximity of intrusions (Step-shaped temperature distributions)



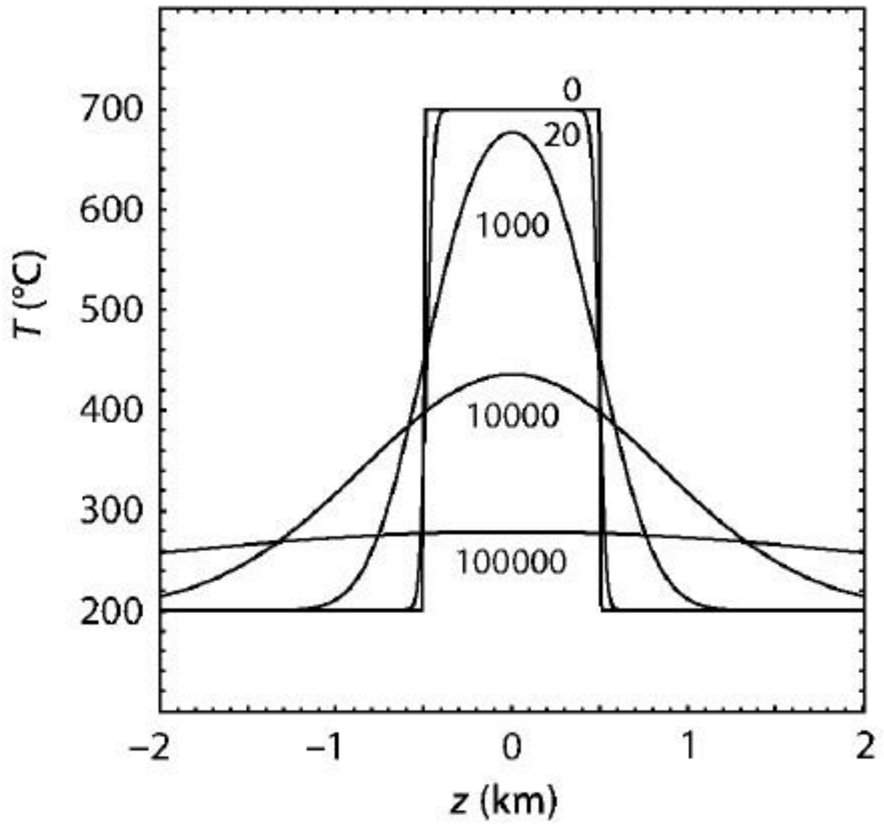
For a coordinate system with its origin in the center of a dike with the thickness *l*, the initial conditions may be described by:

$T=T_i$ for $-(l/2) < z < (l/2)$

$T=T_b$ for $(l/2) < z < -(l/2)$

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2}$$

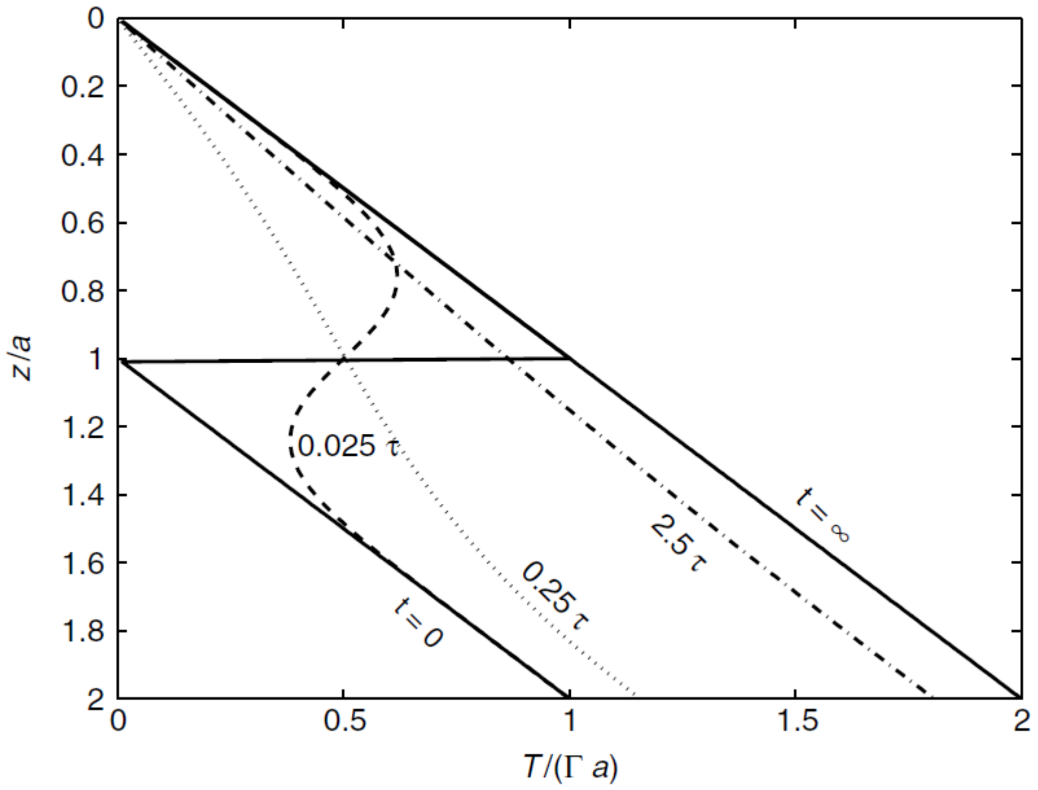
$$T = T_b + \frac{(T_i - T_b)}{2}\left(\mathrm{erf}\left(\frac{0.5l - z}{\sqrt{4\kappa t}}\right) + \mathrm{erf}\left(\frac{0.5l + z}{\sqrt{4\kappa t}}\right)\right)$$



Transient Effects

Effect of Overthrusting: Stacking of Two Slabs

- Underthrusting of a cold slab under the lithosphere will initially cool the lithosphere, followed by thermal re-equilibration.
- Thermal re-equilibration after the stacking of two slabs requires a time more than twice the thermal time constant $\tau = a^2/\kappa$.
  E.g.: For a 100 km thick slab with $\tau \approx$300My, thermal re-equilibration takes >2.5 x $\tau \approx$ 750 Myr.



$T(z,t=0)=0$ for $z<a$ $\quad T(z,t=0)=\Delta T=-\Gamma a$ for $a<z<2a$.

$$T(z,t)=\Delta T\sum_{n=1}^{\infty}\frac{(-)^n}{k_n}\sin(k_n z/a)\sin(k_n)\exp(-k_n^2\kappa t/a^2)\quad z<a \qquad k_n=(2n-1)\pi/4$$

$$T(z,t)=\Delta T\sum_{n=1}^{\infty}\frac{(-)^n}{k_n}\cos(k_n(z-2a)/a)\cos(k_n)\exp(-k_n^2\kappa t/a^2)\quad a<z<2a$$

During the time of re-equiliabration, the temperature in the overriding slab is lower than it was initially:

$$\frac{q(t)}{\lambda\Gamma}=\left(1-\sum_{n=1}^{\infty}(-)^n\sin(k_n)\exp(-\kappa k_n^2 t/a^2)\right)$$

z=depth
a=thickness of the slab (100 km)
Γ= geothermal gradient
$\Gamma\lambda$=fixed heat flux at the base

$\tau = a^2/\kappa$ $\quad\tau$=317 Myr

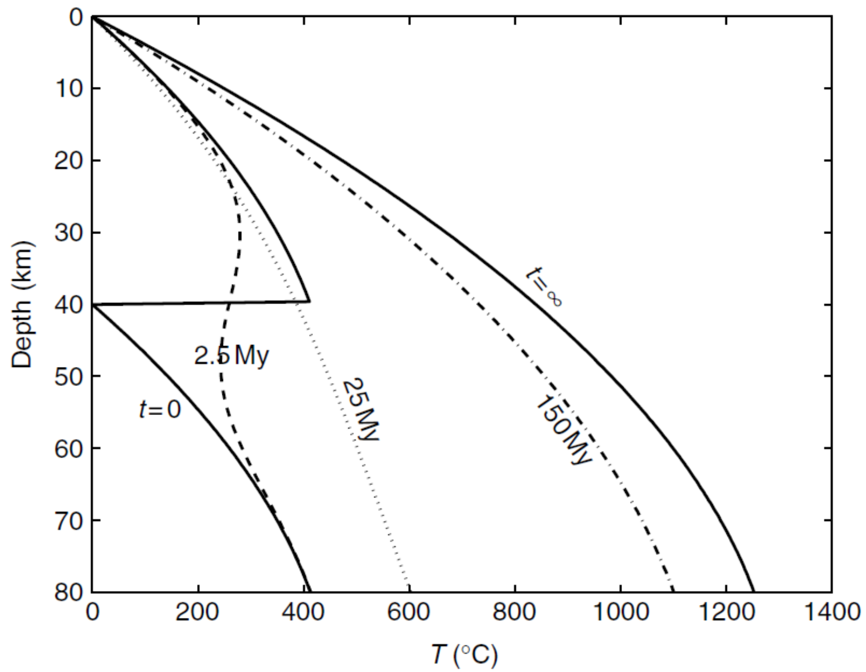
t=0.025τ=8 Myr

Temperature starts to increase again at ~0.25τ

Transient Effects

Effect of Overthrusting: Crustal Scale Thrusting

- During continental collision, one crustal block can thrust over another
- The increase in temperature following the superposition of two crustal blocks can be ~800 K for A of 0.8 mWm^3
- It requires more than 25 Myr for $T > T_0$



When one block overrides another one, both with the same thickness a, and with uniform heat generation A and assuming no heat flux at the base (Q_m=0), $T(z)$ is:

$$T = T_0 + \frac{(Q_m + Ay_c)}{K} y - \frac{A}{2K} y^2$$ or $$T(z) = \frac{2Aaz}{\lambda} - \frac{Az^2}{2\lambda}$$ (steady state conditions)

The initial temperature perturbation and the transient temperature are:

$$\Delta T(z, t=0) = -\Gamma z = \frac{-Aaz}{\lambda}, \; 0 < z < a$$

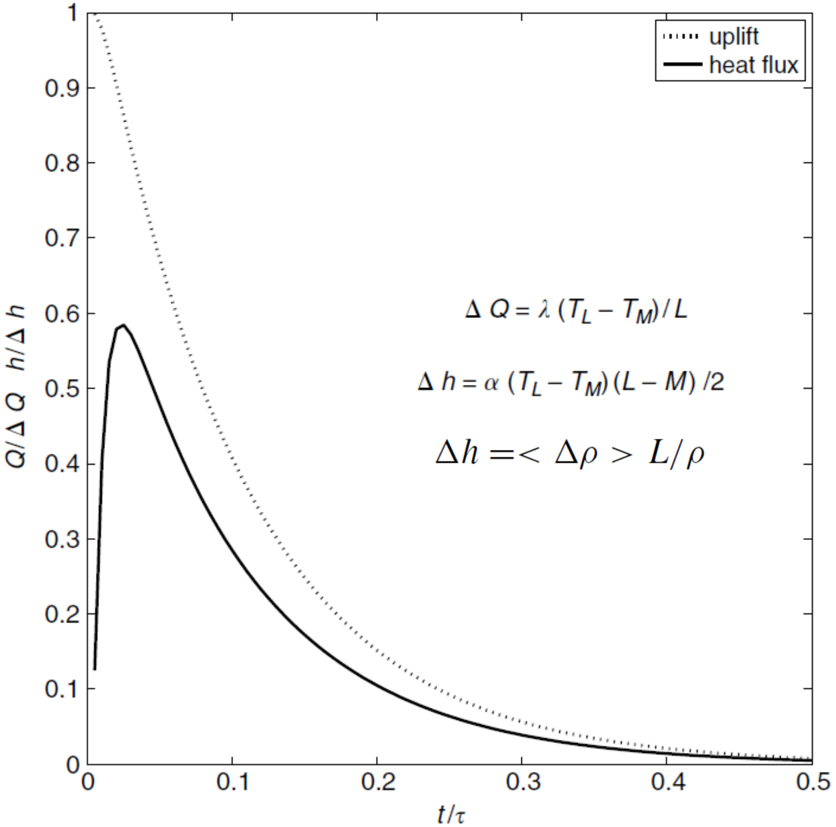
$$T(z,T) = \frac{\Gamma a}{2} \sum_{n=1}^{\infty} \frac{(-)^n}{k_n} \sin(k_n) \sin(k_n z/a) \exp(-k_n^2 \kappa t/a^2) + \Gamma a \sum_{n=1}^{\infty} \frac{(-)^n}{k_n^2} \cos(k_n) \sin(k_n z/a) \exp(-k_n^2 \kappa t/a^2)$$

$$\Delta T(z, t=0) = -\frac{3\Gamma a}{2} = \frac{-3Aa^2}{2\lambda}, \; a < z < 2a$$

$$T(z,t) = \frac{\Gamma a}{2} \sum_{n=1}^{\infty} \frac{(-)^n}{k_n} \cos(k_n) \cos(k_n(z-2a)/a) \exp(-k_n^2 \kappa t/a^2) + \Gamma a \sum_{n=1}^{\infty} \frac{(-)^n}{k_n^2} \sin(k_n) \cos(k_n(z-2a)/a) \exp(-k_n^2 \kappa t/a^2)$$

with $k_n = (2n-1)\pi/4$

Mantle Delamination



T_M= Moho T
T_L= T at the base of the lithosphere
L=thickness of the lithosphere
M or z_m= *Moho depth*

Thermal perturbation in the mantle lithosphere and transient temperature perturbation are:

$$T(z,t=0) = (T_L - T_M)\frac{L-z}{L-z_M},\ z_M < z < L,$$

$$T(z,t) = (T_M - T_L)\sum_{n=1}^{\infty}\left(\left(\frac{z_M}{L}-1\right)\cos\frac{n\pi z_M}{L} - \frac{1}{(n\pi)}\sin\frac{n\pi z_M}{L}\right)\times\frac{1}{n\pi}\sin\frac{n\pi z}{L}\exp(-n^2\pi^2\kappa t/L^2)$$

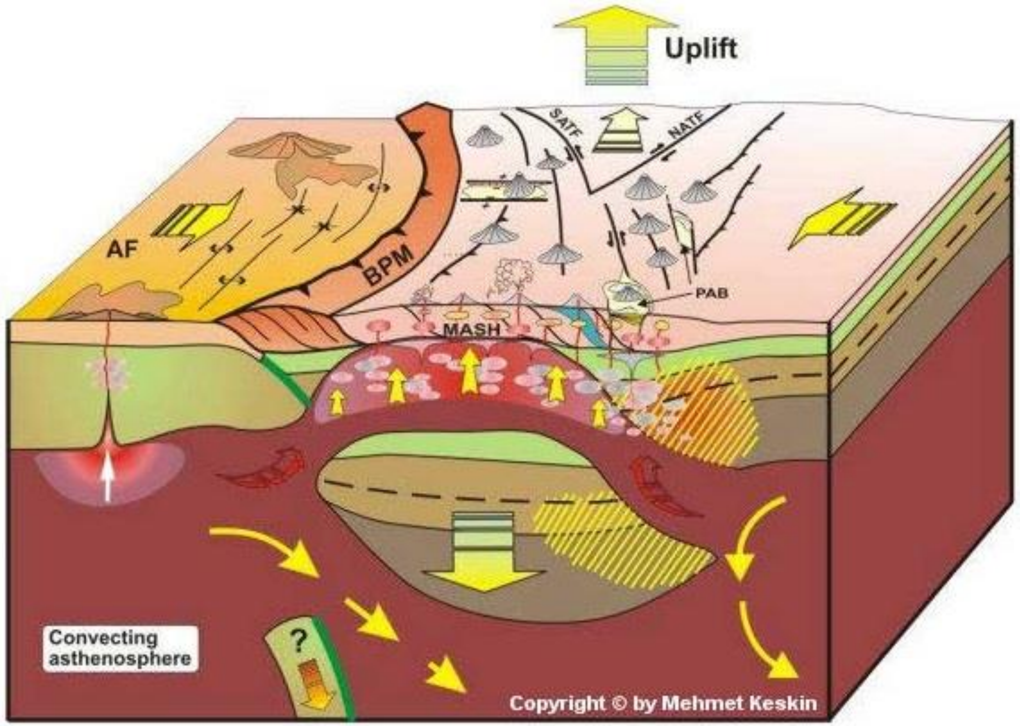
Surface heat flux and average change in density of a lithosoheric column are:

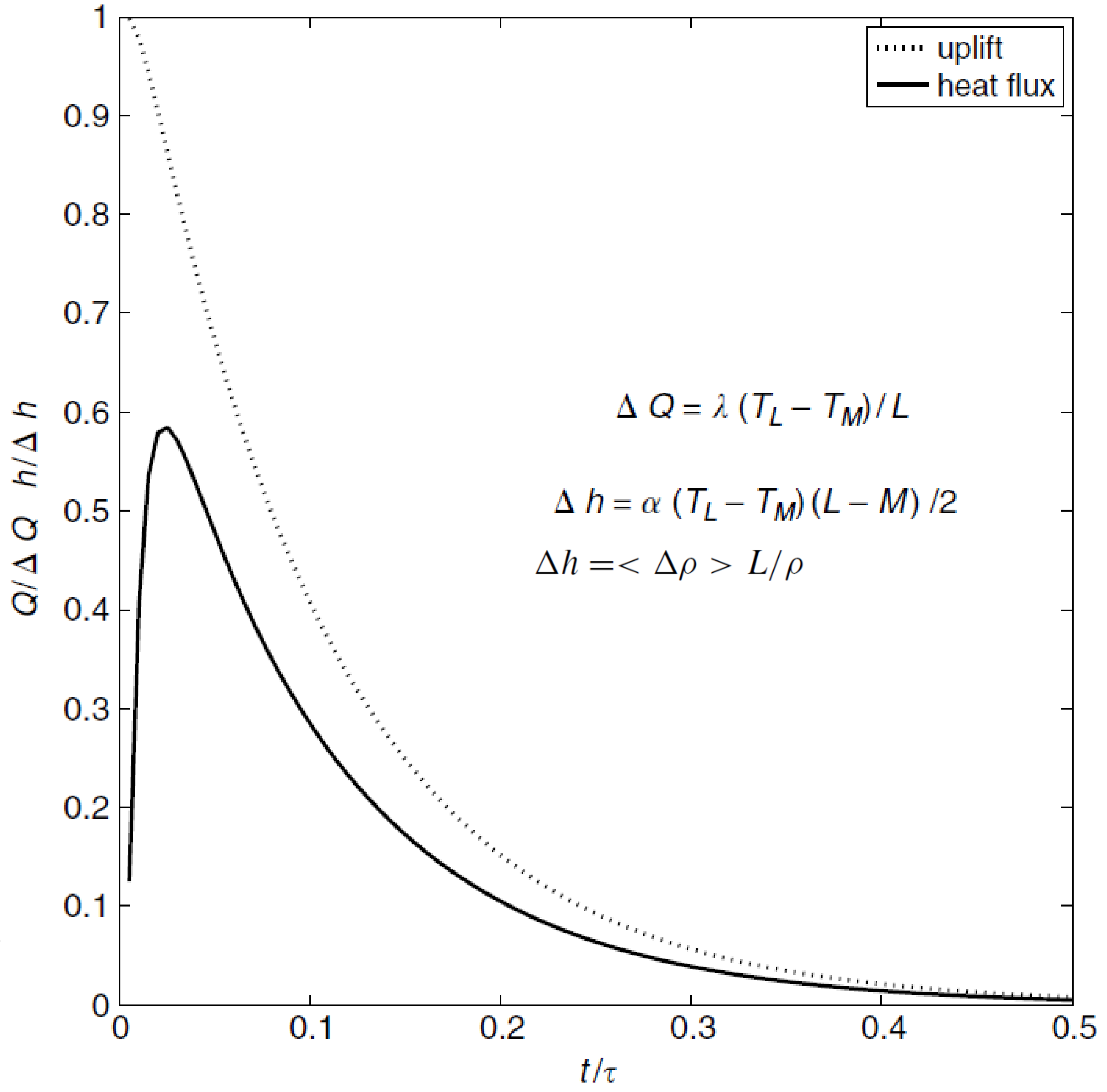
$$q(z,t) = \frac{\lambda(T_M - T_L)}{L}\sum_{n=1}^{\infty}\left(\left(\frac{z_M}{L}-1\right)\cos\frac{n\pi z_M}{L} - \frac{1}{n\pi}\sin\frac{n\pi z_M}{L}\right)\times\exp(-n^2\pi^2\kappa t/L^2)$$

$$\frac{<\Delta\rho>}{\rho} = \frac{\alpha}{2}(T_L - T_M)\frac{(L-z_M)}{L}\sum_{n=1}^{\infty}\frac{2}{(2n-1)^2\pi^2}\times\left(\left(\frac{z_M}{L}-1\right)\cos\frac{(2n-1)\pi z_M}{L} - \frac{1}{(2n-1)\pi}\sin\frac{(2n-1)\pi z_M}{L}\right)\times\exp(-(2n-1)^2\pi^2\kappa t/L^2)$$

Mantle Delamination

The rapid peeling off of the mantle lithosphere and its replacement by asthenospheric material cause rapid plateau uplift.

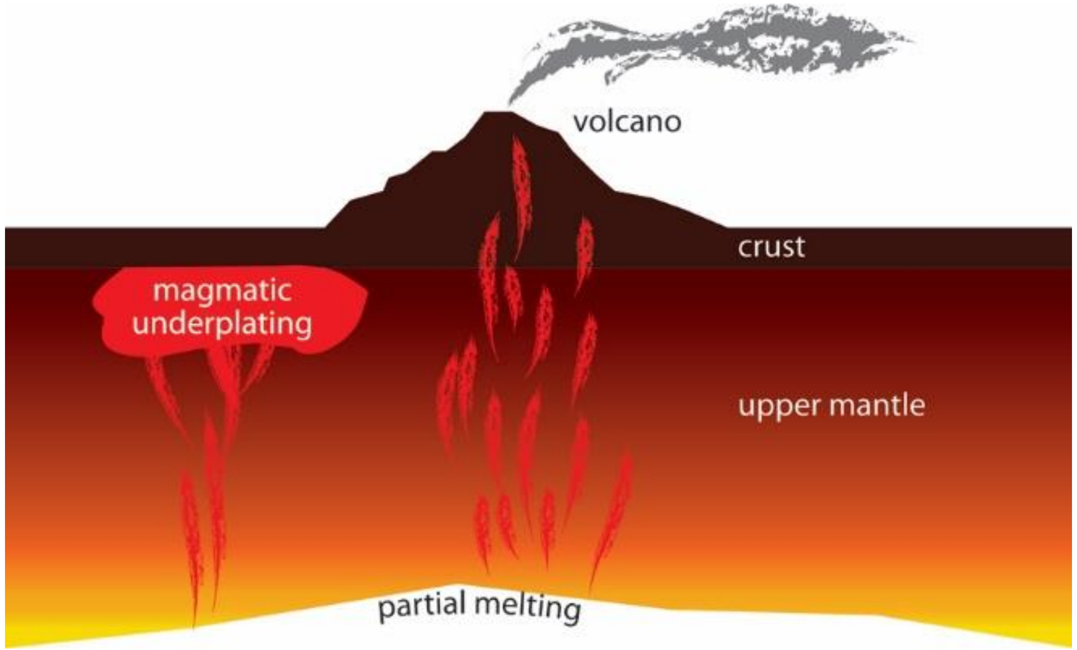




- With Moho temperature of ~ 600°C and a 150 km thick lithosphere, an uplift ≈ 1.1 km can be achieved almost instantly.
- Relaxation of the topography to half the initial level requires ~ 80 My.
- The peak in surface heat flux perturbation lags 25 My behind the uplift and is 7.5 mW m^{-2} ≈ 0.5 × λ × $(T_M - T_L)/L$.

τ=714 Myr

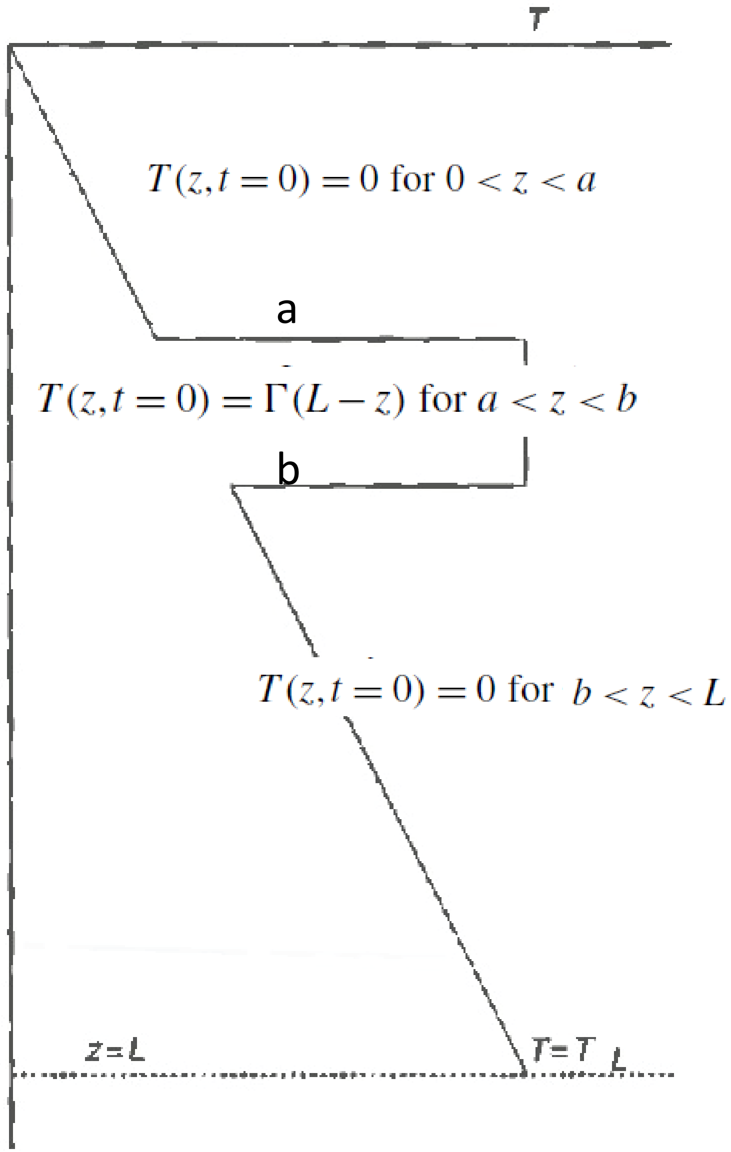
Magmatic Underplating



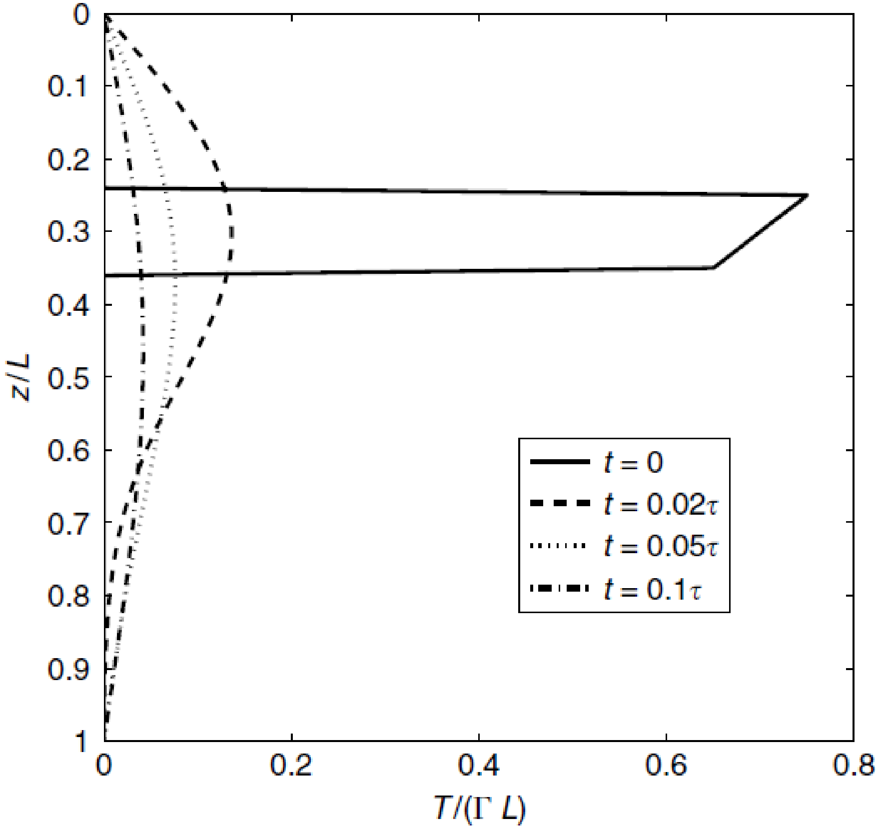
$$T(z,t) = \sum_{n=1}^{\infty} A_n \sin(n\pi z/L) \exp(-n^2\pi^2 t/\tau)$$

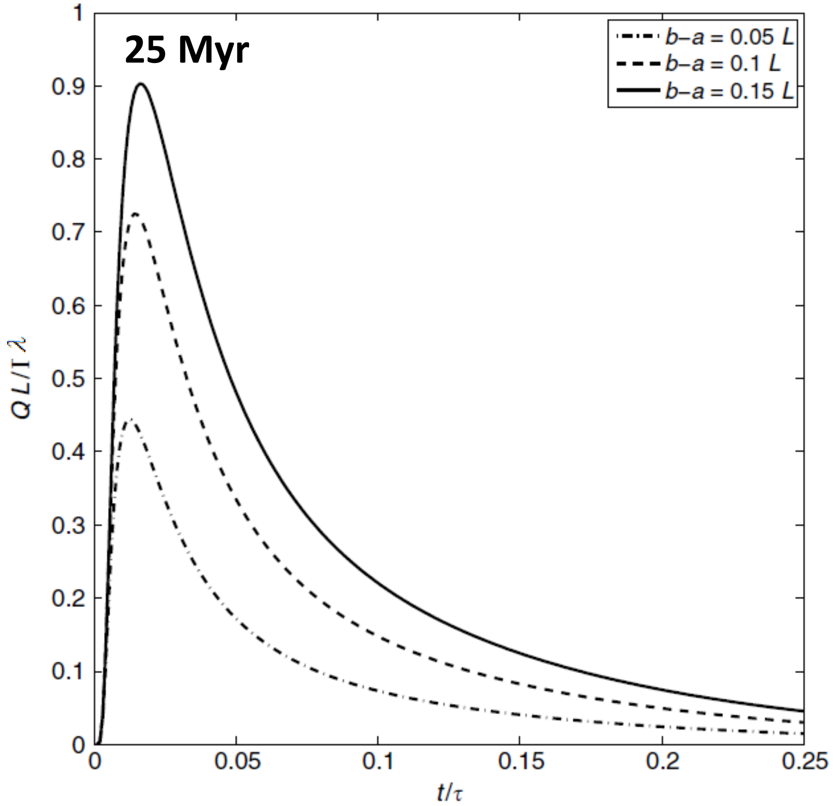
$$\frac{A_n}{2\Gamma} = \frac{(L-a)(\cos(n\pi a/L) - (L-b)\cos(n\pi b/L))}{n\pi} + \frac{L^2}{n^2\pi^2}(\sin(n\pi a/L) - \sin(n\pi b/L))$$

$$q(t) = \frac{\lambda}{L} \sum_{n=1}^{\infty} A_n n\pi \exp(-n^2\pi^2 t/\tau)$$



Magmatic Underplating





The transient surface heat flux reaches its peak a short time after underplating occurs (≈25 My, depending on the intrusion depth) and its amplitude can be significant.

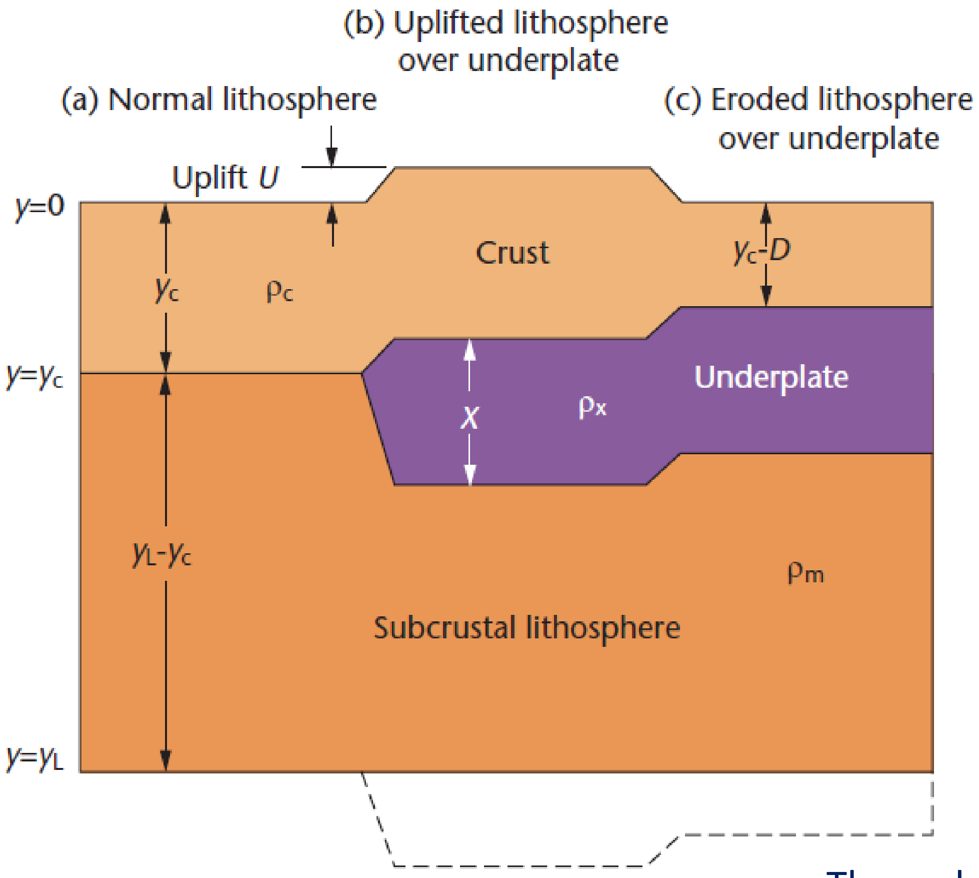
t_{max} = time taken to reach the maximum temperature at dimensionless distance; $y^* = y/a$

Maximum temperature $T^*_{max} \approx \sqrt{\frac{2}{\pi e}}\frac{1}{y^*}$ a=half-width of the body $t^*_{max} \approx \frac{y^{*2}}{2}$

Temperature perturbation decays quite rapidly and is large only close to the intruding layer

Magmatic Underplating (Uplift)

- The density of igneous rocks, generated by adiabatic decompression of the mantle, ranges between 2990 and 3070 kgm^{-3} (< 3300 kgm^{-3}). Thus, the replacement of the lithospheric mantle with these igneous rocks causes uplift.
- The amplitude of the thermal uplift following underplating depends on the thickness of the layer, but it is usually modest. For an excess temperature of 600 K, the surface uplift will be 18×10^{-3} times the layer thickness (e.g., the underplated layer must be at least 60 km thick to cause an uplift of the order of 1000 m).



$$h(t) = \alpha \int_0^L T(z,t)dz = 2\alpha L \sum_{n=1}^{\infty} \frac{A_{2n-1}}{2n-1} \exp(-(2n-1)^2 \pi^2 t/\tau)$$ *h(t)*=thermal uplift

Uplift due to the density difference

Pressure at the depth y_L:

Normal Lithosphere

$$y_c \rho_c g + (y_L - y_c)\rho_m g$$

Uplifted Lithosphere

$$y_c \rho_c g + X\rho_x g + (y_L - y_c - X + U)\rho_m g$$

$$U = X\frac{(\rho_m - \rho_x)}{\rho_m} = X\left(1 - \frac{\rho_x}{\rho_m}\right)$$

(Uplifted lithosphere)

$$D = X\left(\frac{\rho_m - \rho_x}{\rho_m - \rho_c}\right)$$

(in case of erosion $y_c = y_c - D$)

The rock uplift for an underplate thickness of 5 km is one tenth of the underplate thickness

References

Main Readings:

Books:

- Jaupart and Mareschal, 2007, Heat Flow and Thermal Structure of the Lithosphere, Treatise of Geophysics, vol.6, 217-251.
- Beardsmore and Cull, 2001: Crustal Heat Flow, Chapter 3, Thermal Gradient, 47-89.