Condensed Matter Physics I
II test - 14 January 2016
(2.5 hours)

- Solve all the exercises.
- Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.

Exercise 1: *Weak potential*

Consider a 1D lattice with spacing a and with a crystalline potential composed by a series of rectangular wells surrounding the atoms, with depth U_0 and width $a/4$.

1. Write the expression of the crystalline potential.
2. Considering the weak potential model (or nearly free electrons), sketch the first few bands in the first Brillouin zone (reduced representation).
3. Calculate the first energy gap opening at the Brillouin zone boundary.
4. Calculate also the second and the third gap and compare the magnitudes of the first three gaps.
5. Evaluate the first for U_0=4 eV and a=5 Å.

Exercise 2: *Semimetals*

(Hint: To solve this exercise you should use what you know about the free electron model)

Elemental Bismuth is an example of *semimetal*. It has a small overlap between the bottom of the conduction band and the top of the valence band, so that there are partially filled electron and hole bands. Consider the simple situation shown in the Figure, and bands with isotropic dispersion: one conduction-like band partially filled by electrons in its bottom and one valence-like band partially empty (filled by holes at its top). The extrema of the bands and the effective masses for electrons (holes) are E_e (E_h) and m_e (m_h) respectively. E_e and E_h (and also E_F) are very close each other.

1. Write the density of states $g_e(E)$ and $g_h(E)$ for electron and holes at T=0 for E close to E_F.
2. Using 1., write the total $g(E)$ (available electronic levels) for E close to E_F, and make a sketch of it.
3. Using $g_e(E)$ and $g_h(E)$, calculate the density of electrons n_e in the conduction-like band and the density of holes n_h in the valence-like band.
4. If n_h=n_e, calculate E_F in terms of E_e, E_h, m_e and m_h.
5. Calculate $g(E_F)$.
6. Write the electronic contribution to the heat capacity of this semimetal at very low temperature.