

# CLUSTER DYNAMICS FROM GALAXIES AND ICM - INTRACLUSTER MEDIUM

ICM Hydrostatic equation for gas  $\frac{d\phi}{dz} = -\frac{1}{\rho} \frac{dP}{dz}$

$$\frac{d\phi}{dz} = \frac{GM(z)}{z^2} = -\frac{1}{\rho} \frac{dP}{dz} = -\frac{1}{\rho} \frac{d}{dz} \left( \rho \frac{kT}{\mu m_p} \right)$$

$\phi$  potential  $\leftrightarrow$  whole cluster mass

$\rho, P$  density, pressure of ICM

$\mu$  mean molecular weight ( $\sim 0.58$ )

$m_p$  proton mass

$$\frac{GM(z)}{z^2} = -\frac{1}{\rho} \frac{k}{\mu m_p} \left( \rho \frac{dT}{dz} + T \frac{d\rho}{dz} \right)$$

$$M(z) = -\frac{kT}{G\mu m_p} z \left( \frac{d \ln \rho}{d \ln z} + \frac{d \ln T}{d \ln z} \right)$$

Observationally, the gas distribution is fitted with the ISOTHERMAL  $\beta$ -model (CAVALIERE & FUSCO FEMIANO 76)

$$3D \rho_x(z) = \rho_{x0} \left( 1 + \left( \frac{z}{z_{gx}} \right)^2 \right)^{-3/2} \beta_{fit, gas}$$

$\beta_{fit, gas} = \frac{2}{3}$

$$2D \Sigma_x(R) = \Sigma_{gx} \left( 1 + \left( \frac{R}{z_{gx}} \right)^2 \right)^{-3/2} \beta_{fit, gas} + \frac{1}{2}$$

$\downarrow$  projected onto the sky!

$$\Sigma(R) = \int_{-\infty}^{+\infty} \rho(z) dz = 2 \int_0^{\infty} \frac{\rho(z) dz}{\sqrt{z^2 + R^2}} \quad z = \sqrt{z^2 + R^2}$$

emissivity  $E_x \propto \rho_x^2 \Rightarrow$

X-ray surface brightness  $I_x = I_0 \left( 1 + \left( \frac{R}{z_{gx}} \right)^2 \right)^{-3\beta_{fit, gas} + 1/2}$

$\rho_x \rightarrow R^{-2}$   
 $\Rightarrow \rightarrow \infty \rightarrow R^{-1}$

GALAXIES

Jeans' equation  
(See Binney and Tremaine)

$$\Pi(r) = - \frac{\sigma_r^2(r)}{G} \left[ \frac{d \ln \rho}{d \ln r} + \frac{d \ln \sigma_r^2}{d \ln r} + 2\beta \right]$$

velocity anisotropy parameter

$$\beta(r) = \left( 1 - \frac{\sigma_{\theta\phi}^2}{\sigma_{\text{total}}^2} \right)$$

Observationally

3D  $\rho(r) = \rho_0 \left( 1 + \left( \frac{r}{r_0} \right)^2 \right)^{-3/2 \beta_{\text{fit pol}}}$

2D  $\Sigma(r) = \Sigma_0 \left( 1 + \left( \frac{R}{r_0} \right)^2 \right)^{-3/2 \beta_{\text{fit pol}} + 1/2}$

= King model modified  $\beta_{\text{fit pol}} \sim 1$   
= Hubble modified law

$$\rho \propto r^{-3} \rightarrow R^{-2}$$

$$\begin{array}{l} \Pi(r) \\ \text{From gas} \end{array} = \begin{array}{l} \Pi(r) \\ \text{From galaxies} \end{array} \quad \left\{ \begin{array}{l} \text{Assume that} \\ \text{both are isothermal} \\ \frac{dT}{dr} = 0 \quad \frac{d\sigma_r^2}{dr} = 0 \end{array} \right.$$

$$\frac{-kT}{\mu_{\text{mp}}} \frac{d \ln \rho_x}{d \ln r} = - \frac{\sigma_r^2}{G} \frac{d \ln \rho}{d \ln r} - \frac{2\beta \sigma_r^2}{r}$$

$$\frac{\frac{d \ln \rho_x}{d \ln r}}{\frac{d \ln \rho}{d \ln r} + \frac{2\beta}{r}} = \frac{\sigma_r^2}{kT \mu_{\text{mp}}} \approx \beta_{\text{spec}}$$

spectral  $\beta$   
Observations  
 $\beta_{\text{spec}} = \frac{\sigma_{\text{LOS}}^2}{kT \mu_{\text{mp}}}$

$$\sigma_r^2 \sim \sigma_{\text{LOS}}^2$$



For  $z \gg z_0$   
the  $\beta_{CM}$  and  $\beta_{gal}$  profiles  $\rightarrow$

$$\frac{d \ln \rho_x}{dz} \xrightarrow{z \gg z_0} \frac{-3 \beta_{RT, gas} \cdot \frac{1}{z}}{-3 \beta_{RT, gal} \cdot \frac{1}{z} - \frac{2\beta}{z}} =$$

$$= \frac{\beta_{RT, gas}}{\beta_{RT, gal} - \frac{2}{3} \beta}$$

OTHER POSSIBLE PROBLEM FROM THEORY OF VIOLENT RELAX.  $\beta_{spec} = 1$   
THIS IS TRUE FOR CLUSTERS

$$\Rightarrow \beta_{spec} = \frac{\beta_{RT, gas}}{\beta_{RT, gal} - \frac{2}{3} \beta}$$

Observations

$$\beta_{RT, gas} \sim \frac{2}{3}$$

$$\beta \sim 0$$

$$old \beta_{RT, gal} \sim 1$$

$$old \beta_{spec} \gg 1$$

$$\textcircled{\gg 1} = \frac{2}{3}$$

$\beta$  problem!

NOW

in 190  $\beta_{spec}$  with better  $\sigma_{los}$  measures  $\approx 1$

$$\beta_{RT, gal} \sim 0.8$$

$$\textcircled{1} \sim \frac{2/3}{0.8} \approx 1$$

ok NO LONGER  $\beta$  problem!

### 5.3.1 Cooling

The primary cooling process for intracluster gas is the emission of radiation by the processes discussed in Section 5.2.2 above. At temperatures  $T_g \gtrsim 3 \times 10^7$  K, the main emission mechanism is thermal bremsstrahlung, for which the total emissivity is

$$\begin{aligned} \epsilon^{ff} &= 1.435 \times 10^{-27} \bar{g} T_g^{1/2} n_e \sum_i Z_i^2 n_i \text{ ergs cm}^{-3} \text{ s}^{-1} \\ &\approx 3.0 \times 10^{-27} T_g^{1/2} n_p^2 \text{ ergs cm}^{-3} \text{ s}^{-1}, \end{aligned} \quad (5.21)$$

where  $\bar{g}$  is the integrated Gaunt factor, and  $Z_i$  and  $n_i$  are the charge and number density of various ions  $i$ . The second equation follows from assuming solar abundances and  $\bar{g} = 1.1$  in a fully ionized plasma. For  $T_g \lesssim 3 \times 10^7$  K, line cooling becomes very important. Raymond *et al.* (1976) give the cooling rate at lower temperatures; a very crude approximation is (McKee and Cowie, 1977)

$$\epsilon \approx 6.2 \times 10^{-19} T_g^{-0.6} n_p^2 \text{ ergs cm}^{-3} \text{ s}^{-1} \quad 10^5 \text{ K} < T_g < 4 \times 10^7 \text{ K}. \quad (5.22)$$

In assessing the role of cooling in the intracluster gas, it is useful to define a cooling time scale as  $t_{cool} \equiv (d \ln T_g / dt)^{-1}$ . For the temperatures that apply for the intracluster gas in most clusters, equation (5.21) gives a reasonable approximation to the X-ray emission. If the gas cools isobarically, the cooling time is

$$t_{cool} = 8.5 \times 10^{10} \text{ yr} \left( \frac{n_p}{10^{-3} \text{ cm}^{-3}} \right)^{-1} \left( \frac{T_g}{10^8 \text{ K}} \right)^{1/2}, \quad (5.23)$$

which is longer in most clusters than the Hubble time (age of the universe). Thus cooling is not very important in these cases. However, at the centers of some clusters the cooling time is shorter than the Hubble time, and these clusters are believed to have cooling flows (Section 5.7).

### 5.3.2 Infall and compressional heating

The heating of the intracluster gas will now be considered. The major point of this discussion is that, although the gas is quite hot, no major ongoing heating of the gas is generally necessary. This is true because the cooling time in the gas is long (equation 5.23), and the thermal energy in the gas is comparable to or less than its gravitational potential energy. Almost any method of introducing the gas into the cluster, either from outside the cluster or from galaxies within the cluster, will heat it to temperatures on the order of those observed.

Heating of the gas due to infall into the cluster and compression will be considered here. First, imagine that the cluster was formed before the intracluster gas fell into the cluster, and that the intracluster gas makes a negligible contribution to the mass of the cluster (Section 4.4). If the gas was initially cold, and located at a large distance from the cluster, then its initial energy can be ignored. If the cluster potential remains fixed while the gas falls into the cluster and the gas neither loses energy by radiation nor exchanges its energy with other components of the cluster, then the total energy of the gas will remain zero. After falling into the cluster, the gas will collide with other elements of gas, and its kinetic energy will be converted to thermal energy. Thus infall and compression can produce temperatures on the order of

$$\frac{3}{2} \frac{k T_g}{\mu m_p} \approx -\phi, \quad (5.24)$$

where  $\phi$  is the gravitational potential in the cluster. At the center of an isothermal cluster,  $\phi \approx -9\sigma_r^2$ , where  $\sigma_r$  is the line-of-sight velocity dispersion of the cluster. If this is substituted in equation (5.24), the derived temperature is

$$T_g \approx 5 \times 10^8 \text{ K} \left( \frac{\sigma_r}{10^3 \text{ km/s}} \right)^2, \quad (5.25)$$

which is a factor of 5–10 times larger than the observed temperatures (equation 4.10). Of course, the gas that falls into a cluster was presumably bound to the cluster before it fell in, so that equation (5.25) overestimates the temperature. A similar calculation for gas bound to the cluster is given in Shibazaki *et al.* (1976).

The temperature may also be lower because of cooling during the infall, or because the gas fell in at the same time that the cluster was forming and thus experienced a smaller potential on average. If the gas fell in at the same time that the cluster collapsed and was heated by the rapid variation of the potential during violent relaxation (Section 2.9.2), then it might have the same energy per unit mass as the matter in galaxies,

$$\frac{3}{2} \frac{kT_g}{\mu m_p} \approx \frac{3}{2} \sigma_r^2, \quad (5.26)$$

which gives equation (5.14) for the temperature. This is in reasonable agreement with the intracluster gas temperatures determined from X-ray spectra (equation 4.10).

These crude estimates are meant only to illustrate the point that the observed gas temperatures are consistent with heating due to infall into the cluster. More detailed models for infall are discussed in Section 5.10.1.

### 5.3.3 Heating by ejection from galaxies

The presence of a nearly solar abundance of iron in the intracluster gas (Sections 4.3.2 and 5.2.3) suggests that a reasonable fraction of the gas may have come from stars in galaxies within the clusters. The gas ejected from galaxies is heated in two ways. First, the gas may have some energy when it is ejected. Let  $\epsilon_{ej}$  be the total energy per unit mass of gas ejected from a galaxy in the rest frame of that galaxy, but not including the cluster gravitational potential, and define  $3kT_{ej}/2 \equiv \mu m_p \epsilon_{ej}$ . Second, the gas will initially be moving relative to the cluster center of mass at the galaxy's velocity. The ejected gas will collide with intracluster gas and thermalize its kinetic energy. On average this will give a temperature

$$kT_g \approx \mu m_p \sigma_r^2 + kT_{ej}. \quad (5.27)$$

If the ejection energy can be ignored, the temperature is given by equation (5.14), in reasonable agreement with the observations (equation 4.10).

In a steady-state wind outflow from a galaxy, one expects  $kT_{ej} \gtrsim \mu m_p \sigma_*^2$ , where  $\sigma_* \approx 200$  km/s is the velocity dispersion of stars within the galaxy. If the ejection temperature is near the lower limit given by this expression, then this form of heating will not be very important because  $\sigma_*^2 \ll \sigma_r^2$ . However, the ejection temperature could be considerably higher. For example, supernovae within galaxies could both produce the heavy elements seen in cluster X-ray spectra and heat the gas in galaxies until it was ejected. Supernovae eject highly enriched gas at velocities of  $v_{SN} \approx 10^4$  km/s. The highly enriched, rapidly moving supernova ejecta would collide with the interstellar medium in a galaxy and heat the gas. If  $M_{SN}$  is the mass of ejecta from a supernova and  $M_{ej}$  is the resulting total gas mass ejected from the galaxy, then  $T_{ej} \approx 2 \times 10^9 \text{ K} (v_{SN}/10^4 \text{ km/s})^2 (M_{SN}/M_{ej})$ , which will be significant if the supernova ejecta are diluted by less than a factor of about 100.

### 5.3.4 Heating by galaxy motions

Although ongoing heating of the intracluster gas may not be necessary to account for the observed gas temperatures, the estimates given above and the history of the gas are sufficiently uncertain that one cannot rule out ongoing heating as an important processes. One way in which intracluster gas could be heated would be through friction between the gas and the galaxies that are constantly moving throughout the cluster (Ruderman and Spiegel, 1971; Hunt, 1971; Yahil and Ostriker, 1973; Schipper, 1974; Livio *et al.*, 1978; Rephaeli and Salpeter, 1980). The calculation of the magnitude of this drag force and of the consequent heating of the intracluster gas is complicated by the following problems. First, the motion of an average cluster galaxy through the intracluster medium is likely to be just transonic  $M \approx 1$ , where  $M \equiv v/c_s$  is the Mach number,  $v$  is the galaxy velocity, and  $c_s = 1480(T_g/10^8 \text{ K})^{1/2}$  km/s is the sound speed in the gas. If equation (4.10) for the observed gas temperatures is assumed, and the average galaxy velocity is taken to be  $\sqrt{3}\sigma_r$ , then the average Mach number is  $\langle M \rangle \approx 1.5$ . Thus the galaxy motion cannot be treated as being either highly supersonic (strong shocks, etc.) or very subsonic (incompressible, etc.). In some cases shocks will be formed by the motion, and in some cases no shocks form. Second, the mean free path  $\lambda_i$  of ions in the intracluster medium due to Coulomb collisions (equation 5.34) is similar to the radius of a galaxy  $R_{gal} \approx 20$  kpc (Nulsen, 1982). Thus it is unclear whether the intracluster gas should be treated as a collisionless gas or as a fluid, and the role of transport processes such as viscosity (Section 5.4.4) is uncertain. For example,

# CLUSTER FORMATION

Lynden-Bell (1967). Theory of the “Violent Relaxation”.  
The variation of energy of a galaxy depends only on the variation of the global potential (see Binney & Tremaine text, too).

$dE/dt \sim d\Phi/dt$   $\Phi$ =global cluster potential

→ we expect velocity equipartition among galaxies  
And Maxwellian distribution of velocities (Gaussian in 1D, i.e. line-of-sight).

In the case galaxies and gas form the cluster simultaneously, we expect density energy equipartition between galaxies and gas, e.g. Sarazin (1986).

$$\beta_{\text{spec}} = \sigma_v^2 / (kT \mu m_p) = 1$$

Old data →  $\beta_{\text{spec}} > 1$ ...now  $\sim 1$

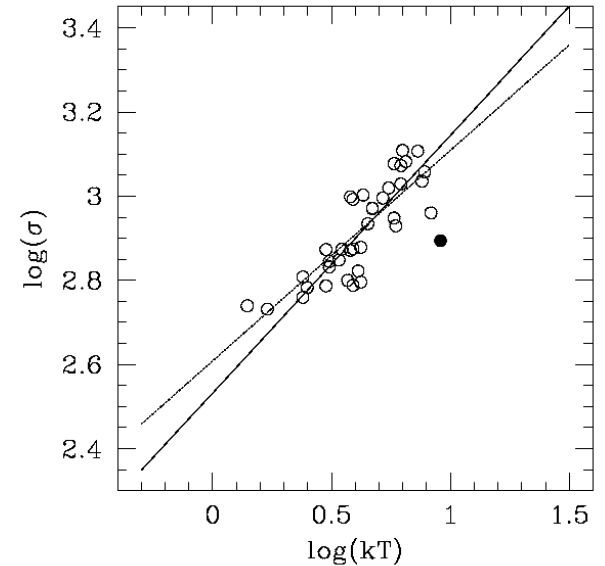
# Velocity Dispersion and X-ray Temperature

## $\beta_{\text{spec}}$ VALUE

$\sigma_v$  and  $T_x$  give the measures of the energy per unit mass of two different cluster components (galaxies and gas)

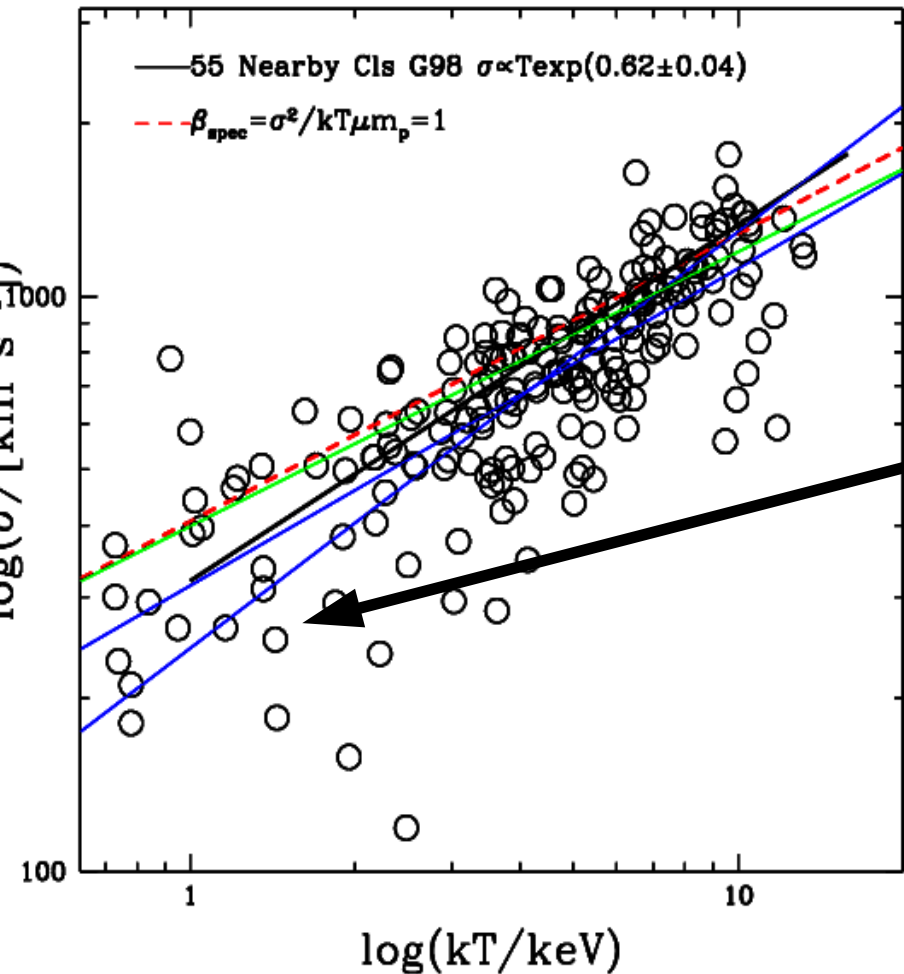
$$\beta_{\text{spec}} = \sigma_v^2 / [kT / (\mu m_p)]$$

**Model of gals/ICM specific energy equipartition  $\beta_{\text{spec}}=1$**



MG+1996,1998

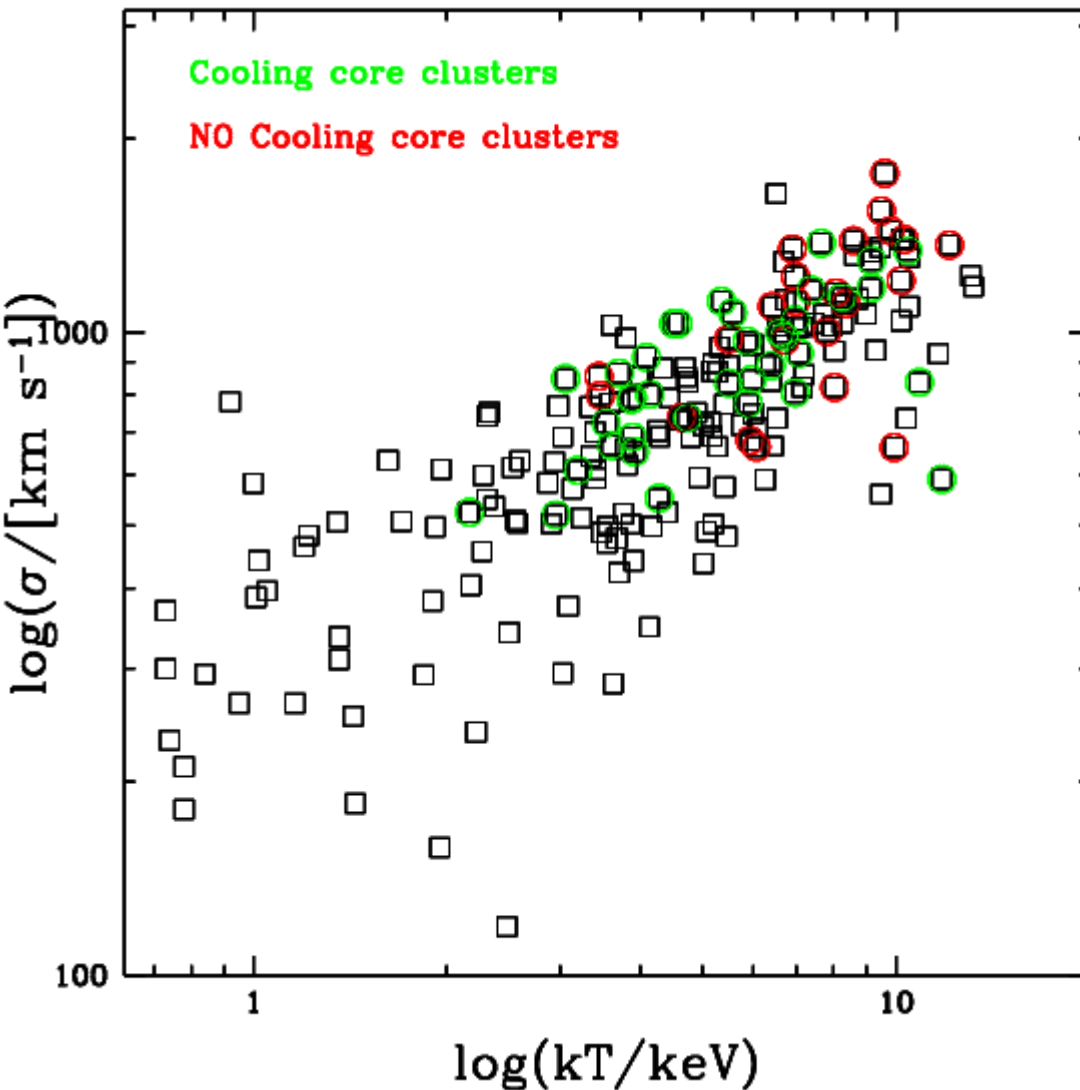
Some new data in 2009



Dynam. friction slows down group gals?  
extra-heating model for gas e.g. gal. winds?

**NEED TO BE RE-ANALYZED!**

# Clusters far from dynamical equilibrium deviate from $\sigma_v$ -Tx Relation? **No!**



Using cooling times by  
Allen & Fabian 98; Peres et al. 98

**61 Cls with a cool core  
~RELAXED CLUSTERS**

**24 Cls without a cool core  
NON RELAX. CLUSTERS**

Distributions are different at the 95%

**NON RELAXED  
CLUSTERS HAVE  
LARGER  $\sigma_v$  AND Tx.**