

$\vec{J} = \vec{J}_1 + \vec{J}_2 \Rightarrow \left[\int \rho^{-1} \vec{E} = \left(\int \rho_1^{-1} \right) + \left(\int \rho_2^{-1} \right) \vec{E} \right]$ ADDITIVITY of CURRENTS

$\rho_{xx}^{-1} = \frac{\rho_1}{\rho_1^2 + R_1^2 H^2} + \frac{\rho_2}{\rho_2^2 + R_2^2 H^2} = \frac{\rho(H) + R^2 H^2}{\rho(H) + R^2 H^2}$

$\rho_{xy}^{-1} = \frac{R_1 H}{\rho_1^2 + R_1^2 H^2} + \frac{R_2 H}{\rho_2^2 + R_2^2 H^2} = \frac{RH}{\rho(H) + R^2 H^2}$ (1+) (2-)

$R_1 = \frac{1}{\mu_1 c}$ $R_2 = -R_1 = -\frac{1}{\mu_2 c}$
 $\rho_1 = \frac{1}{\mu_1 e^2 \mu_1}$ $\rho_2 = \frac{1}{\mu_2 e^2 \mu_2}$

$\rho(H) = ?$
 $R = ?$

Remarks: $\mu_i \equiv \omega_i / \omega_i$

1) $\frac{\frac{1}{\mu_1 e^2 \mu_1}}{\frac{1}{\mu_1^2 e^2 \mu_1^2} + \frac{H^2}{\mu_1^2 e^2 c^2}} + \frac{\frac{1}{\mu_2 e^2 \mu_2}}{\frac{1}{\mu_2^2 e^2 \mu_2^2} + \frac{H^2}{\mu_2^2 e^2 c^2}} = \frac{\rho}{\rho^2 + R^2 H^2}$

2) $\frac{\frac{1}{\mu_1 c}}{\frac{1}{\mu_1^2 e^2 \mu_1^2} + \frac{H^2}{\mu_1^2 e^2 c^2}} + \frac{-\frac{1}{\mu_2 c}}{\frac{1}{\mu_2^2 e^2 \mu_2^2} + \frac{H^2}{\mu_2^2 e^2 c^2}} = \frac{R}{\rho^2 + R^2 H^2}$

1) $\frac{\cancel{\mu_1} / \mu_1}{\frac{c^2 + H^2 e^2 \mu_1^2}{\mu_1^2 e^2 c^2 \mu_1^2}} + \frac{\cancel{\mu_2} / \mu_2}{\frac{c^2 + H^2 e^2 \mu_2^2}{\mu_2^2 e^2 c^2 \mu_2^2}} = \frac{\rho}{\rho^2 + R^2 H^2}$

$\frac{\mu_1 e^2 c^2 \mu_1}{c^2 + H^2 e^2 \mu_1^2} + \frac{\mu_2 e^2 c^2 \mu_2}{c^2 + H^2 e^2 \mu_2^2} = \frac{\rho}{\rho^2 + R^2 H^2}$

Props $\left[\rho = \tilde{\rho} \cdot \frac{1}{\mu e^2} ; R = \tilde{R} \cdot \frac{1}{\mu c} \right]$ (\tilde{R} is adimensional)

$\frac{\rho}{\rho^2 + R^2 H^2} = \frac{\tilde{\rho} / \mu e^2}{\frac{\tilde{\rho}^2}{\mu^2 e^4} + \frac{\tilde{R}^2 H^2}{\mu^2 e^2 c^2}} = \frac{\tilde{\rho} \cdot \mu e^2 c^2}{\tilde{\rho}^2 \cdot c^2 + \tilde{R}^2 e^2 H^2}$

$\frac{R}{\rho^2 + R^2 H^2} = \frac{\tilde{R} / \mu c}{\frac{\tilde{\rho}^2}{\mu^2 e^4} + \frac{\tilde{R}^2 H^2}{\mu^2 e^2 c^2}} = \frac{\tilde{R} \cdot \mu e^3 c}{\tilde{\rho}^2 c^2 + \tilde{R}^2 e^2 H^2}$

Ex. 1)

$$1) \frac{\mu_1}{c^2 + A^2 \mu_1^2} + \frac{\mu_2}{c^2 + A^2 \mu_2^2} = \frac{\tilde{\rho}}{\tilde{\rho}^2 c^2 + A^2 \tilde{R}^2} \rightarrow \frac{1}{\tilde{\rho}^2 c^2 + A^2 \tilde{R}^2} = \frac{1}{\tilde{\rho}^2}$$

with $A^2 \equiv e^2 H^2$

$$2) \frac{\mu_1^2}{c^2 + A^2 \mu_1^2} - \frac{\mu_2^2}{c^2 + A^2 \mu_2^2} = \frac{\tilde{R}}{\tilde{\rho}^2 c^2 + A^2 \tilde{R}^2}$$

1) in 2):
ds

$$\frac{\mu_1^2}{c^2 + A^2 \mu_1^2} - \frac{\mu_2^2}{c^2 + A^2 \mu_2^2} = \frac{\tilde{R}}{\tilde{\rho}} \left(\frac{\mu_1}{c^2 + A^2 \mu_1^2} + \frac{\mu_2}{c^2 + A^2 \mu_2^2} \right)$$

$$\begin{aligned} (\mu_1^2 - \mu_2^2) c^2 &= \frac{\tilde{R}}{\tilde{\rho}} (\mu_1 c^2 + A^2 \mu_1 \mu_2^2 + \mu_2 c^2 + A^2 \mu_1^2 \mu_2) \\ &= \frac{\tilde{R}}{\tilde{\rho}} [(\mu_1 + \mu_2) c^2 + (\mu_1 + \mu_2) A^2 \mu_1 \mu_2] \end{aligned}$$

$$\left| \frac{\tilde{\rho}}{\tilde{R}} = \frac{c^2 + A^2 \mu_1 \mu_2}{c^2 (\mu_1 - \mu_2)} \right| \quad (\mu_1 \neq \mu_2)$$

Rewrite 1) as:

$$\begin{aligned} \frac{\mu_1 c^2 + A^2 \mu_1 \mu_2^2 + \mu_2 c^2 + A^2 \mu_1^2 \mu_2}{(c^2 + A^2 \mu_1^2)(c^2 + A^2 \mu_2^2)} &= \frac{1}{\tilde{R}} \cdot \frac{\tilde{\rho}}{(\frac{\tilde{\rho}}{\tilde{R}})^2 c^2 + A^2} \\ \frac{(\mu_1 + \mu_2)(c^2 + A^2 \mu_1 \mu_2)}{(c^2 + A^2 \mu_1^2)(c^2 + A^2 \mu_2^2)} &= \frac{1}{\tilde{R}} \cdot \frac{c^2 + A^2 \mu_1 \mu_2}{c^2 (\mu_1 - \mu_2)} \cdot \frac{1}{\left[\frac{c^2 + A^2 \mu_1 \mu_2}{c^2 (\mu_1 - \mu_2)} \right]^2 c^2 + A^2} \end{aligned}$$

$$\begin{aligned} \boxed{\tilde{R}} &= \frac{(c^2 + A^2 \mu_1^2)(c^2 + A^2 \mu_2^2) \cdot c^2 (\mu_1 - \mu_2)}{c^2 (\mu_1 - \mu_2) [(c^2 + A^2 \mu_1 \mu_2)^2 + 2 c^2 A^2 \mu_1 \mu_2] c^2 + A^2 c^4 (\mu_1^2 + \mu_2^2 - 2 \mu_1 \mu_2)} \\ &= \frac{(c^2 + A^2 \mu_1^2)(c^2 + A^2 \mu_2^2) c (\mu_1 - \mu_2)}{(c^2 + A^2 \mu_1^2)(c^2 + A^2 \mu_2^2) c^2 (\mu_1 + \mu_2)} = \boxed{\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}} \end{aligned}$$

Since $\tilde{R} = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}$

\Rightarrow ~~subst.~~ in $\tilde{\rho} = \frac{c^2 + A^2 \mu_1 \mu_2}{c^2 (\mu_1 - \mu_2)}$

$$\tilde{\rho} = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \cdot \frac{c^2 + A^2 \mu_1 \mu_2}{c^2 (\mu_1 - \mu_2)} = \frac{1}{\mu_1 + \mu_2} + \frac{e^2 H^2}{c^2} \frac{\mu_1 \mu_2}{\mu_1 + \mu_2}$$

$\Delta \tilde{\rho}(H)$
always > 0

$\Delta \rho(H) = \frac{1}{m e^2} \frac{e^2 H^2}{c^2} \frac{\mu_1 \mu_2}{\mu_1 + \mu_2}$

1.2

$$R = \frac{1}{m e c} \tilde{R} = \frac{1}{m e c} \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}$$

if the mobilities are =
the total Hall effect is 0.
(consistent with perfect e-h
symmetry)

1.3

$\mu_1 = \mu_2$

Sheet 2), eq. 2) $\Rightarrow \tilde{R} = 0 \Rightarrow R = 0$

and

$\tilde{\rho}$ from 1): $\frac{1}{\tilde{\rho} c^2} = 2 \frac{\mu_1}{c^2 + A^2 \mu_1^2}$

$\tilde{\rho} = \frac{c^2 + A^2 \mu_1^2}{2 \mu_1 c^2} = \frac{1}{2 \mu_1} + \frac{A^2 \mu_1}{2 c^2}$

as it can be
deduced
also from last
eqs of (1.1)

$A^2 \equiv e^2 H^2$

$\rho = \tilde{\rho} \cdot \frac{1}{m e^2} = \frac{1}{2 m e^2} + \frac{e^2 H^2 m}{2 c^2}$