Binney - Tremaine

FOR Eq. 4-107 b (p. 223)

$$\rho = 4\pi F \int_0^{\sqrt{2\psi}} \left(\psi - \frac{1}{2}v^2\right)^{n-3/2} v^2\, dv$$

$v^2 = 2\psi\cos^2\theta$ $\quad v=0 \longrightarrow \cos\theta = 0 \quad \theta = \frac{\pi}{2}$

$v = \sqrt{2\psi} \longrightarrow \cos\theta = 1 \quad \theta = 0$

$$2v\,dv = -2\psi\, 2\cos\theta\,\mathrm{sen}\,\theta\, d\theta = -4\psi\,\mathrm{sen}\,\theta\cos\theta\, d\theta$$

$v\,dv$ $\qquad \sqrt{2}$

$$v^2 dv = -2\psi\,\mathrm{sen}\,\theta\cos\theta\, d\theta \cdot \sqrt{2\psi}\cos\theta =$$

$$= -2\sqrt{2}\,\psi^{3/2}\,\mathrm{sen}\,\theta\cos^2\theta\, d\theta$$

$\mathrm{sen}^{2n-3}\theta$

$$\rho = 4\pi F \int_{\pi/2}^{0} (-1)\left[\psi(1-\cos^2\theta)\right]^{n-3/2}(-2\sqrt{2})\psi^{3/2} \cdot \mathrm{sen}\,\theta\cos^2\theta\, d\theta =$$

$$= 4\pi F \int_0^{\pi/2} \psi^n (\mathrm{sen}\,\theta)^{2n-2}\cos^2\theta\,(+2\sqrt{2})\, d\theta =$$

$$= 8\sqrt{2}\pi F \left[\int_0^{\pi/2} \psi^n (\mathrm{sen}^{2n-2}\theta)(1-\mathrm{sen}^2\theta)\, d\theta\right] =$$

$$= 8\sqrt{2}\pi F \left[\int_0^{\pi/2} \psi^n (\mathrm{sen}\,\theta)^{2n-2} d\theta - \int_0^{\pi/2} \psi^n (\mathrm{sen}\,\theta)^{2n}\, d\theta\right] =$$

$$= C_n \psi^n$$

$$C_n = \underbrace{8\sqrt{2}}_{2^{7/2}}\pi F \left[\int_0^{\pi/2} (\mathrm{sen}\,\theta)^{2n-2} d\theta - \int_0^{\pi/2} (\mathrm{sen}\,\theta)^{2n} d\theta\right]$$

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