FROM EQ. 7.14 → 7.17

$$\frac{d\bar{V}_M}{dt} = -16\pi^2 \ln\Lambda\, G^2 m(M+m)\left[\int_0^{V_M} \frac{m_0}{(2\pi\sigma^2)^{3/2}} e^{-\frac{V^2}{2\sigma^2}} V_m^2\, dV_m\right] \frac{\bar{V}_M}{V_M^3} =$$

$$= -16\pi^2 \ln\Lambda\, G^2 m(M+m)\, m_0 \left[\int \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-\frac{V^2}{2\sigma^2}} \frac{V_m^2}{2\sigma^2}\, 2\sigma^2 \, d\left(\frac{V_m}{\sqrt{2}\sigma}\right)\sqrt{2}\sigma\right] \cdot \frac{\bar{V}_M}{V_M^3} =$$

$$X = \frac{V_M}{\sqrt{2}\sigma}$$

$$= -16\pi^2 \ln\Lambda\, G^2 m(M+m)\, m_0 \left[\underbrace{\sqrt{\frac{\cancel{4}\cdot\cancel{2}\cdot\cancel{\sigma^6}}{\cancel{8}\pi^3\cancel{\sigma^6}}}}_{\frac{1}{\pi}\cdot\frac{1}{\sqrt{\pi}}} \int_0^X e^{-x'^2} x'^2\, dx'\right]$$

$$\frac{1}{\pi}\cdot\frac{1}{\sqrt{\pi}} \left(-\frac{1}{2}\right)\int_0^X \underbrace{e^{-x'^2}(-2x')}_{f'}\, \underbrace{x'}_{g}\, dx'$$

$$\int f'g = fg - \int fg'$$

$$-\frac{1}{4}\cdot 2 \cdot \left(\frac{1}{\sqrt{\pi}}\left[e^{-x'^2}(-2x')\right]_0^X - \frac{1}{\sqrt{\pi}}\int_0^X e^{-x'^2}\cdot 1\, dx'\right) =$$

$$= 4\pi \ln\Lambda\, G^2 m(M+m) \cdot \frac{\bar{V}_M}{V_M^3}\left[\underbrace{\frac{2}{\sqrt{\pi}}\int_0^X e^{-x'^2} dx}_{\mathrm{erf}(X)} - \frac{2X}{\sqrt{\pi}} e^{-X^2}\right]$$

eq. 7.17