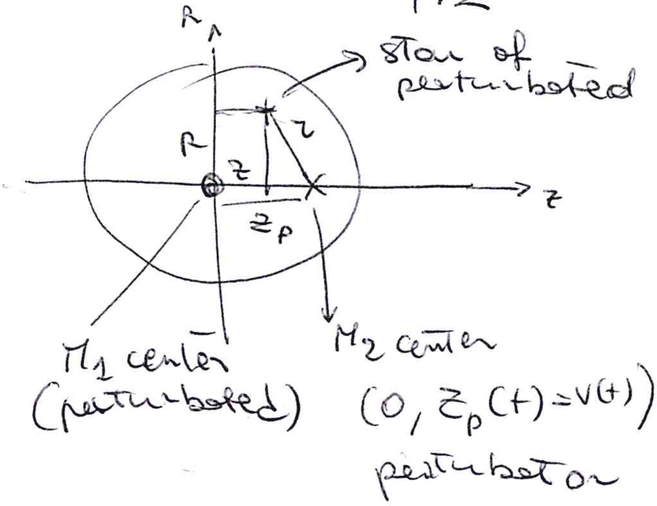
PENETRATING ENCOUNTERS 7.2

FROM. EQ. 7.56 → 7.58 → 7.59



$$\Delta V_R = -\frac{R}{V}\int_{-\infty}^{+\infty} \frac{d\phi}{dr}\,\frac{dz_p}{r}$$

$$\phi_{\text{Plummer}} = -\frac{GM_2}{\sqrt{r^2+a^2}} = \phi$$

→ per perturbatore

$$\frac{d\phi}{dr} = -GM_2\left(-\frac{1}{2}\right)\left(r^2+a^2\right)^{-3/2} 2r = GM_2\,\frac{r}{(r^2+a^2)^{3/2}}$$

$$\Delta V_R = -\frac{R}{V}\,GM_2\int_{-\infty}^{\infty}\frac{\cancel{r}}{(r^2+a^2)^{3/2}}\,\frac{dz_p}{\cancel{r}} \qquad r=\sqrt{(z_p-z)^2+R^2}$$

$$\Delta V_R = -\frac{RGM_2}{V}\int_{-\infty}^{\infty}\frac{dX}{(X^2+R^2+a^2)^{3/2}} \qquad X = z_p - z$$

$$\Delta V_R = -\frac{RGM_2}{V}\,\frac{2}{(R^2+a^2)} \qquad \text{eq. 7.58}$$

$$\rho(r)_{\text{Plummer}} = \frac{3M_1}{4\pi a^3}\left(1+\frac{r^2}{a^2}\right)^{-5/2} \quad \text{density}$$

X Perturbatore NOTA $a\uparrow$ $\rho\uparrow$

$$\Sigma(R) = 2\int_R^{\infty}\frac{\rho(r)\,r\,dr}{\sqrt{r^2-R^2}} = \frac{6M_1}{4\pi a^3}\int_R^{\infty}\frac{\left(1+\frac{r^2}{a^2}\right)^{-5/2} r\,dr}{\sqrt{r^2-R^2}} =$$

↓ projected density

Abel integral

$$= \frac{3}{2\pi}\,\frac{M a^3}{a^3\cdot a}\int_R^{\infty}\frac{\left(1+\frac{r^2}{a^2}\right)^{-5/2}\frac{r}{a}\,d\left(\frac{r}{a}\right)}{\sqrt{\frac{r^2}{a^2}-\frac{R^2}{a^2}}} =$$

$$x = \frac{r}{a}$$

$$= \frac{3M_1}{2\pi a^2} \int_{\frac{R}{a}}^{\infty} \underbrace{\frac{(1+x^2)^{-5/2} \cdot x \, dx}{\sqrt{x^2 - \frac{R^2}{a^2}}} = \frac{3M_1}{2\pi a^2} \frac{2}{3\left[\left(\frac{R}{a}\right)^2+1\right]^2}}_{\text{wolfram}} =$$

$$= \frac{M_1 a^2}{\pi (R^2+a^2)}$$

ΔE 7.57 $\Delta E = \pi \int_0^{\infty} [\Delta V_R(R)]^2 \, \Sigma(R) R \, dR$

$$\Delta E = \pi \frac{4G^2M_2^2}{V^2} \int_0^{\infty} \frac{R^2}{(R^2+a^2)^2} \frac{M_1 a^2}{(R^2+a^2)^2 \pi} R \, dR =$$

$$= \frac{4G^2M_1M_2^2a^2}{V^2} \underbrace{\int_0^{\infty} \frac{R^3 \, dR}{(R^2+a^2)^4}}_{\substack{\text{wolfram} \\ \frac{1}{2a^4}}} = \frac{G^2M_2^2M_1}{3V^2a^2} \qquad 7.58$$

NOTA $a \uparrow$ $\rho \uparrow$ $\Delta E \downarrow$

~~perturbato~~ meno "perturbato"

a parità di M_2, V

cfr. 7.58 ($b=0$) with 7.54 (tidal approximation) ($b > r$)

interpolation