

“Complementi di Fisica”

Lecture 31

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In this lecture

- **Contents**

- Drift of electrons and holes in practice (numbers...):
 - conductivity σ as a function of impurity concentration
 - “band bending”: representation of macroscopic electric fields
- Diffusion in practice (numbers...):
 - Diffusion coefficient D
 - Induced (“Built-in”) electric field
 - Einstein relation between σ and D

- **Reference textbooks**

- D.A. Neamen, *Semiconductor Physics and Devices*, McGraw-Hill, 3rd ed., 2003, p.154-188 (“5 Carrier Transport Phenomena”)
- R.Pierret, *Advanced Semiconductor Fundamental*, Prentice Hall, 2nd ed., p.175-215 (“6 Carrier Transport”)



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From the Boltzmann Equation...

- The continuity equations for the electrical current density in semiconductors can be obtained from the Boltzmann equation:

$$\frac{\partial f}{\partial t} = -\vec{v}_g \cdot \vec{\nabla}_{\vec{r}} f - \frac{\vec{F}}{\hbar} \cdot \vec{\nabla}_{\vec{k}} f - \frac{f - f_0}{\tau} \quad \vec{F} = -e\vec{E}$$

Force on electrons

- Multiplying by the group velocity and integrating over the momentum space $dk_x dk_y dk_z$:

$$\int \vec{v}_g \frac{\partial f}{\partial t} d^3\vec{k} = - \int \vec{v}_g (\vec{v}_g \cdot \vec{\nabla}_{\vec{r}} f) d^3\vec{k} - \int \vec{v}_g \left(\frac{\vec{F}}{\hbar} \cdot \vec{\nabla}_{\vec{k}} f \right) d^3\vec{k} - \int \vec{v}_g \frac{f - f_0}{\tau} d^3\vec{k}$$

- One obtains the expression for current density (detailed derivation: see FELD p.187-194, MOUT p.100-104)



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Drift-diffusion continuity equation

For electrons (similar for holes):

$$\tau_n \frac{\partial J_n}{\partial t} + \vec{J}_n = qn\mu_n \left(\vec{E} + \frac{1}{n} \frac{k_B T}{q} \vec{\nabla}_{\vec{r}} n + \frac{k_B}{q} \vec{\nabla}_{\vec{r}} T \right)$$

Drift

Diffusion

already discussed in the Drude model

relaxation time τ is small:
This new term can be neglected if frequency is not too high (few hundred MHz)

Temperature gradient:
also a temperature gradient can drive an electric current!
Absent at thermal equilibrium



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Drift and diffusion: Einstein relation

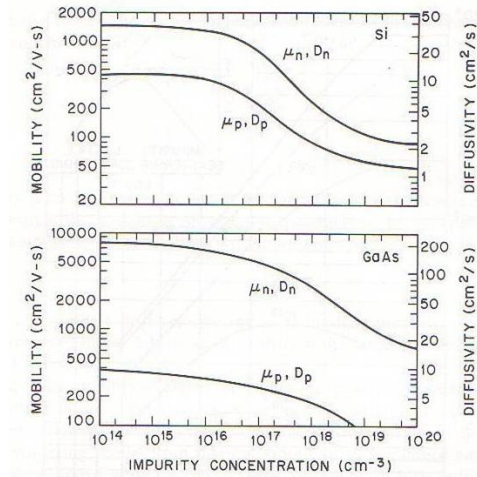
- Drift (mobility) and diffusion (diffusivity) coefficients are correlated!

thermal velocity $\downarrow$ $D \equiv v_{th} l$ drift velocity $\downarrow$ $v_n = -\mu_n E$ external field $\downarrow$ $v_p = \mu_p E$

$$D_n = \left(\frac{kT}{q} \right) \mu_n \quad D_p = \left(\frac{kT}{q} \right) \mu_p$$

- Why ?

- See previous lecture
- Also: no current at equilibrium, see next



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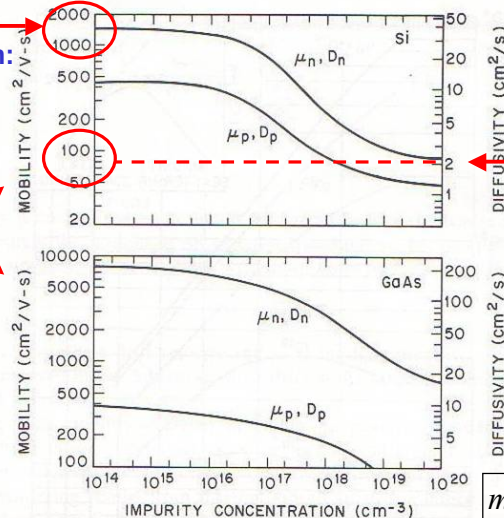
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Mobility and diffusivity vs. concentration

Low doping concentration:
Lattice scattering limit

mobility μ
($\text{cm}^2 \text{ V}^{-1} \text{ s}^{-1}$)



$T \sim 300 \text{ K}$

High doping concentration:
Impurity scattering limit

diffusivity D
($\text{cm}^2 \text{ s}^{-1}$)

Electrons and holes:

$$m_n^* < m_p^* \Rightarrow \mu_n > \mu_p$$



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“Band bending”: “built-in” electrical field

Non-uniform doping: “built-in” field

p-type doping, thermal equilibrium
(no “external” el.field applied!):

$$\begin{aligned}
 J_p &= |q|\mu_n p \mathcal{E} - |q|D_n \frac{dp}{dx} = 0 \\
 &= |q|\mu_n p \mathcal{E} - |q|\frac{kT}{|q|}\mu_n \frac{dp}{dx} = \\
 &= |q|\mu_n p \left(\mathcal{E} - V_{th} \frac{1}{p} \frac{dp}{dx} \right) = 0
 \end{aligned}$$

$\mathcal{E}$ is the “built-in” electric field:

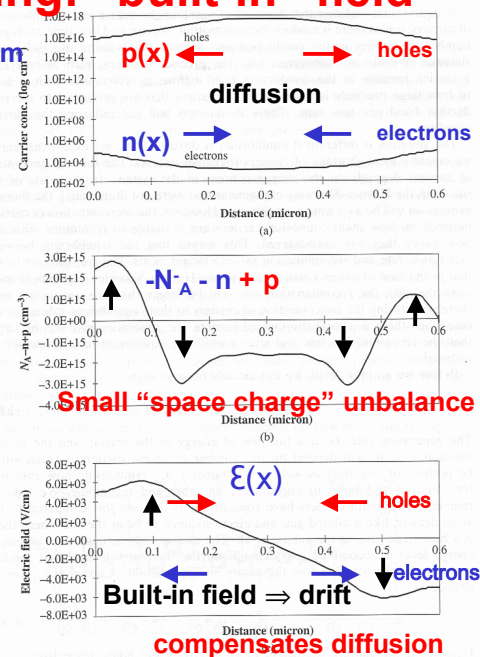
$$\mathcal{E} = V_{th} \frac{1}{p} \frac{dp}{dx} \approx V_{th} \frac{1}{N_A} \frac{dN_A}{dx}$$

$$V_{th} \equiv \frac{kT}{q} \quad \text{“thermal voltage equivalent”}$$



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L.Lancieri - Compler



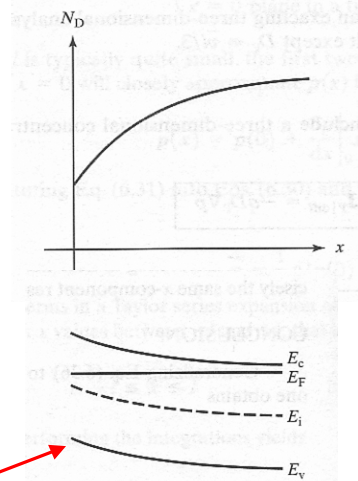
Non-uniform doping in equilibrium

- Under thermal and diffusive equilibrium conditions the Fermi level (= **total chemical potential**) inside a material (or group of materials in intimate contact) is invariant as a function of position

$$\frac{dE_F}{dx} = \frac{dE_F}{dy} = \frac{dE_F}{dz} = 0$$

- A non-zero (“built-in”) electric field is established in nonuniformly doped semiconductors under equilibrium conditions

$$\begin{aligned}\mathcal{E}_x &= (1/q)(dE_C/dx) = \\ &= (1/q)(dE_i/dx) = \\ &= (1/q)(dE_V/dx)\end{aligned}$$



“band bending”, as for an external field; here the field is “built-in”^(b)



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Einstein relation - 1

Relation between electron concentration, donor concentration, Fermi level E_F and intrinsic Fermi level E_i in the “quasi-neutrality” approximation (local neutrality cannot be exact, otherwise there would be no built-in field!):

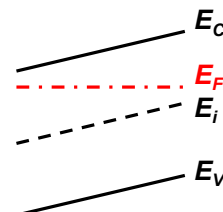
$$n(x) = n_i e^{(E_F - E_i(x))/kT} \approx N_D(x)$$

At equilibrium, E_F is constant $\Rightarrow$ “band bending”:

$$E_F - E_i = kT \ln \left(\frac{N_D(x)}{n_i} \right) \Rightarrow -\frac{dE_i}{dx} = \frac{kT}{N_D(x)} \frac{dN_D(x)}{dx}$$

Induced (“built-in”) electric field:

$$\mathcal{E}_x = -\frac{dV}{dx} = -\frac{1}{-|q|} \frac{dE_i}{dx} = -\frac{kT}{|q|} \frac{1}{N_D(x)} \frac{dN_D(x)}{dx}$$



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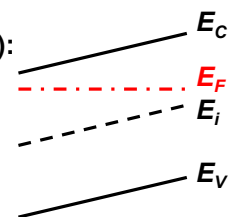
Einstein relation - 2

At equilibrium the total electric current (diffusion + drift) must be zero; using for the electric field $\mathcal{E}_x$ the expression just derived one obtains:

$$\begin{aligned}
 J_{n,x} = 0 &= |q| n \mu_n \mathcal{E}_x + |q| D_n \frac{dn}{dx} \approx \\
 &\approx |q| N_D(x) \mu_n \mathcal{E}_x + |q| D_n \frac{dN_D(x)}{dx} = \\
 &= |q| \cancel{N_D(x)} \mu_n \left(-\frac{kT}{|q|} \frac{1}{\cancel{N_D(x)}} \frac{dN_D(x)}{dx} \right) + |q| D_n \frac{dN_D(x)}{dx}
 \end{aligned}$$

⇒ Einstein relation for electrons (similar for holes):

$$\frac{D_n}{\mu_n} = \frac{kT}{|q|} \quad \left(\frac{D_p}{\mu_p} = \frac{kT}{|q|} \right)$$



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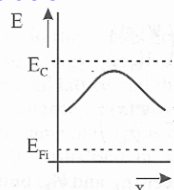
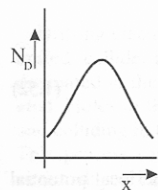
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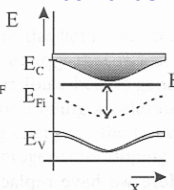


Non-uniform doping: an example

Donor concentration:



“band bending”:

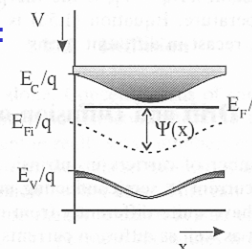


⇒ Built-in electric field:

$$\begin{aligned}
 \mathcal{E}_x &= (1/q)(dE_i/dx) = \\
 &= (1/q)(d(E_F - E_i)/dx) = \\
 &= -d\Psi/dx
 \end{aligned}$$

Electric potential

$$V(x) = \Psi(x) = (1/q)(E_F - E_i(x))$$



Equilibrium:
 $E_F = \text{constant}$

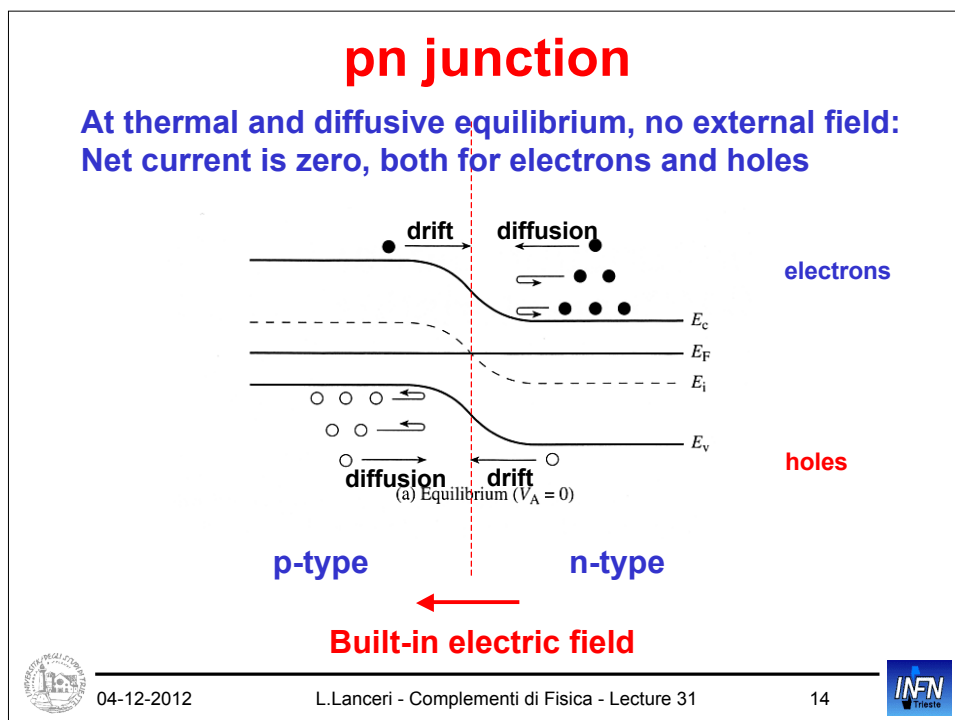
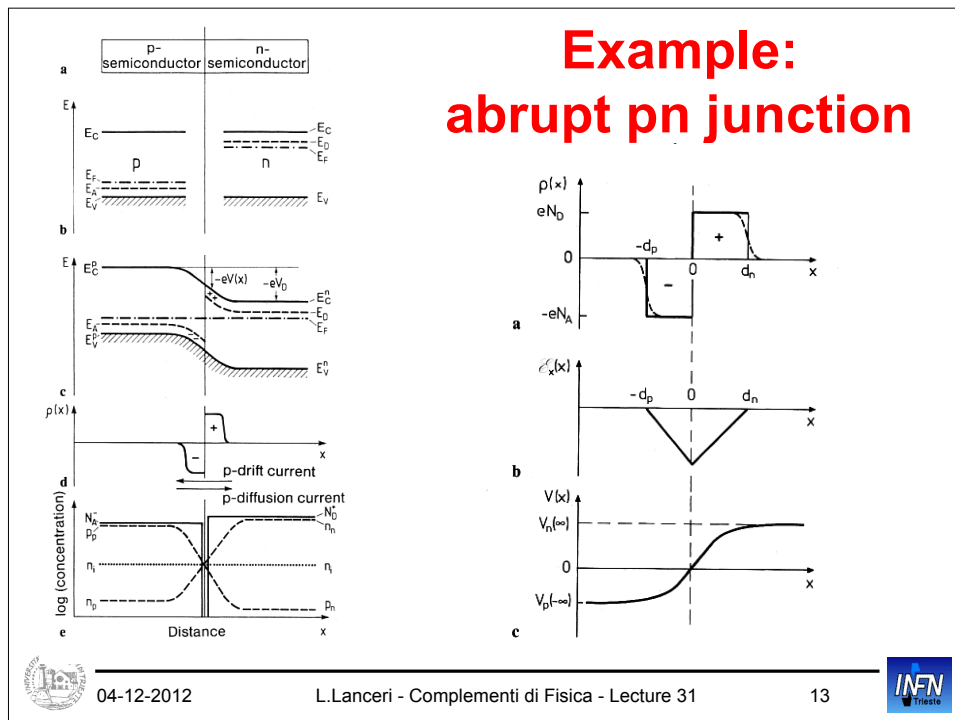


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Einstein relation – numerical examples

- In the “non-degenerate” limit:

$$\frac{D_n}{\mu_n} = \frac{kT}{q} \quad \frac{D_p}{\mu_p} = \frac{kT}{q}$$

- Typical sizes of mobility and diffusivity:

$$T = 300K \Rightarrow \frac{kT}{q} \approx 0.026 \text{ V}$$

$$\mu_n = 1000 \text{ cm}^2 / \text{Vs} \Rightarrow D_n \approx 26 \text{ cm}^2 / \text{s}$$



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Current density equations

- When an *external electric field* $\mathcal{E}$ is present in *addition to the concentration gradient*: *no equilibrium!*
 - Both drift and diffusion currents will flow**
 - The total current density is *different from zero* in this case!**

- For electrons and holes: $J = J_n + J_p$

$$J_n = J_{n,drift} + J_{n,diff} = \underbrace{|q|\mu_n n \mathcal{E}}_{\text{drift}} + \underbrace{|q|D_n \frac{dn}{dx}}_{\text{diffusion}}$$

$$J_p = J_{p,drift} + J_{p,diff} = |q|\mu_p p \mathcal{E} - |q|D_p \frac{dp}{dx}$$



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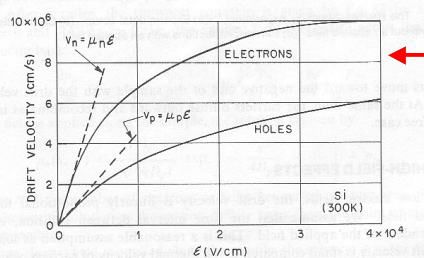


High field effects

Drift velocity saturation

Avalanche processes

Drift velocity saturation

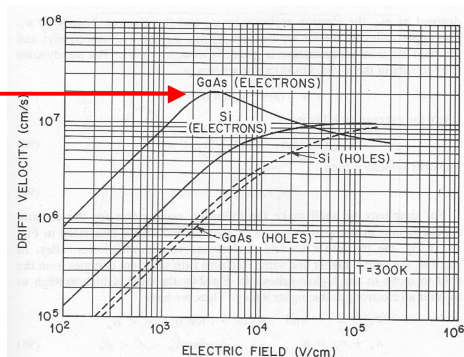


Silicon:

- electrons and holes drift velocity
- increases linearly at small fields
- saturates ($\sim$ thermal velocity) at high fields

GaAs:

- electrons and holes drift velocity
- Peculiar behavior
- see next slide for an explanation



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Two-valley semiconductors

GaAs:

Two-valley model of

E-k band diagram:

Different effective masses

Different mobilities

$$\mu = \frac{q\tau_c}{m^*}$$

$$\sigma = q(\mu_1 n_1 + \mu_2 n_2) = qn\bar{\mu}$$

$$\bar{\mu} \equiv (\mu_1 n_1 + \mu_2 n_2)/(n_1 + n_2)$$

$$v_n = \bar{\mu}E$$

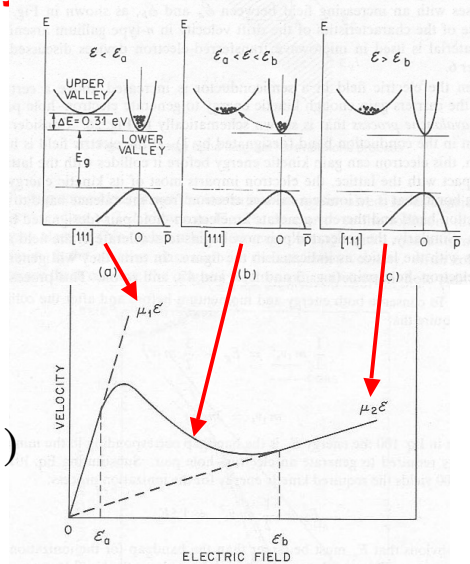


Fig. 25 One possible velocity-field characteristic of a two-valley semiconductor



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Avalanche processes

To start an avalanche:
enough kinetic energy
to create an e-h pair

Order of magnitude estimate
from energy and momentum
conservation (process 1 → 2):

$$\frac{1}{2} m_1 v_s^2 \approx E_g + 3 \frac{1}{2} m_1 v_f^2$$

$$m_1 v_s \approx 3 m_1 v_f$$

$$E_0 = \frac{1}{2} m_1 v_s^2 \approx 1.5 E_g$$

$$G_A = \frac{1}{q} (\alpha_n |J_n| + \alpha_p |J_p|)$$

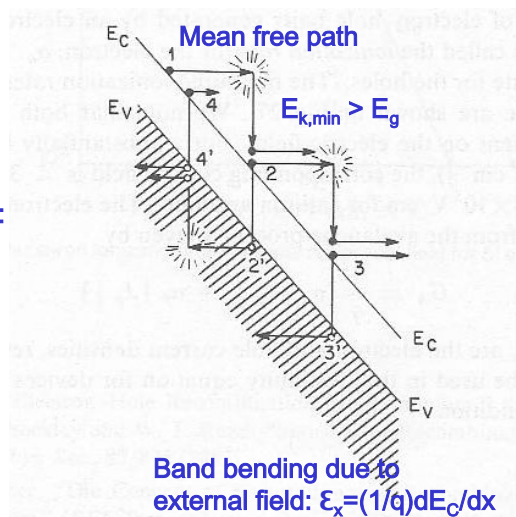


Fig. 26 Energy band diagram for avalanche process.



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Ionization rates and generation rate

α_n, α_p ionization rates (cm^{-1})

e-h generation rate ($\text{cm}^{-3}\text{s}^{-1}$)

$$G_A = \frac{1}{q} (\alpha_n |J_n| + \alpha_p |J_p|)$$

In Silicon:

$E_0 = 3.2 E_g$ (el.)

$E_0 = 4.4 E_g$ (h.)

$\mathcal{E} \sim 3 \times 10^5 \text{ V/cm}$

For $\alpha \sim 10^4 \text{ cm}^{-1}$

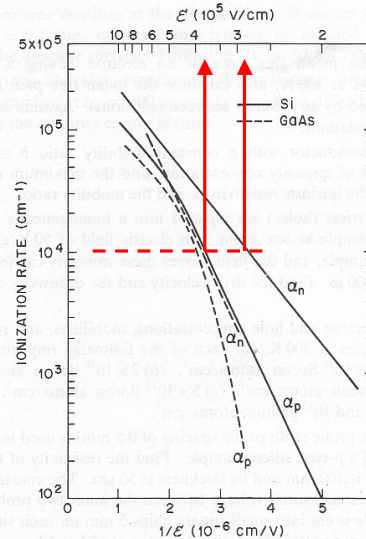


Fig. 27 Measured ionization rates versus reciprocal field for Si and GaAs.¹³



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Lecture 31 - exercises

- **Exercise 1:** Find the electron and hole concentrations, mobilities and resistivities of silicon samples at 300K, for each of the following impurity concentrations: (a) 5×10^{15} boron atoms/ cm^3 ; (b) 2×10^{16} boron atoms/ cm^3 together with 1.5×10^{16} arsenic atoms/ cm^3 ; and (c) 5×10^{15} boron atoms/ cm^3 , together with 10^{17} arsenic atoms/ cm^3 , and 10^{17} gallium atoms/ cm^3 .
- **Exercise 2:** For a semiconductor with a constant mobility ratio $b \equiv \mu_n/\mu_p > 1$ independent of impurity concentration, find the maximum resistivity ρ_m in terms of the intrinsic resistivity ρ_i and of the mobility ratio.
- **Exercise 3:** A semiconductor is doped with N_D ($N_D \gg n_i$) and has a resistance R_1 . The same semiconductor is then doped with an unknown amount of acceptors N_A ($N_A \gg N_D$), yielding a resistance of $0.5R_1$. Find N_A in terms of N_D if the ratio of diffusivities for electrons and holes is $D_n/D_p = 50$.



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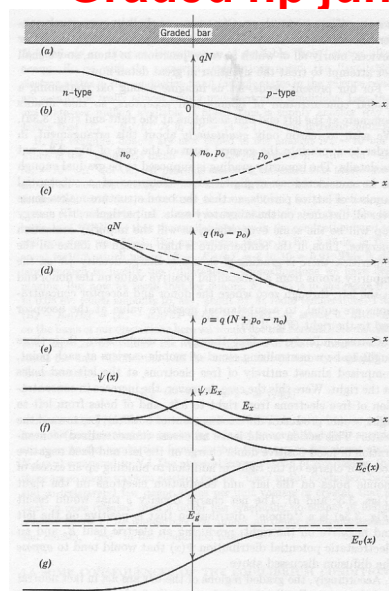
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Back-up slides

Graded np junction



$$N = N_D - N_A$$

N: net fixed charge

n_0, p_0 : majority carrier concentrations

net mobile charge

net total charge

**potential Ψ
El. field E_x**

electron energy-band diagram



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