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# Testi del Syllabus

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Resp. Did. **MEZZETTI EMILIA** **Matricola: 002830**

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Docente **MEZZETTI EMILIA, 9 CFU**

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Anno offerta: **2018/2019**

Insegnamento: **534SM - ADVANCED GEOMETRY 3**

Corso di studio: **SM34 - MATEMATICA**

Anno regolamento: **2018**

CFU: **9**

Settore: **MAT/03**

Tipo Attività: **C - Affine/Integrativa**

Anno corso: **1**

Periodo: **Secondo Semestre**

Sede: **TRIESTE**

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## Testi in italiano

**Lingua insegnamento** English

**Contenuti (Dipl.Sup.)** An introduction to Algebraic Geometry: Zariski topology, affine and projective varieties, morphisms and rational maps, dimension, tangent spaces and singularities. Selected topics in Commutative Algebra.

**Testi di riferimento**

I. R. Shafarevich: Basic Algebraic Geometry 1: Varieties in Projective Space, Third edition. Springer, Heidelberg, 2013

J. Harris: Algebraic geometry. A first course, Graduate Texts in Mathematics, 133, Springer-Verlag, New York, 1995.

R. Hartshorne: Algebraic geometry, Graduate Texts in Mathematics, No. 52. Springer-Verlag, New York-Heidelberg, 1977. (First chapter)

S.D. Cutkosky, Introduction to Algebraic Geometry, Graduate Studies in Mathematics 188, AMS 2018

**Obiettivi formativi**

**KNOWLEDGE AND UNDERSTANDING**  
By the end of the course the student is expected to be familiar with the fundamental objects of classical algebraic geometry, both affine and projective, and of basic concepts of commutative algebra.

**CAPACITY TO APPLY KNOWLEDGE AND UNDERSTANDING**  
By the end of the course the student is expected to be able to apply the notions of basic algebraic geometry acquired to solve problems and exercises of medium difficulty. The exercises can also be proposed as easy theoretical results.

**JUDGMENT AUTONOMY**  
By the end of the course the student is expected to be able to recognize and apply the most basic techniques of algebraic geometry and also to recognize the situations and problems in which these techniques can be used advantageously.

**COMMUNICATIVE SKILLS**  
By the end of the course the student is expected to be able to express

himself with proficient command of language and exposure security on the topics of the course.

#### LEARNING CAPACITY

By the end of the course the student is expected to be able to consult the standard texts of algebraic geometry and commutative algebra.

<b>Prerequisiti</b>	Linear algebra, affine and projective geometry; a basic knowledge of plane algebraic curves is recommended.
<b>Metodi didattici</b>	Lectures and problem sessions. During the course some exercises will be assigned as homework, to be delivered in written form. The solutions will be discussed in class.
<b>Altre informazioni</b>	Information about the progress of the program and teaching materials will be posted on the site <a href="http://moodle2.units.it">http://moodle2.units.it</a>
<b>Modalità di verifica dell'apprendimento</b>	The exam program coincides with the arguments of the lectures. The exam will be held in oral form only, but the students who will not deliver the assigned exercises will have to take a written test, consisting in solving exercises modeled on those assigned during the course. The oral exam aims to carry out an assessment of the student's familiarity with the program, comprehension of the contents (definitions and proofs) and command of language.
<b>Programma esteso</b>	Affine and projective spaces and their subspaces. Affine algebraic sets. Zariski topology. Noetherian rings, Hilbert basis theorem, weak and strong form. Projective algebraic sets. Zariski topology on projective space. Projective version of Hilbert Nullstellensatz. Products of projective spaces. Zariski topology. Irreducible topological spaces. Noetherian topological spaces, irreducible components. Affine varieties, projective and quasi-projective (qp) varieties. Coordinates ring of an affine variety, field of rational functions, local ring and its maximal ideal, ring of regular functions. Ring of homogeneous coordinates of an open set in a projective variety. Morphisms between qp varieties, pullback and functorial properties. Isomorphisms. Local morphisms. Projective closure of an affine set. Products and universal property. Immersiones and Segre varieties. Separateness and completeness of qp varieties. Veronese map and intersections of projective varieties with hypersurfaces. Rational maps between qp varieties. Blowup of the affine plane. Combinatorial dimension of a topological space. Finite morphisms between varieties and closed morphisms. Noether Normalization Lemma. Dimension theory. Dimension of the intersection of an affine variety with an affine hypersurface. Projective varieties of codimension one. Dimension of the product of qp varieties, dimension of the affine cone of a projective variety. Singular points and regular points of an affine hypersurface. Tangent space. Differential of a morphism between varieties and Jacobian matrix. Zariski tangent space at a point of a qp variety. Jacobian criterion. Singular and regular points of a affine hypersurface. Tangent space.



## Testi in inglese

English

An introduction to Algebraic Geometry: Zariski topology, affine and projective varieties, morphisms and rational maps, dimension, tangent spaces and singularities. Selected topics in Commutative Algebra.

I. R. Shafarevich: Basic Algebraic Geometry 1: Varieties in Projective Space, Third edition. Springer, Heidelberg, 2013

J. Harris: Algebraic geometry. A first course, Graduate Texts in Mathematics, 133, Springer-Verlag, New York, 1995.

R. Hartshorne: Algebraic geometry, Graduate Texts in Mathematics, No. 52. Springer-Verlag, New York-Heidelberg, 1977. (first chapter)

S.D. Cutkosky, Introduction to Algebraic Geometry, Graduate Studies in Mathematics 188, AMS 2018

#### KNOWLEDGE AND UNDERSTANDING

By the end of the course the student is expected to be familiar with the fundamental objects of classical algebraic geometry, both affine and projective, and of basic concepts of commutative algebra.

#### CAPACITY TO APPLY KNOWLEDGE AND UNDERSTANDING

By the end of the course the student is expected to be able to apply the notions of basic algebraic geometry acquired to solve problems and exercises of medium difficulty. The exercises can also be proposed as easy theoretical results.

#### JUDGMENT AUTONOMY

By the end of the course the student is expected to be able to recognize and apply the most basic techniques of algebraic geometry and also to recognize the situations and problems in which these techniques can be used advantageously.

#### COMMUNICATIVE SKILLS

By the end of the course the student is expected to be able to express himself with proficient command of language and exposure security on the topics of the course.

#### LEARNING CAPACITY

By the end of the course the student is expected to be able to consult the standard texts of algebraic geometry and commutative algebra.

Linear algebra, affine and projective geometry; a basic knowledge of plane algebraic curves is recommended.

Lectures and problem sessions. During the course some exercises will be assigned as homework, to be delivered in written form. The solutions will be discussed in class.

Information about the progress of the program and teaching materials will be posted on the site <http://moodle2.units.it>

The exam program coincides with the arguments of the lectures. The exam will be held in oral form only, but the students who will not deliver the assigned exercises will have to take a written test, consisting in solving exercises modeled on those assigned during the course. The oral exam aims to carry out an assessment of the student's familiarity with the program, comprehension of the contents (definitions and proofs) and command of language.

Affine and projective spaces and their subspaces. Affine algebraic sets. Zariski topology.

Noetherian rings, Hilbert basis theorem, weak and strong form.

Projective algebraic sets. Zariski topology on projective space. Projective version of Hilbert Nullstellensatz.

Products of projective spaces. Zariski topology.

Irreducible topological spaces. Noetherian topological spaces, irreducible components.

Affine varieties, projective and quasi-projective (qp) varieties.

Coordinates ring of an affine variety, field of rational functions, local ring

and its maximal ideal, ring of regular functions.  
Ring of homogeneous coordinates of an open set in a projective variety.  
Morphisms between  $q$ -varieties, pullback and functorial properties.  
Isomorphisms. Local morphisms.  
Projective closure of an affine set.  
Products and universal property.  
Immersions and Segre varieties.  
Separateness and completeness of  $q$ -varieties. Veronese map and intersections of projective varieties with hypersurfaces.  
Rational maps between  $q$ -varieties.  
Blowup of the affine plane.  
Combinatorial dimension of a topological space. Finite morphisms between varieties and closed morphisms.  
Noether Normalization Lemma.  
Dimension theory. Dimension of the intersection of an affine variety with an affine hypersurface.  
Projective varieties of codimension one.  
Dimension of the product of  $q$ -varieties, dimension of the affine cone of a projective variety.  
Singular points and regular points of an affine hypersurface. Tangent space. Differential of a morphism between varieties and Jacobian matrix.  
Zariski tangent space at a point of a  $q$ -variety. Jacobian criterion.  
Singular and regular points of a affine hypersurface.  
Tangent space.