APPLIED MATHEMATICS 30/01/2018

1) You arrive at a bus stop at 10 o'clock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30. What is the probability that you will have to wait longer than 10 minutes? If at 10:15 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

Let X be the random variable time of arrival of the bus. We set the time 0 at 10 o'clock. X has distribution $\mathrm{U}(0,a)$ with $a = 30$ min.

We have

$$\mathbb{P}\left(X > \frac{a}{3}\right) = \frac{\frac{2a}{3}}{a} = \frac{2}{3}$$

and

$$\begin{aligned}\mathbb{P}\left(X \geq \frac{a}{2} + \frac{a}{3} = \frac{5a}{6} | X > \frac{a}{2}\right) &= \frac{\mathbb{P}\left(X > \frac{a}{2} \cap X \geq \frac{5a}{6}\right)}{\mathbb{P}\left(X \geq \frac{a}{2}\right)} \\ &= \frac{\mathbb{P}\left(X \geq \frac{5a}{6}\right)}{\mathbb{P}\left(X \geq \frac{a}{2}\right)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}.\end{aligned}$$

2) A certain component is critical to the operation of an electrical system and must be replaced immediately upon failure. If the mean lifetime of this type of component is 100 hours and its standard deviation is σ, how σ should be so that the probability that the system is in continual operation for the next 2000 hours with 20 components is at least .95? Assume that the component lifetimes are independent random variables normally distributed.

Let $X_1, X_2, ..., X_n$ be the component lifetimes of the $n = 20$ components. $X_1, X_2, ..., X_n$ are IID with distribution $\mathrm{N}\left(\mu, \sigma^2\right)$, with $\mu = 100$ h. Then

$$S_n = X_1 + X_2 + ... + X_n$$

has distribution $\mathrm{N}\left(n\mu, (\sqrt{n}\sigma)^2\right)$. We want

$$\mathbb{P}(S_n \geq a) \geq 95\%$$

with

$$\mathbb{P}(S_n \geq a) = \Phi\left(Z = \frac{S_n - n\mu}{\sqrt{n}\sigma} \geq \frac{a - n\mu}{\sqrt{n}\sigma} = 0\right) = 50\%.$$

There does not exist σ such that $\mathbb{P}(S_n \geq a) \geq 95\%$.

3) In a recent study, 79 of 140 meteorites were observed to enter the atmosphere with a velocity of less than 25 miles per second. If we take $\widehat{p} = 79/140$ as an estimate of the probability that an arbitrary meteorite that enters the

atmosphere will have a speed less than 25 miles per second, what can we say, with 99 percent confidence, about the maximum error of our estimate?

A 99% confidence interval has the form

$$\widehat{p} \pm k \frac{\widehat{\sigma}}{\sqrt{n}}$$

with $k = 2.58$ and

$$\widehat{\sigma} = \sqrt{\widehat{p}(1 - \widehat{p})} = 0.4959$$

The maximum error is

$$k \frac{\widehat{\sigma}}{\sqrt{n}} = 0.1079.$$