CIRCUITI A MICROONDE

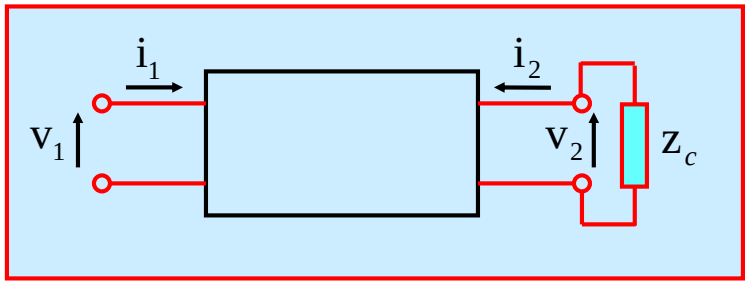
Parte 4

Generatori e carichi

Prof E. Valentinuzzi

2 – bocche con carico

Matrice z



$$\begin{cases} v_1 = z_{11} i_1 + z_{12} i_2 \\ v_2 = z_{21} i_1 + z_{22} i_2 \end{cases} \qquad z_c = -\frac{v_2}{i_2}$$

$$z_i = \frac{v_1}{i_1} = z_{11} + z_{12}\frac{i_2}{i_1} = z_{11} + z_{12}\left(\frac{z_{21}}{\frac{v_2}{i_2} - z_{22}}\right) = z_{11} - \frac{z_{12} z_{21}}{z_{22} + z_c}$$

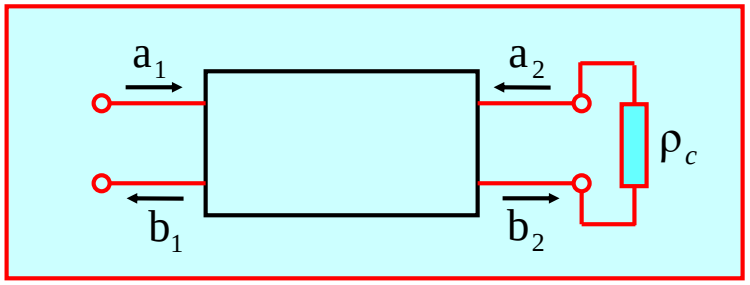
Matrice c

$$\begin{cases} v_1 = c_{11} v_2 - c_{12} i_2 \\ i_1 = c_{21} v_2 - c_{22} i_2 \end{cases} \qquad z_i = \frac{v_1}{i_1} = \frac{c_{11} z_c + c_{12}}{c_{21} z_c + c_{22}}$$

2 – bocche con carico

Matrice t

$$\begin{cases} a_1 = t_{11} b_2 + t_{12} a_2 \\ b_1 = t_{21} b_2 + t_{22} a_2 \end{cases} \qquad \rho_c = \frac{a_2}{b_2}$$



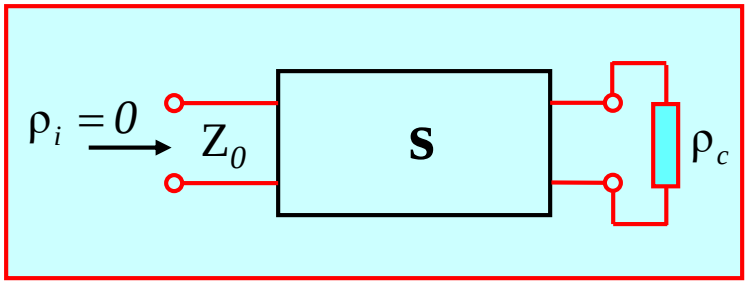
$$\rho_i = \frac{b_1}{a_1} = \frac{t_{22}\rho_c + t_{21}}{t_{12}\rho_c + t_{11}}$$

Matrice s

$$\begin{cases} b_1 = s_{11} a_1 + s_{12} a_2 \\ b_2 = s_{21} a_1 + s_{22} a_2 \end{cases}$$

$$\rho_i = \frac{b_1}{a_1} = s_{11} + s_{12}\frac{a_2}{a_1} = s_{11} + s_{12}\left(\frac{s_{21}}{\frac{1}{\rho_c} - s_{22}}\right) = s_{11} + \frac{s_{12}s_{21}\rho_c}{1 - s_{22}\rho_c}$$

Adattatore



$$\rho_i = s_{11} + \frac{s_{12}s_{21}\rho_c}{1 - s_{22}\rho_c} = 0$$

$$s_{11} - \left(s_{11}s_{22}\rho_c - s_{12}s_{21}\rho_c\right) = s_{11} - \rho_c \det \mathbf{s} \quad \Rightarrow \quad \frac{s_{11}}{\det \mathbf{s}} = \rho_c$$

$$\mathbf{s}\,\tilde{\mathbf{s}}^{*} = \mathbf{e}$$

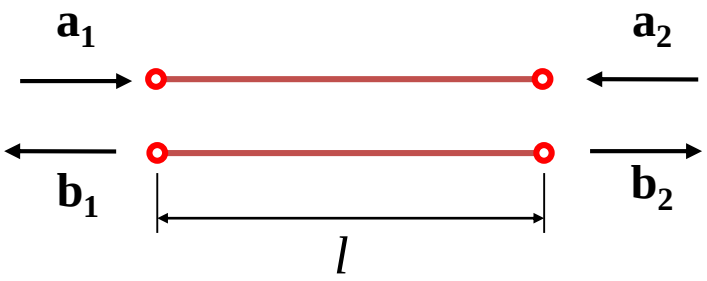
(assenza di perdite)

$$\tilde{\mathbf{s}}^{*} = \mathbf{s}^{-1} \qquad \begin{pmatrix} s_{11}^{*} & s_{21}^{*} \\ s_{12}^{*} & s_{22}^{*} \end{pmatrix} = \frac{1}{\det \mathbf{s}} \begin{pmatrix} s_{22} & -s_{12} \\ -s_{21} & s_{11} \end{pmatrix} \qquad \frac{s_{11}}{\det \mathbf{s}} = s_{22}^{*}$$

$$\boxed{\rho_c = s_{22}^{*}}$$

Qualche esempio

Tratto di linea di lunghezza l



$$s_{11} = 0$$

$$s_{12} = e^{-j\beta l}$$

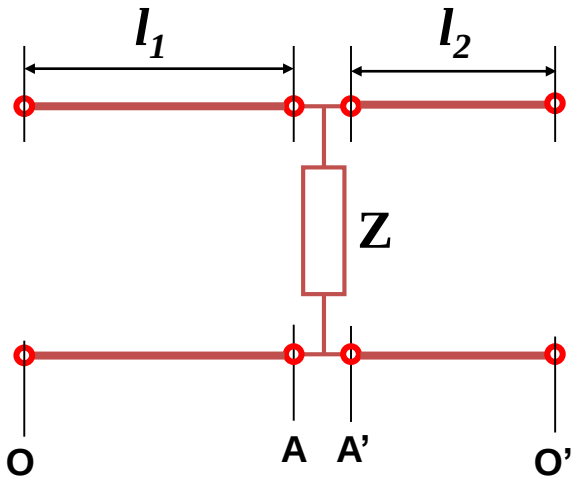
$$s_{21} = e^{-j\beta l}$$

$$s_{22} = 0$$

$$\mathbf{s} = \begin{pmatrix} 0 & e^{-j\beta l} \\ e^{-j\beta l} & 0 \end{pmatrix}$$

$$\mathbf{t} = \begin{pmatrix} e^{j\beta l} & 0 \\ 0 & e^{-j\beta l} \end{pmatrix}$$

Impedenza Z in parallelo



Tra A e A':

$$\mathbf{Z} = \begin{vmatrix} Z & Z \\ Z & Z \end{vmatrix} \Rightarrow \begin{vmatrix} z & z \\ z & z \end{vmatrix} \qquad \left(z = \frac{Z}{Z_0} \right)$$

$$\mathbf{s} = \begin{vmatrix} \dfrac{-1}{1+2z} & \dfrac{2z}{1+2z} \\ \dfrac{2z}{1+2z} & \dfrac{-1}{1+2z} \end{vmatrix}$$

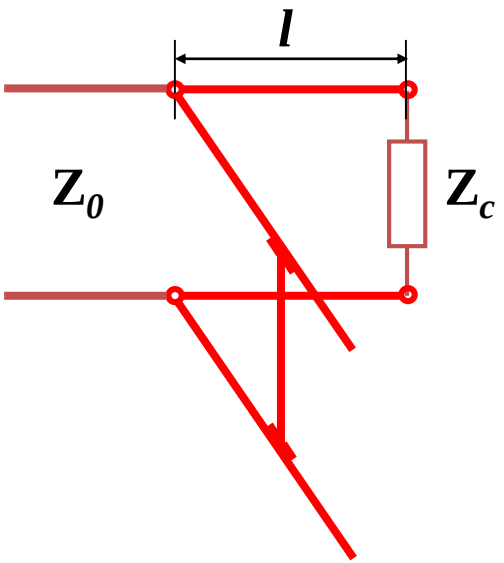
Spostamento sezioni di riferimento:

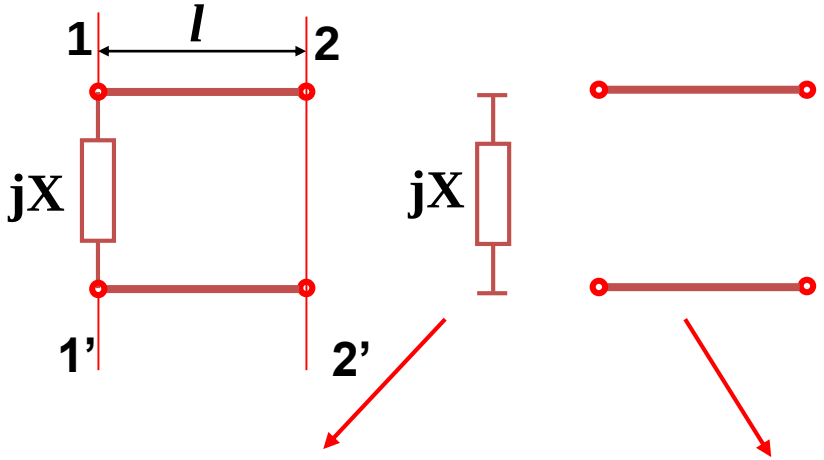
$$\boldsymbol{\varphi} = \begin{vmatrix} e^{-j\beta_1 l_1} & 0 \\ 0 & e^{-j\beta_2 l_2} \end{vmatrix}$$

Tra O e O':

$$\mathbf{s} = \begin{vmatrix} \dfrac{-1}{1+2z} e^{-j2\beta l_1} & \dfrac{2z}{1+2z} e^{-j\beta(l_1+l_2)} \\ \dfrac{2z}{1+2z} e^{-j\beta(l_1+l_2)} & \dfrac{-1}{1+2z} e^{-j2\beta l_2} \end{vmatrix}$$

Adattatore a singolo stub





$$\mathbf{s} = \begin{vmatrix} \dfrac{-1}{1+2\,\mathrm{jx}} & \dfrac{2\,\mathrm{jx}}{1+2\,\mathrm{jx}}e^{-j\beta l} \\ \dfrac{2\,\mathrm{jx}}{1+2\,\mathrm{jx}}e^{-j\beta l} & \dfrac{-1}{1+2\,\mathrm{jx}}e^{-j2\beta l} \end{vmatrix}$$

$$\mathbf{s}' = \begin{vmatrix} \dfrac{-1}{1+2\,\mathrm{jx}} & \dfrac{2\,\mathrm{jx}}{1+2\,\mathrm{jx}} \\ \dfrac{2\,\mathrm{jx}}{1+2\,\mathrm{jx}} & \dfrac{-1}{1+2\,\mathrm{jx}} \end{vmatrix} \qquad \mathbf{s}'' = \begin{vmatrix} 0 & e^{-j\beta l} \\ e^{-j\beta l} & 0 \end{vmatrix}$$

$$\mathrm{s}_{22}^{*} = \frac{-1}{1-2\,\mathrm{jx}}e^{j2\beta l}$$

$$\sqrt{1+4\,\mathrm{x}^2} = \frac{1}{|\rho_c|} \qquad \mathrm{x} = \pm\frac{1}{2}\sqrt{\frac{1}{|\rho_c|^2}-1} \qquad \beta l = \frac{1}{2}\arg(\rho_c) - \frac{1}{2}\left[-\pi + \tan^{-1}(2\,\mathrm{x})\right]$$

Esempio

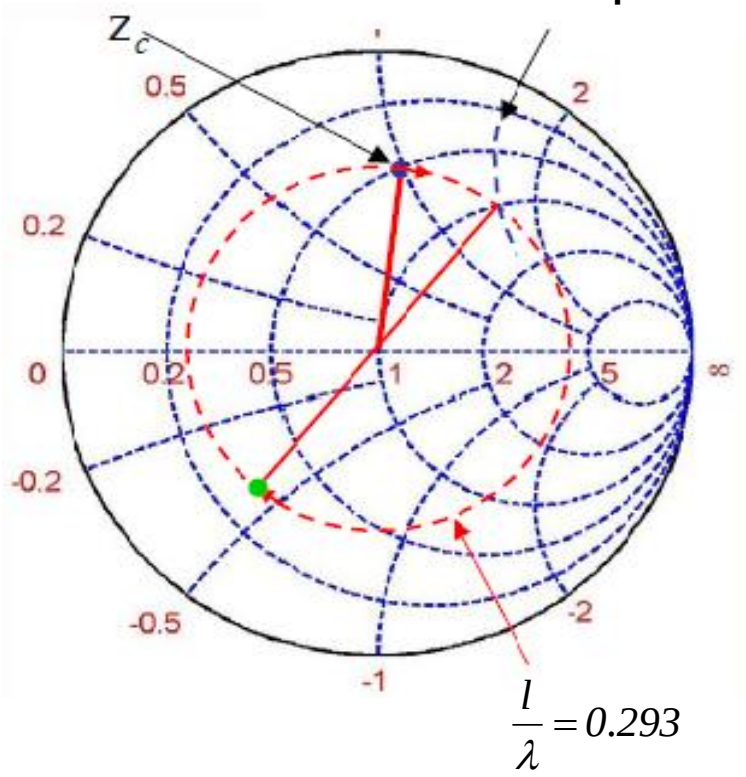
Suscettanza da compensare

$$Z_c = 25 + j50\ \Omega$$

$$Z_0 = 50\ \Omega$$

$$z_c = 0.5 + j1$$

$$\rho_c = \frac{z_c - 1}{z_c + 1} = \frac{j1 - 0.5}{1.5 + j1} = 0.62 e^{j1.4465}$$

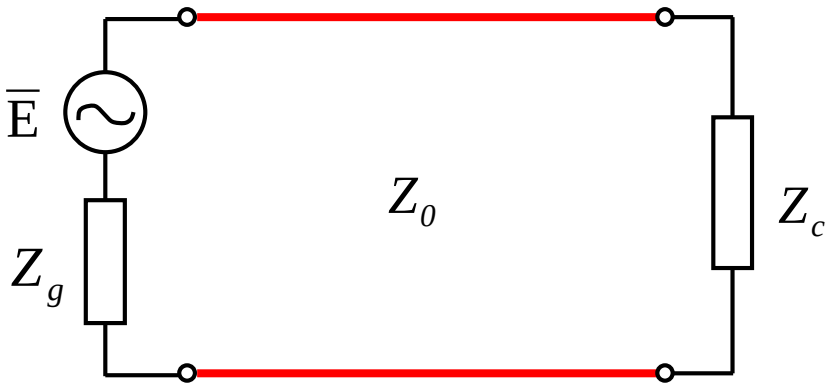


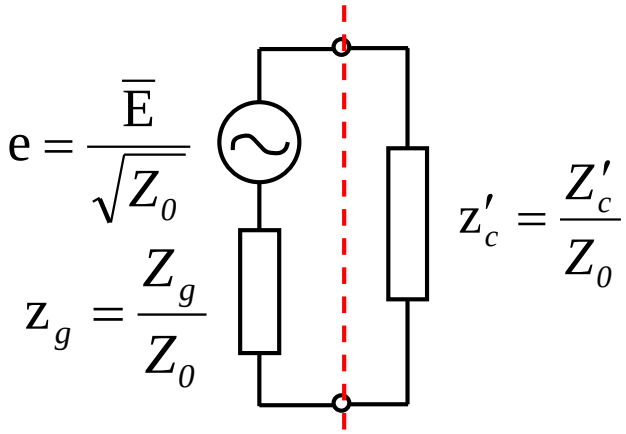
$$\mathrm{x} = \frac{1}{2}\sqrt{\frac{1}{|\rho_c|^2} - 1} = 0.6324$$

$$\beta l = \frac{1}{2}1.4465 - \frac{1}{2}\left[-\pi + \tan^{-1}(2 * 0.6324)\right] = 1.8432$$

$$\frac{l}{\lambda} = \frac{1.8432}{2\pi} = 0.293$$

Generatori e carichi



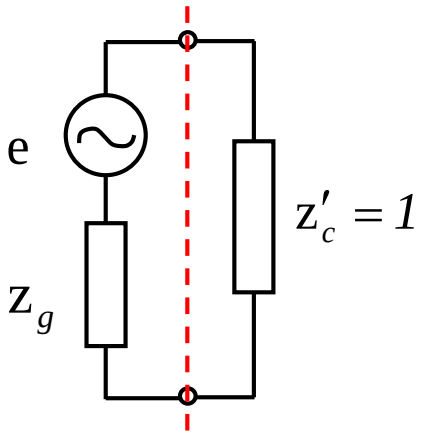


$Z'_c =$ Impedenza di carico vista dal generatore

La potenza erogata dal generatore dipende dal carico

Potenza progressiva

$$Z_c = Z_0 \qquad \left(z_c = z'_c = 1\right)$$



Sulla linea si ha soltanto l'onda incidente

Alla potenza corrispondente erogata al carico si dà il nome di

Potenza progressiva (P_p)

$$P_p = \frac{1}{2}\frac{e^2}{\left|1+z_g\right|^2}$$

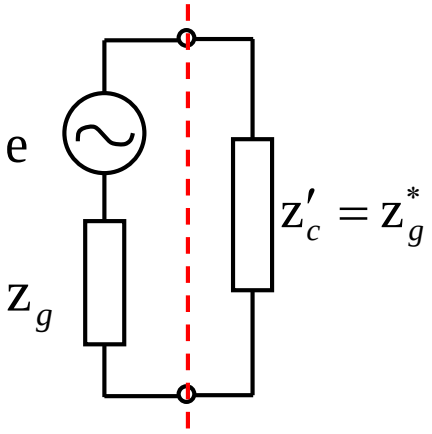
In funzione di $\rho_g = \dfrac{z_g - 1}{z_g + 1}$;

$$\left|1+z_g\right|^2 = \left|1+\frac{1+\rho_g}{1-\rho_g}\right|^2 = \left|\frac{2}{1-\rho_g}\right|^2$$

$$P_p = \frac{e^2}{8}\left|1-\rho_g\right|^2$$

Potenza disponibile

$$z_c' = z_g^*$$



Massima potenza erogata al carico

(P_d = potenza disponibile)

$$P_d = \frac{e^2}{8}\frac{1}{\mathrm{Re}(z_g)}$$

$$P_d = \frac{e^2}{8}\frac{1}{\frac{1}{2}\left(\frac{1+\rho_g}{1-\rho_g}+\frac{1+\rho_g^*}{1-\rho_g^*}\right)} = \frac{e^2}{8}\frac{1}{\frac{1}{2}\frac{(1-\rho_g^*+\rho_g-|\rho_g|^2+1-\rho_g+\rho_g^*-|\rho_g|^2)}{|1-\rho_g|^2}}$$

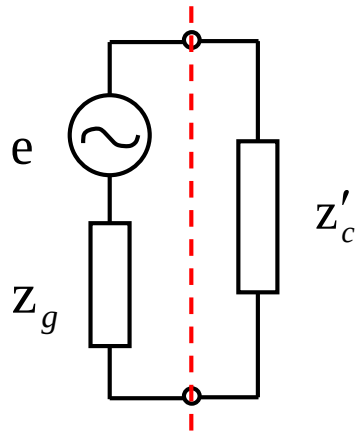
$$P_d = \frac{e^2}{8}\frac{|1-\rho_g|^2}{1-|\rho_g|^2} = \frac{P_p}{1-|\rho_g|^2}$$

$$Se \quad \rho_g = 0 \quad \Rightarrow \quad P_d = P_p$$

(generatore adattato)

Potenza su carico qualsiasi

z'_c *qualsiasi*



Potenza erogata al carico

$$P_c = \frac{e^2}{2}\frac{\mathrm{Re}(z'_c)}{\left|z'_c + z_g\right|^2}$$

$$P_c = \frac{e^2}{2}\frac{\mathrm{Re}(z'_c)}{\left|z'_c + z_g\right|^2} = \frac{e^2}{2}\frac{1-\left|\rho'_c\right|^2}{\left|1-\rho'_c\right|^2}\frac{1}{\left(\frac{1+\rho'_c}{1-\rho'_c}+\frac{1+\rho_g}{1-\rho_g}\right)\left(\frac{1+\rho'^{*}_c}{1-\rho'^{*}_c}+\frac{1+\rho^{*}_g}{1-\rho^{*}_g}\right)}$$

$$= \frac{e^2}{2}\frac{1-\left|\rho'_c\right|^2}{\left|1-\rho'_c\right|^2}\frac{1}{\frac{\left|1-\rho_g+\rho'_c-\rho'_c\rho_g+1-\rho'_c+\rho_g-\rho'_c\rho_g\right|^2}{\left|1-\rho'_c\right|^2\left|1-\rho_g\right|^2}} = \frac{e^2}{8}\left|1-\rho_g\right|^2\frac{1-\left|\rho'_c\right|^2}{\left|1-\rho'_c\rho_g\right|^2}$$

Potenza su carico qualsiasi

$$P_c = \frac{e^2}{8}\left|1-\rho_g\right|^2 \frac{1-\left|\rho'_c\right|^2}{\left|1-\rho'_c\rho_g\right|^2}$$

$$= P_p \frac{1-\left|\rho'_c\right|^2}{\left|1-\rho'_c\rho_g\right|^2} = P_d \frac{\left(1-\left|\rho_g\right|^2\right)\left(1-\left|\rho'_c\right|^2\right)}{\left|1-\rho'_c\rho_g\right|^2}$$

Per definire il comportamento di un generatore sono sufficienti :

$$P_d \quad e \quad \rho_g$$

oppure $P_p \quad e \quad \rho_g$

Grandezze preferibili ad e e z_g

Potenza utilizzata

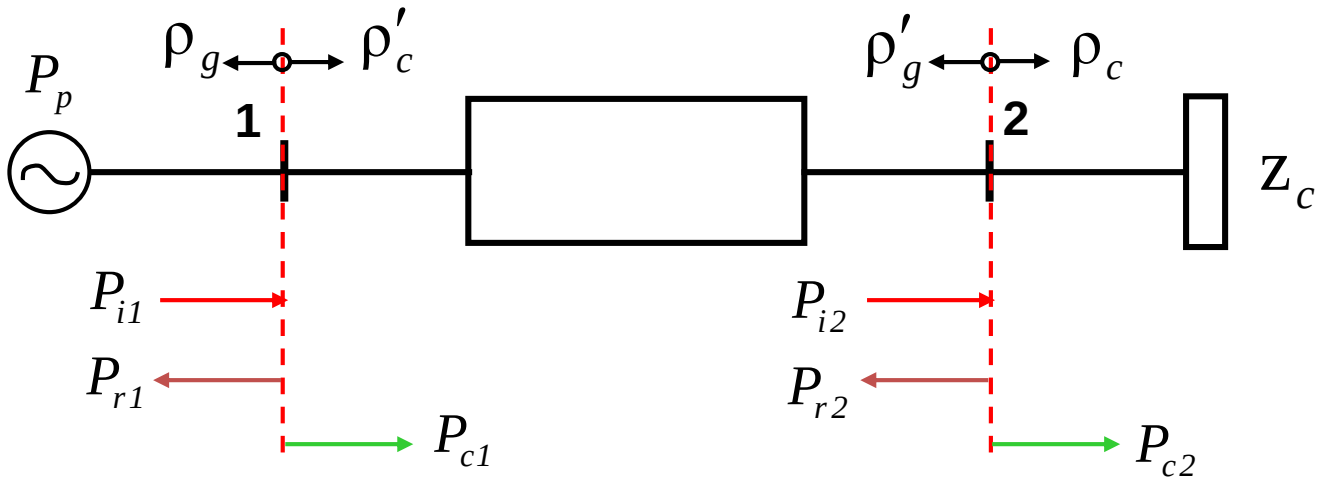
$$P_c = P_p \frac{1-\left|\rho_c'\right|^2}{\left|1-\rho_c'\rho_g\right|^2} = P_d \frac{\left(1-\left|\rho_g\right|^2\right)\left(1-\left|\rho_c'\right|^2\right)}{\left|1-\rho_c'\rho_g\right|^2}$$

Ma: $P_c = P_i - P_r = P_i\left(1-\left|\rho_c'\right|^2\right)$

$$P_i = \frac{P_p}{\left|1-\rho_c'\rho_g\right|^2} \qquad P_r = P_p \frac{\left|\rho_c'\right|^2}{\left|1-\rho_c'\rho_g\right|^2}$$

Se il generatore è adattato $\left(\rho_g = 0\right)$ ⇒ $P_i = P_p = P_d$

Potenza attraverso una giunzione a 2 bocche



Ricordare: $$\rho'_c = s_{11} + \frac{s_{12}s_{21}\rho_c}{1 - s_{22}\rho_c} = \frac{s_{11} - \rho_c \det \mathbf{s}}{1 - s_{22}\rho_c}$$

Potenza attraverso una giunzione a 2 bocche

$$P_{c1} = P_p \frac{1-\left|\rho_c'\right|^2}{\left|1-\rho_c'\rho_g\right|^2}$$

$$\rho_c' = \frac{s_{11}-\rho_c \det \mathbf{s}}{1-s_{22}\rho_c} \qquad \left|\rho_c'\right|^2 = \frac{\left|s_{11}-\rho_c \det \mathbf{s}\right|^2}{\left|1-s_{22}\rho_c\right|^2}$$

$$1-\left|\rho_c'\right|^2 = \frac{\left|1-s_{22}\rho_c\right|^2-\left|s_{11}-\rho_c \det \mathbf{s}\right|^2}{\left|1-s_{22}\rho_c\right|^2}$$

$$\left|1-\rho_c'\rho_g\right|^2 = \left|1-\rho_g \frac{s_{11}-\rho_c \det \mathbf{s}}{1-s_{22}\rho_c}\right|^2$$

$$P_{c1} = P_p \frac{\dfrac{\left|1-s_{22}\rho_c\right|^2-\left|s_{11}-\rho_c \det \mathbf{s}\right|^2}{\left|1-s_{22}\rho_c\right|^2}}{\dfrac{\left|1-s_{22}\rho_c-s_{11}\rho_g+\rho_g\rho_c \det \mathbf{s}\right|^2}{\left|1-s_{22}\rho_c\right|^2}} = P_p \frac{\left|1-s_{22}\rho_c\right|^2-\left|s_{11}-\rho_c \det \mathbf{s}\right|^2}{\left|1-s_{22}\rho_c-s_{11}\rho_g+\rho_g\rho_c \det \mathbf{s}\right|^2}$$

$$= \Delta$$

Potenza attraverso una giunzione a 2 bocche

$$P_{c1} = P_p \frac{\left|1 - s_{22}\rho_c\right|^2 - \left|s_{11} - \rho_c \det \mathbf{s}\right|^2}{\Delta} = P_p \frac{\left|1 - s_{22}\rho_c\right|^2}{\Delta} - P_p \frac{\left|s_{11} - \rho_c \det \mathbf{s}\right|^2}{\Delta}$$

$$P_{i1} \qquad P_{r1}$$

$$P_{c2} = \frac{1}{2}\left|b_2\right|^2\left(1 - \left|\rho_c\right|^2\right)$$

$$\frac{1}{2}\left|b_2\right|^2 = \frac{1}{2}\left|a_1\right|^2 \frac{\left|s_{21}\right|^2}{\left|1 - s_{22}\rho_c\right|^2} = P_{i1} \frac{\left|s_{21}\right|^2}{\left|1 - s_{22}\rho_c\right|^2}$$

$$P_{c2} = P_{i1} \frac{\left|s_{21}\right|^2\left(1 - \left|\rho_c\right|^2\right)}{\left|1 - s_{22}\rho_c\right|^2} = P_p \frac{\left(1 - \left|\rho_c\right|^2\right)\left|s_{21}\right|^2}{\Delta}$$

$$P_{c2} = P_{i2} - P_{r2} = P_p \frac{\left|s_{21}\right|^2}{\Delta} - P_p \frac{\left|\rho_c\right|^2 \left|s_{21}\right|^2}{\Delta}$$

Potenza attraverso una giunzione a 2 bocche

Potenza assorbita dalla rete a 2 porte

$$P_{c1} = P_p \frac{\left|1 - s_{22}\rho_c\right|^2 - \left|s_{11} - \rho_c \det \mathbf{s}\right|^2}{\Delta} \qquad P_{c2} = P_p \frac{\left|s_{21}\right|^2 \left(1 - \left|\rho_c\right|^2\right)}{\Delta}$$

$$P_a = P_{c1} - P_{c2} =$$

$$= P_p \frac{\left|1 - s_{22}\rho_c\right|^2 - \left|s_{11} - \rho_c \det \mathbf{s}\right|^2 - \left|s_{21}\right|^2 \left(1 - \left|\rho_c\right|^2\right)}{\Delta}$$

Alcuni casi particolari

Generatore adattato $(\rho_g = 0)$

$$\Delta = \left|1 - s_{22}\rho_c - s_{11}\rho_g + \rho_g\rho_c \det \mathbf{s}\right|^2 = \left|1 - s_{22}\rho_c\right|^2$$

$$P_{c1} = P_p\left(1 - \frac{\left|s_{11} - \rho_c \det \mathbf{s}\right|^2}{\left|1 - s_{22}\rho_c\right|^2}\right) \qquad P_{c2} = P_p \frac{\left|s_{21}\right|^2\left(1 - \left|\rho_c\right|^2\right)}{\left|1 - s_{22}\,\rho_c\right|^2}$$

Carico adattato $(\rho_c = 0)$

$$\Delta = \left|1 - s_{11}\rho_g\right|^2$$

$$P_{c1} = P_p \frac{1 - \left|s_{11}\right|^2}{\left|1 - s_{11}\rho_g\right|^2} \qquad P_{c2} = P_p \frac{\left|s_{21}\right|^2}{\left|1 - s_{11}\rho_g\right|^2}$$

Completo adattamento

Generatore adattato $(\rho_g = 0)$

Carico adattato $(\rho_c = 0)$

Caso molto importante

$$\begin{cases} P_{c1} = P_p\left(1 - |s_{11}|^2\right) \\ P_{i1} = P_p \\ P_{r1} = P_p |s_{11}|^2 \end{cases} \qquad \begin{cases} P_{c2} = P_p |s_{21}|^2 \\ P_{i2} = P_{c2} \\ P_{r2} = 0 \end{cases}$$

$$P_a = P_p\left(1 - |s_{11}|^2 - |s_{21}|^2\right)$$

Attenuazione di inserzione

Rapporto tra:

a) Potenza assorbita dal carico per connessione diretta al generatore

b) Potenza assorbita con l'interposizione del 2 - porte

$$= P_p \frac{1-|\rho_c|^2}{|1-\rho_c\rho_g|^2} \qquad = P_p \frac{\left(1-|\rho_c|^2\right)|s_{21}|^2}{\Delta}$$

$$\Delta = |1-s_{22}\rho_c-s_{11}\rho_g+\rho_g\rho_c \det \mathbf{s}|^2$$

$$\alpha_i = \frac{|1-s_{22}\rho_c-s_{11}\rho_g+\rho_g\rho_c \det \mathbf{s}|^2}{|1-\rho_c\rho_g|^2|s_{21}|^2}$$

Nel caso di completo adattamento $\alpha_{i0} = \dfrac{1}{|s_{21}|^2}$