Ising model

Recover what we have done in class for the 3D Ising model on a square lattice with nearest neighbor interactions only, and without external magnetic field.

Recover the main results following the assignment given in class, and then add these questions:

1)

Consider a single spin trajectory $s_1(t)$ as a function of time. This individual spin will flip at random times between state $+1$ and -1 and vice versa. Obtain distribution (histogram) of times between flipping events. Start with $T > T_c$ and attempt to approach T_c from above. Explain your observations. What happens when $T < T_c$?

2)

Study and plot (in the same plot) the magnetization per site M vs MC steps for temperatures T = 1.4, 2.269 and 3.4 for a lattice size of L = 50.

3)

Study and plot (in the same plot) the magnetization per site M vs energy per site E for temperatures T = 1.4, 2.269 and 3.4 for a lattice size of L = 50: what can you observe?

4)

C. The correlation function

In systems where a physical magnitude relies on position, one generally asks, given a measure at point $\mathbf{r}_i$ what is the relation between another measure at a position $\mathbf{r}_j$. This is given by the spatial correlation function and if the system presents translational and rotational symmetry (such as the Ising model), the correlation function does not depend on the absolute positions, but on the distance between them $r = |\mathbf{r}_i - \mathbf{r}_j|$. The correlation function we are interested in is the spin-spin correlation function that is given by,

$$G(r,T) = \langle s(0)s(r)\rangle - \langle s(0)\rangle^2, \qquad (36)$$

where $\langle s(0)\rangle = \langle s(r)\rangle = M/N$ is the magnetization per site. Because of the fact that for a given temperature, M reaches a constant value, the behavior of the correlation function is carried by the first term of eq. (36). Thus we will consider the correlation function only as,

$$G(r,T) = \langle s(0)s(r)\rangle. \qquad (37)$$

We are limited to obtain the correlation function up to $L/2$, where L is the lattice size. This came as a price of the periodic boundary conditions we are using. For example, if we were to calculate the correlation function up to the value $r = L$ we would find that the correlation function would be equal to 1 there, which is wrong because we would be computing the correlation function at $r = 0$.

The process for numerically computing the correlation function is the following: For each spin in the lattice, we determine the value of the local correlation function in $r = n$ taking the average magnetic state of the nearest neighbors found advancing n steps in one direction (not mixing $\hat{e}_i$ with $\hat{e}_j$, i.e. not moving in diagonals). The global correlation function is taken as the average of all the local correlation functions. The process is repeated for multiple simulations of the Ising model.

Correlation function G(r, T) for temperatures T = 1.4, 2.269, 3.4 for a lattice size of L = 50 (or better, for 128).

From 1-4, What can you infer about the behavior of the system approaching the critical temperature?