From: An Introduction to Computer Simulation Methods Third Edition (revised)
by Harvey Gould, Jan Tobochnik, and Wolfgang Christian

Ch 11: Numerical and Monte Carlo Methods

Problem 11.14. Generating normal distributions

Fernández and Criado have suggested another method of generating normal distributions that is much faster than the Box-Muller method. We will just summarize the algorithm; the proof that the algorithm leads to a normal distribution is given in their paper.

i. Begin with N numbers, v_i, in an array. Set all the $v_i = \sigma$, where σ is the desired standard deviation for the normal distribution.

ii. Update the array by randomly choosing two different entries, v_i and v_j from the array. Then let $v_i = (v_i + v_j)/\sqrt{2}$ and use the new v_i to set $v_j = -v_i + v_j\sqrt{2}$.

iii. Repeat step (ii) many times to bring the array of numbers to "equilibrium."

iv. After equilibration, the entries v_i will have a normal distribution with the desired standard deviation and zero mean.

Write a program to produce a series of random numbers according to this algorithm. Your program should allow the user to enter N and σ and a button should be implemented to allow for equilibration before various averages are computed. The desired output is the probability distribution of the random numbers that are produced as well as their mean and standard deviation. First make sure that the standard deviation of the probability distribution approaches the desired input σ for sufficiently long times. What is the order of magnitude of the equilibration time? Does it depend on N? Plot the natural log of the probability distribution versus v^2 and check that you obtain a straight line with the appropriate slope.

Julio Fernández and Carlos Criado, "Algorithm for normal random numbers," Phys. Rev. E **60**, 3361–3365 (1999).