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Ch 6

The bifurcation diagram in Figure 6.2 indicates that the period doubling behavior ends at $r \approx 0.892$. This value of r is known very precisely and is given by $r = r_\infty = 0.892486417967\ldots$ At $r = r_\infty$, the sequence of period doublings accumulate to a trajectory of infinite period. In

We studied in class the logistic map. Its bifurcation diagram indicates that the period doubling behavior ends at $r \approx 0.892$. This value of r is known very precisely and is given by $r = r_\infty = 0.892486417967$ . . . At $r = r_\infty$, the sequence of period doublings accumulate to a trajectory of infinite period.

In the following, we will find further numerical evidence that the general behavior of the logistic map is independent of the details of the form $x_{n+1} = f(x_n) = 4rx_n(1 - x_n)$ of f(x).

***Problem 6.8.** Other one-dimensional maps

It is easy to modify your programs to consider other one-dimensional maps. Determine the qualitative properties of the one-dimensional maps:

$$f(x) = xe^{r(1-x)} \tag{6.12}$$

$$f(x) = r \sin \pi x. \tag{6.13}$$

Do they also exhibit the period doubling route to chaos? The map in (6.12) has been used by ecologists (cf. May) to study a population that is limited at high densities by the effect of epidemics. Although it is more complicated than (6.5), its advantage is that the population remains positive no matter what (positive) value is taken for the initial population. There are no restrictions on the maximum value of r, but if r becomes sufficiently large, x eventually becomes effectively zero. What is the behavior of the time series of (6.12) for $r = 1.5, 2$, and 2.7? Describe the qualitative behavior of $f(x)$. Does it have a maximum?

The sine map (6.13) with $0 < r \leq 1$ and $0 \leq x \leq 1$ has no special significance, except that it is nonlinear.

DO the analysis of these maps at your best…