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Ch 11: Numerical and Monte Carlo Methods

Comparison of different deterministic methods for numerical integration

The assignment suggests to use routines from the Open Source Physics library and some java-based project, but don't worry: you can easily implement your own subroutines and complement what we have done in class.

First of all, implement the Romberg's method for numerical integration (you can easily find the algorithm searching on the web).

Implement also your own ODE solver (ordinary differential equation), such as the Euler method that we have also discussed in class. If you are finding the value of the $y = \int_a^b f(x)dx$, then we can solve the integral as an ordinary differential equation as dy/dx=f(x), y(a)=0. We can then use any of the numerical techniques such as Euler's methods to find the value of **y(b)** which would be the approximate value of the integral.

Problem 11.6. Understanding errors in integration routines

a. Use the `ode`, `simpson`, `trapezoid`, and `rhomberg` methods in the `Integral` class of the Open Source Physics library to estimate the integral of $\sin^2(2\pi x)$ between $x = 0$ and 1 with a tolerance of 0.01. The exact answer is 0.5. Do all four methods return results within the tolerance? Are some results much more accurate than the tolerance? Change the tolerance to 0.1. How do the results change? Notice that for some of the methods, the results are much better than might be expected because the positive and negative errors cancel. Why does the trapezoid method always give the exact answer?

b. Integrate the same function from $x = 0.2$ to 1.0. How accurate are the results now? Explore how the results change with the input tolerance. Does the behavior of the `ode` integrator differ from the others. Why? How do you think the tolerance parameter is used for each of the methods?

c. Integrate $f(x) = x^n$ for various values of n. How do the different integrators compare? Why does the trapezoid integrator do worse than the others for $n = 2$?