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Ch 7 Random Processes

(here reference is made to some java projects; disregard this suggestion and write your own subroutines in your favorite programming language)

Problem 7.14. Random walks with steps of variable length

a. Consider a random walk in one dimension with jumps of all lengths. The probability that the length of a single step is between a and $a + \Delta a$ is $f(a)\Delta a$, where $f(a)$ is the probability density. If the form of $f(a)$ is given by $f(a) = C\,e^{-a}$ for $a > 0$ with the normalization condition $\int_0^\infty p(a)da = 1$, the code needed to generate step lengths according to this probability density is given by (see Section 12.5)

```
stepLength = -Math.log(1 - Math.random());
```

Modify `Walker` and `WalkerApp` to simulate walks of variable length with this probability density. Note that the bin width Δa is one of the input parameters. Consider $N \geq 100$ and visualize the motion of the walker. Generate many walks of N steps and determine $p(x)\Delta x$, the probability that the displacement is between x and $x + \Delta x$ after N steps. Plot $p(x)$ versus x and confirm that the form of $p(x)$ is consistent with a Gaussian distribution.

b. Assume that the probability density $f(a)$ is given by $f(a) = C/a^2$ for $a \geq 1$. Determine the normalization constant C using the condition $C\int_1^\infty a^{-2}\,da = 1$. In this case, we will learn in Section 12.5 that the statement

```
stepLength = 1.0/(1.0 - Math.random());
```

generates values of a according to this form of $f(a)$. Do a Monte Carlo simulation as in part (a) and determine $p(x)\Delta x$. Is the form of $p(x)$ a Gaussian? This type of random walk for which $f(a)$ decreases as a power law, $a^{-1-\alpha}$, is known as a *Levy flight* for $\alpha \leq 2$.