

# LEZIONE 11-12

# Vortice Forzato (moto rigido)

$$\frac{dh_0}{dr} - T \frac{ds}{dr} = V_a \frac{dV_a}{dr} + \frac{V_t}{r} \frac{d}{dr} (V_t \cdot r)$$

$$\frac{dh_0}{dr} - T \frac{ds}{dr} = 0$$

$$\frac{V_t}{r} = \text{cost}$$

$$\frac{V_t}{r} = \frac{V_{ti}}{r_i} \quad \rightarrow \quad V_t = V_{ti} \frac{r}{r_i}$$

# Vortice Forzato (moto rigido)

$$\frac{dh_0}{dr} - T \frac{ds}{dr} = V_a \frac{dV_a}{dr} + \frac{V_t}{r} \frac{d}{dr} (V_t \cdot r)$$

$$\frac{dh_0}{dr} - T \frac{ds}{dr} = 0$$

$$\frac{d}{dr} \left( \frac{V_a^2}{2} \right) + \frac{V_{ti}}{r_i} \frac{d}{dr} \left( \frac{V_{ti}}{r_i} r^2 \right) = 0$$

$$\frac{d}{dr} \left( \frac{V_a^2}{2} \right) = -2 \left( \frac{V_{ti}}{r_i} \right)^2 \cdot r$$

$$\left( \frac{V_a^2}{2} \right) = -2V_{ti}^2 \left( \frac{r}{r_i} \right)^2 + C$$

# Vortice Forzato (moto rigido)

$$\frac{dh_0}{dr} - T \frac{ds}{dr} = V_a \frac{dV_a}{dr} + \frac{V_t}{r} \frac{d}{dr} (V_t \cdot r)$$

$$\frac{dh_0}{dr} - T \frac{ds}{dr} = 0$$

$$V_a = V_{ai}$$

$$r = r_i$$

$$\Rightarrow C = \frac{V_{ai}^2}{2} + V_{ti}^2$$

$$\frac{V_{ti}}{V_{ai}} = \operatorname{tg} \alpha_i$$

$$\left( \frac{V_a}{V_{ai}} \right)^2 = 1 - 2 \tan^2 \alpha_i \left[ \left( \frac{r}{r_i} \right)^2 - 1 \right]$$

# Vortice Forzato

$$h = h_i + \frac{V_i^2 - V^2}{2}$$

$$\frac{V_t}{r} = \text{cost}$$

$$\frac{h}{h_i} = 1 + \frac{1}{2h_i} \left( V_{ti}^2 - V_t^2 + V_{ai}^2 - V_a^2 \right)$$

$$\frac{h}{h_i} = 1 + \frac{V_{ti}^2}{2h_i} \left[ 1 - \left( \frac{V_t}{V_{ti}} \right)^2 \right] + \frac{V_{ai}^2}{2h_i} \left[ 1 - \left( \frac{V_a}{V_{ai}} \right)^2 \right]$$

# Vortice Forzato

$$\frac{h}{h_i} = 1 + \frac{V_{ti}^2}{2h_i} \left[ 1 - \left( \frac{V_t}{V_{ti}} \right)^2 \right] + \frac{V_{ai}^2}{2h_i} \left[ 1 - \left( \frac{V_a}{V_{ai}} \right)^2 \right]$$

$$\frac{V_t}{V_{ti}} = \frac{r}{r_i}$$

$$\frac{V_{ti}}{V_{ai}} = \operatorname{tg} \alpha_i$$

$$\left( \frac{V_a}{V_{ai}} \right)^2 = 1 - 2 \tan^2 \alpha_i \left[ \left( \frac{r}{r_i} \right)^2 - 1 \right]$$

$$\frac{h}{h_i} = 1 + \frac{V_{ti}^2}{2h_i} \left[ \left( \frac{r}{r_i} \right)^2 - 1 \right]$$

# Vortice Forzato

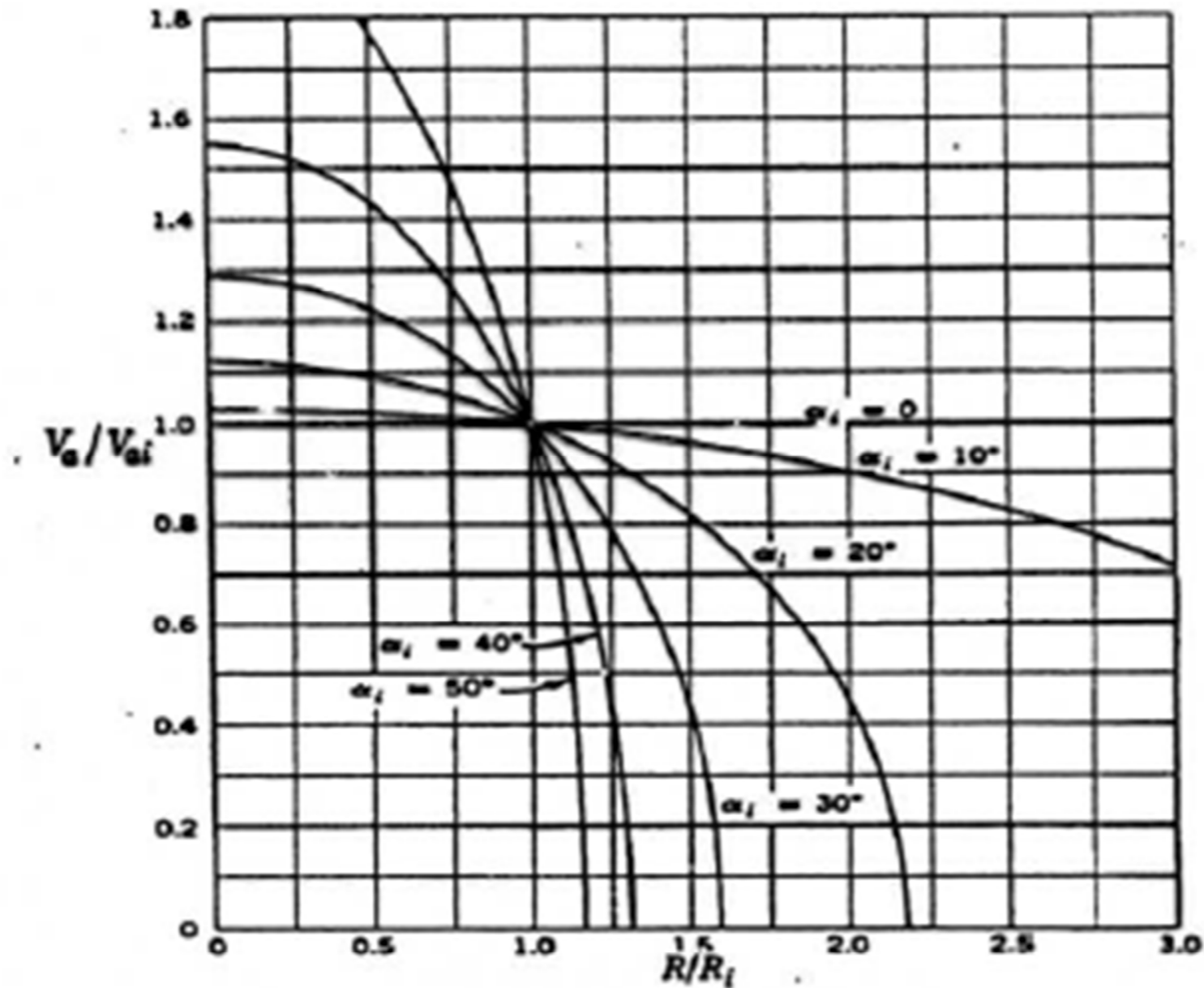
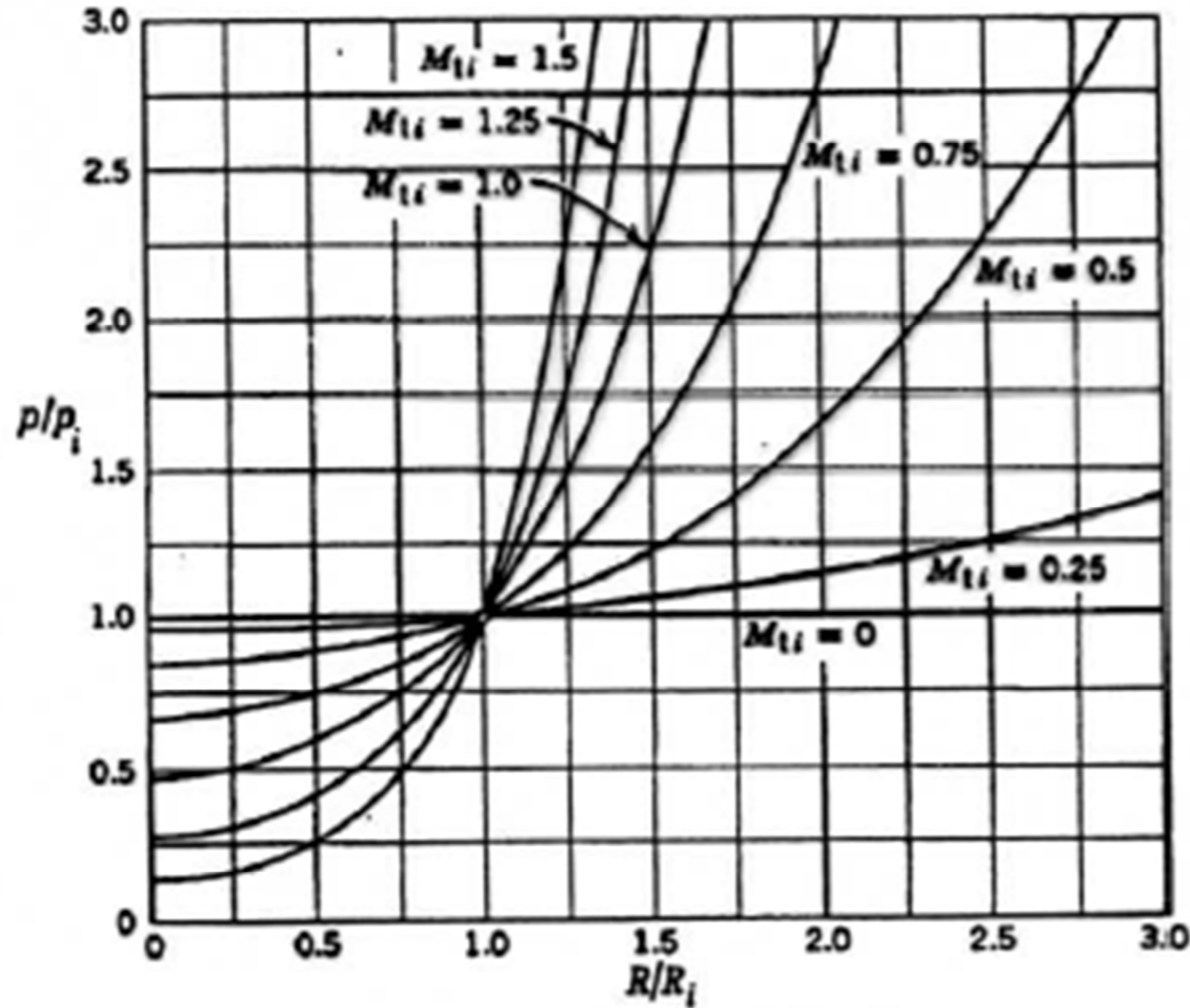


Figura 5.24: *Velocità assiale in funzione del raggio per un flusso a vortice forzato*

# Vortice Forzato



$$\frac{p}{p_i} = \left( \frac{h}{h_i} \right)^{\frac{k}{k-1}}$$

$$\frac{h}{h_i} = 1 + \frac{k-1}{2} M_{ti}^2 \left[ \left( \frac{r}{r_i} \right)^2 - 1 \right]$$

25: Pressione in funzione del raggio per un flusso a vortice forzato



# Vortice Forzato

Imponendo:  $\frac{dh_0}{dr} = 0$

cosa accade nella sezione a valle?

Sezione 1

$$h_{01} = \cos t \quad \frac{V_t}{r} = K$$
$$V_{t1} = K_1 \cdot r$$

Sezione 2

$$V_{t2} = K_2 \cdot r$$

# Vortice Forzato

$$\frac{dh_0}{dr} - T \frac{ds}{dr} = V_a \frac{dV_a}{dr} + \frac{V_t}{r} \frac{d}{dr} (V_t \cdot r)$$

Lavoro scambiato:

$$\begin{aligned} h_{02} - h_{01} &= u (V_{t2} - V_{t1}) = \omega r (K_2 r - K_1 r) = \\ &= \omega (K_2 - K_1) r^2 = \Delta h_{012} \end{aligned}$$

ovvero:  $\Delta h \propto r^2$   $h_{02} \neq \text{cost}$

$$\begin{aligned} \frac{d\Delta h_{012}}{dr} &= \frac{d(h_{02} - h_{01})}{dr} = \frac{dh_{02}}{dr} = V_a \frac{dV_a}{dr} + \frac{V_t}{r} \frac{d}{dr} (V_t r) = \\ &= 2\omega (K_2 - K_1) r = \frac{d}{dr} \left( \frac{V_a^2}{2} \right) + K_2 \frac{d}{dr} (K_2 r^2) \end{aligned}$$

# Vortice Forzato

$$V_{a2}^2 = -2 \left[ K_2^2 - \omega (K_2 - K_1) \right] r^2 + C$$

il valore di C posso calcolarlo da:

$$\frac{\dot{m}}{2\pi\rho} = \int_{rh}^{rs} V_{a1} r dr = \int_{rh}^{rs} V_{a2} r dr$$

# Vortice generico

$$V_{t1} = a \cdot r^n - \frac{b}{r} \qquad V_{t2} = a \cdot r^n + \frac{b}{r}$$

$$L_u = h_{02} - h_{01} = u(V_{t2} - V_{t1}) = 2 \cdot b \cdot \omega$$

- $n = 0$       “zero power blending”
- $n = 1$       “fast power blending”

# Vortice generico

$n = 1$  “Palettatura a grado di reazione costante”

$$V_{t1} = a \cdot r - \frac{b}{r}$$

$$V_{t2} = a \cdot r + \frac{b}{r}$$

$$R = 1 - \frac{V_2^2 - V_1^2}{2L_u} = 1 - \frac{V_{t2}^2 + \cancel{V_{a2}^2} - V_{t1}^2 - \cancel{V_{a1}^2}}{2L_u}$$

$$V_{a2} \approx V_{1a}$$

$$R = 1 - \frac{(\cancel{V_{t2}} - \cancel{V_{t1}})(V_{t2} + V_{t1})}{2u(\cancel{V_{t2}} - \cancel{V_{t1}})} = 1 - \frac{(V_{t2} + V_{t1})}{2u} = 1 - \frac{a}{\omega} = \text{cost}$$

# Angolo palettatura costante

$$\frac{V_t}{V_a} = \tan \alpha = \cos t$$

Angolo costante significa bordo d'attacco rettilineo, come si presenta il deflusso?

Come variano  $V_a$  e  $V_t$  al variare di  $r$  rispettando l'equilibrio radiale?

# Angolo palettatura costante

$$\frac{V_t}{V_a} = \tan \alpha = \cos t$$

$$\frac{V}{V_i} = \frac{V_a}{V_{ai}} = \frac{V_t}{V_{ti}} = \left( \frac{r_i}{r} \right)^{\sin^2 \alpha}$$

$$\begin{cases} \frac{dh_0}{dr} = 0 \\ T \frac{ds}{dr} = 0 \end{cases}$$

$$\frac{d}{dr} \left( \frac{V_a^2}{2} \right) + \frac{V_t}{r} \frac{d(rV_t)}{dr} = 0$$

# Angolo palettatura costante

$$V_a = V \cos \alpha$$

$$V_t = V \sin \alpha$$

$$\frac{d}{dr} \left( \frac{V^2 \cos^2 \alpha}{2} \right) + \frac{V \sin \alpha}{r} \frac{d(r V \sin \alpha)}{dr} = 0$$

$$\boxed{\frac{dV}{dr} + \frac{V}{r} \sin^2 \alpha = 0}$$



# Angolo palettatura costante

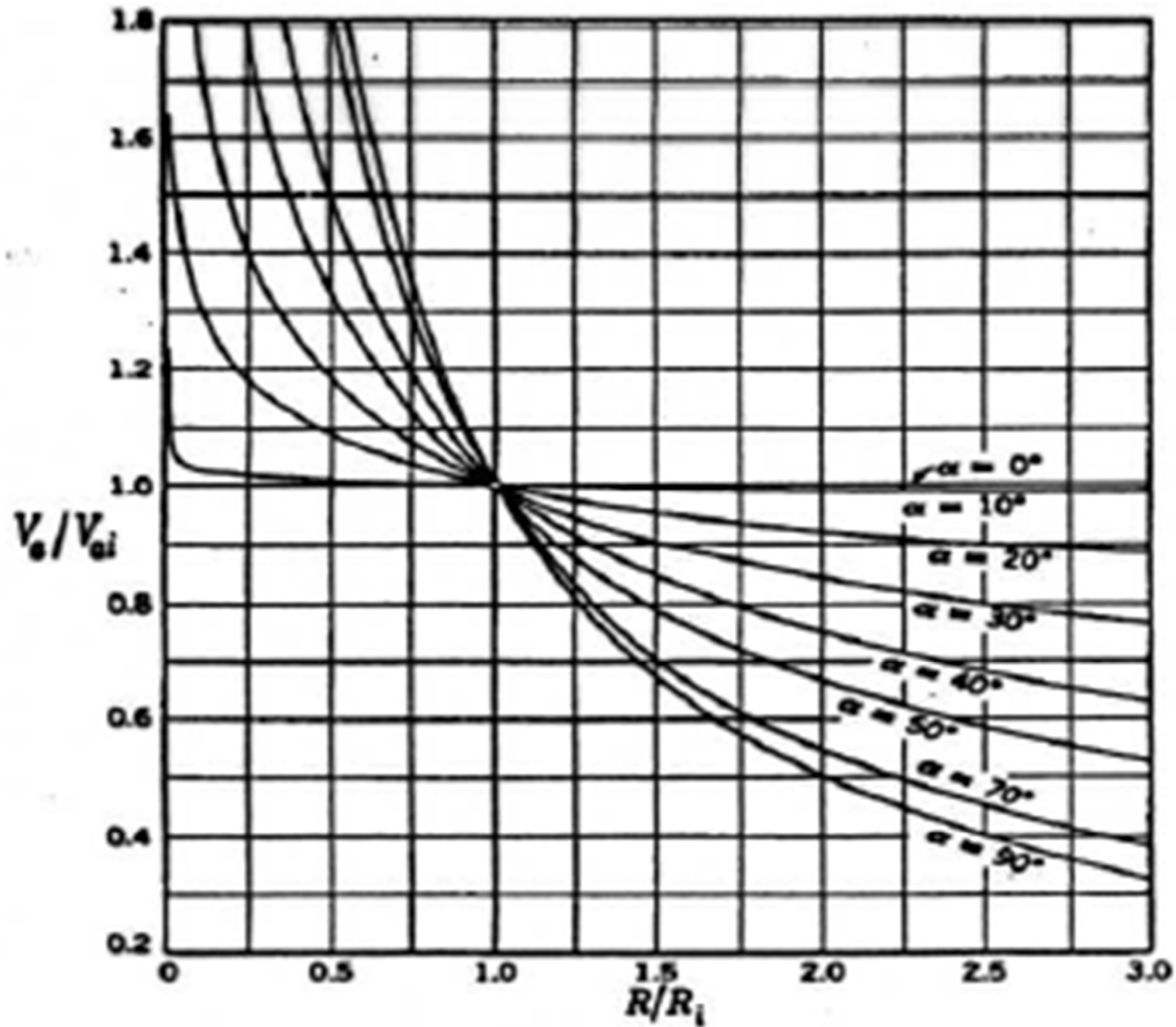
$$y' + \varphi(x)y + \psi(x) = 0 \qquad y = e^{-\int \varphi(x) dx} \left[ C - \int \psi(x) e^{\int \varphi(x) dx} dx \right]$$

in questo caso:  $\varphi(x) = 0$

$$V = e^{-\int \frac{\text{sen}^2 \alpha}{r} dr} \cdot C = r^{-\text{sen}^2 \alpha} \cdot C$$

$$V = V_i \quad \text{se} \quad r = r_i \quad \rightarrow \quad C = \frac{V_i}{r_i^{-\text{sen}^2 \alpha}} \quad \boxed{\frac{h}{h_i} = 1 + \frac{V_{ti}^2}{2h_i \text{sen}^2 \alpha} \left[ 1 - \left( \frac{r_i}{r} \right)^{2\text{sen}^2 \alpha} \right]}$$

# Angolo palettatura costante



# turbina vortice libero

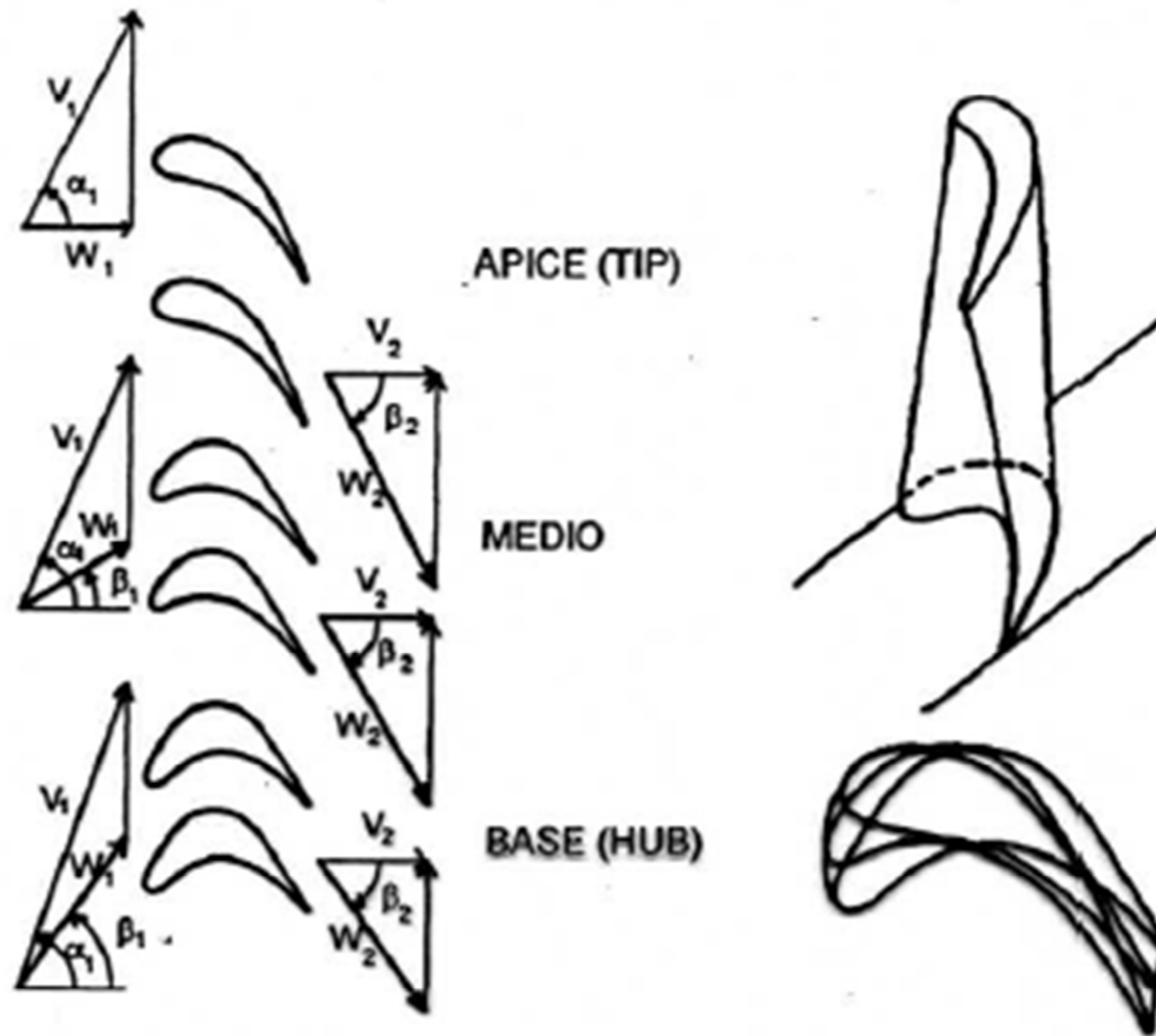


Figura 5.27: Profili e triangoli delle velocità per il rotore di uno stadio di bassa pressione di turbina a vapore o a gas ( $r_{Tip}/r_{Hub} = 1.4$ ).

# compressore vortice libero

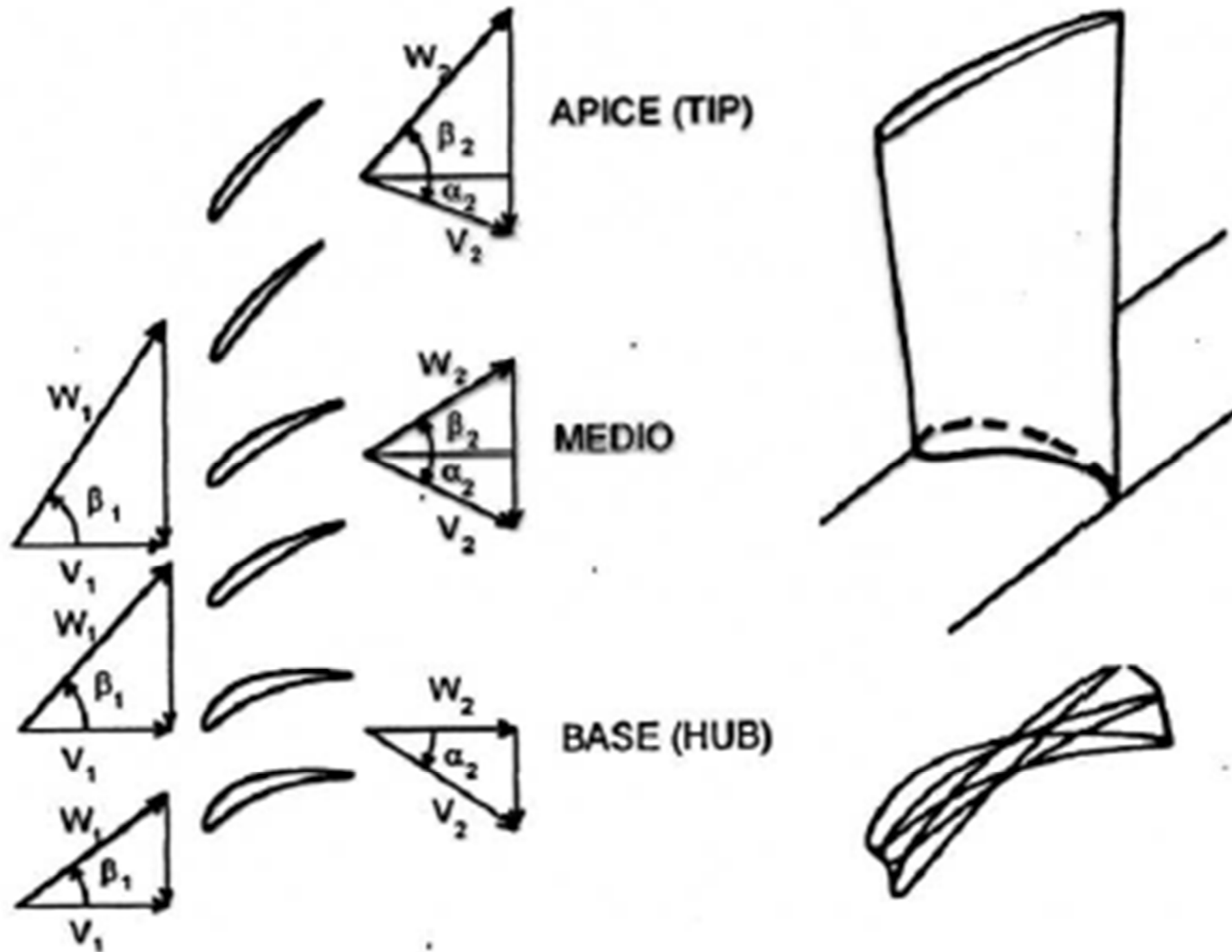


Figura 5.28: Profili e triangoli delle velocità per un rotore di turbo.fan a vortice libero ( $\tau_{Tip}/\tau_{Hub} = 2$ ).