MASTER DEGREE COURSE IN MATHEMATICS, A.Y. 2018/19
ADVANCED GEOMETRY 3 - WORKSHEET 3

To be returned by April 12th.

1. Let $Z = \cup_{\alpha \in I} U_\alpha$ be an open covering of the topological space X. Assume that $U_\alpha \cap U_\beta \neq \emptyset$ for any $\alpha \neq \beta \in I$, and that U_α is irreducible for any $\alpha \in I$. Prove that Z is irreducible.

2. Let X be an irreeducible curve over an algebraically closed field. Prove that every proper closed subset of X is a union of finitely many points. Deduce that any bijection between two irreducible curves is a homeomorphism.

3. Let X be a quasi-affine variety. Prove that X is also a quasi-projective variety.

4. Let $f : \mathbb{A}^2 \to \mathbb{A}^2$ be the map defined by $f(x, y) = (x, xy)$. Describe $f(\mathbb{A}^2)$: is it dense in $\mathbb{A}^2$? Prove that it is not locally closed but it is a finite union of locally closed sets.