Exercises QFT II — 2018/2019

Problem Sheet 4

Problem 7: Full scalar propagator in terms of 1PI 2pt function

Consider the full propagator, i.e. the two-point Green's function $\tilde{G}^{(2)}(p)$ in momentum space, for a scalar self-interacting theory (keep the interaction Lagrangian generic, but proportional to a coupling constant $\lambda \ll 1$). We have seen that

$$-i\tilde{G}^{(2)}(p) = \left(\tilde{\Gamma}^{(2)}(p)\right)^{-1} , \tag{1}$$

where $\tilde{\Gamma}^{(2)}(p)$ is the Fourier transform of $\Gamma^{(2)} \equiv \frac{\delta^2\Gamma}{\delta\psi^2}$.

In perturbation theory one can express $\tilde{\Gamma}^{(2)}(p)$ as the sum (we neglect the $i\epsilon$ term)

$$\tilde{\Gamma}^{(2)}(p) = p^2 - m^2 - i\,\lambda\,\Pi\,. \tag{2}$$

- Justify the expression (2), starting from the fact that $\tilde{G}^{(2)}(p) = \frac{i}{p^2-m^2}$ at leading order in λ.
- By using (2) and (1), show that the full propagator (2pt connected Green's function) can be expressed as

$$\tilde{G}^{(2)}(p) = \frac{i}{p^2-m^2} + \frac{i}{p^2-m^2}\lambda\Pi\frac{i}{p^2-m^2} + \frac{i}{p^2-m^2}\lambda\Pi\frac{i}{p^2-m^2}\lambda\Pi\frac{i}{p^2-m^2} + \ldots \tag{3}$$

  [Use the fact that $\frac{1}{1-X}$ can be expressed as the series $\sum_{n=0}^{\infty} X^n$. In this exercise, X will be proportional to λ.]

- Interpret the relation (3) diagrammatically (the full propagator should be now written in terms of the 1PI 2pt function and the free propagator). [The proper vertices are $i\tilde{\Gamma}^{(n)}$].

Problem 8: Physical mass and two point function

At the first order in perturbation theory, the two-point function of $\lambda\phi^4$-theory is

$$\tilde{\Gamma}^{(2)}(p) = p^2 - m^2 + \frac{\lambda m^2}{32\pi^2}\left(\frac{1}{2-\omega} + \psi(2) - \ln\left(\frac{m^2}{4\pi\mu^2}\right) + \mathcal{O}(2-\omega)\right) \tag{4}$$

In the *minimal subtraction scheme* (MS), the renormalized mass m_r is defined through the relation

$$m_r^2 = m^2 - \frac{\lambda m^2}{32\pi^2}\frac{1}{2-\omega} \tag{5}$$

1. Write $\tilde{\Gamma}^{(2)}(p)$ in terms of m_r instead of m. [Invert the expression (5) and then substitute the found result for m^2 into (4).]
2. Find the expression of the physical mass m_{ph} in terms of m_r.
   [Solve the equation $\tilde{\Gamma}^{(2)}(p^2 = m_{ph}^2) = 0$.]
3. Write $\tilde{\Gamma}^{(2)}(p)$ in terms of m_{ph} instead of m_r and show that the dependence on μ disappears.
4. Repeat points 1,2,3 by using a different subtraction scheme, i.e. the *modified minimal subtraction scheme* ($\overline{MS}$):

$$m_r^2 = m^2 - \frac{\lambda m^2}{32\pi^2}\left(\frac{1}{2-\omega} + \psi(2)\right) \tag{6}$$