Esercizi I Foglio

1) $\sin z = \dfrac{e^{iz} - e^{-iz}}{2i}$ per definizione.

$$\sin(z+w) = \frac{1}{2i}\left(e^{i(z+w)} - e^{-i(z+w)}\right) =$$

$$= \frac{1}{2i}\left(e^{i(z+w)} + e^{i(z-w)} - e^{i(z-w)} - e^{-i(z+w)}\right) =$$

$$= \frac{1}{2i}\left(e^{iz}\left(e^{iw} + e^{-iw}\right) - e^{-iw}\left(e^{iz} + e^{-iz}\right)\right) =$$

$$= \frac{1}{i}\left(e^{iz}\cos w - e^{-iw}\cos z\right) =$$

$$= \frac{1}{i}\left(\left(\cos z + i\sin z\right)\cos w - \left(\cos w - i\sin w\right)\cos z\right) =$$

$$\rightarrow \sin z\cos w + \sin w\cos z$$

La formula per $\cos(z+w)$ si ottiene in

molto analoga oppure prendendo la formula per $\sin(z+w)$ e applicando $\frac{d}{dz}$ ad ambo i membri:

2) $z^{10} = -i = e^{-i\frac{\pi}{2}}$, $|z| = 1$, $z = e^{i\vartheta}$

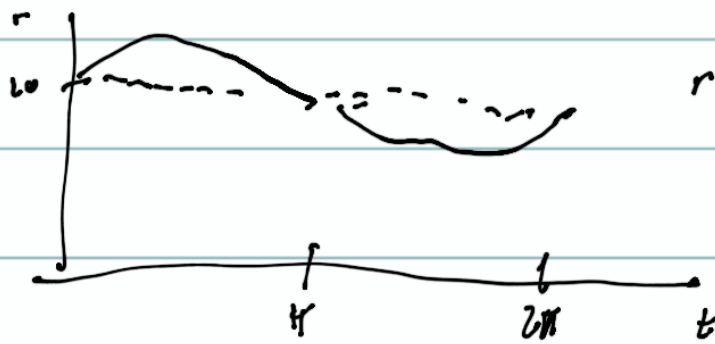
$$10\vartheta = -\frac{\pi}{2} + 2k\pi$$

$$\vartheta_k = -\frac{\pi}{20} + \frac{k\pi}{5} \quad , k = 0, \dots, 9$$

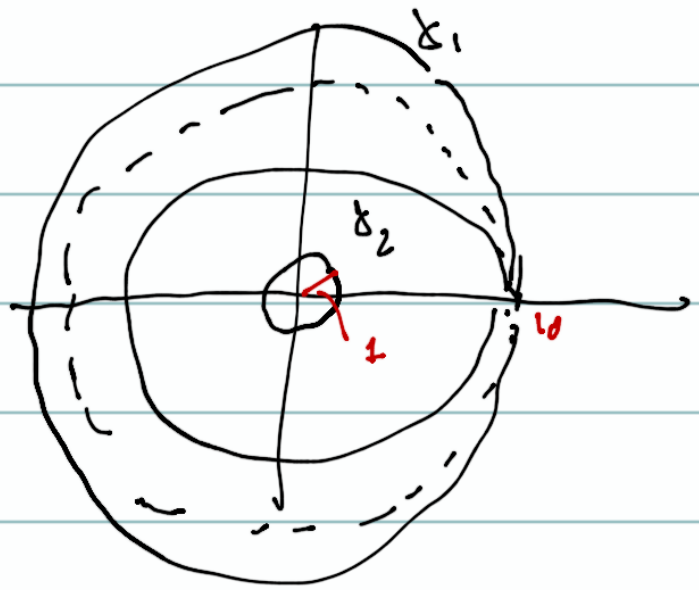
$$z_k = e^{i\vartheta_k} \quad , k = 0, 1, \dots, 9 .$$

3) $r(t) = 10 + \sin t$, $\vartheta(t) = 2t$

$$\gamma(t) = r(t)\, e^{i\vartheta(t)}$$



$\gamma = \gamma_1 + \gamma_2$, $\gamma_1 = \gamma|_{[0,\pi]}$, $\gamma_2 = \gamma|_{[\pi, 2\pi]}$



Sia

$$D_1 = \{ z \in \mathbb{C} \mid z = re^{i\vartheta}, \ 1 < r < 10 + \sin\frac{\vartheta}{2}, \ \vartheta \in [0, 2\pi] \}$$

$$D_2 = \{ z \in \mathbb{C} \mid z = re^{i\vartheta}, \ 1 < r < 10 - \sin\frac{\vartheta}{2}, \ \vartheta \in [0, 2\pi] \}$$?

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$\int_{\partial D_i} \frac{dz}{z} = 0$, $i = 1, 2$, perché $\overline{D_i} \subset \mathbb{C} \setminus \{0\}$.

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$\partial D_i = \gamma_i - \partial B_1(0)$, $i = 1, 2$

$$\frac{1}{2\pi i}\int_{\gamma}\frac{dz}{z}=\frac{1}{2\pi i}\cdot\left(\int_{\gamma_1}+\int_{\gamma_2}\right)=\frac{2}{2\pi i}\int_{\partial B_1}\frac{dz}{z}=2.$$