Exercises QFT II — 2018/2019

Problem Sheet 6

Problem 11: Cutkosky cutting rules

Consider the one-loop Feynman amplitude with four external legs in $\lambda\phi^4$ theory, and concentrate on the s-channel ($s = p^2$).

$$F[p^2] = -\frac{\lambda^2}{2}\int \frac{d^4q}{(2\pi)^4}\,\frac{i}{q^2 - m^2 + i\epsilon}\,\frac{i}{(q-p)^2 - m^2 + i\epsilon}\,. \tag{1}$$

Cutkosky gave the rules, called *cutting rules*, to determine the discontinuity of the Feynman amplitude without explictly computing it: $\mathrm{Disc}F[p^2]$ is given by cutting the diagram into two and replacing each propagator related to the cut internal lines by a delta function imposing the on-shell condition, i.e.

$$\frac{i}{k^2 - m^2 + i\epsilon} \qquad \to \qquad -(2\pi i)\delta(k^2 - m^2)\,. \tag{2}$$

1. Do this substitution in (1) and compute the discontinuity.
2. Is there a discontinuity for $s < 4m^2$?

Problem 12: Superficial degree of divergence

Consider the scalar theory with both cubic and quartic interactions:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{m^2}{2}\phi - \frac{\lambda_3}{3!}\phi^3 - \frac{\lambda_4}{4!}\phi^4\,. \tag{3}$$

Remeber that superficial degree of divergence is given by

$$D = I(d-2) + d(1-V) \tag{4}$$

where I is the number of internal lines, V the number of vertices and d is the spacetime dimensions. In the present example, $V = V_3 + V_4$, where V_3 is the number of three point vertices and V_4 of four point vertices.

- Show that the degree of divergence can be expressed as

$$D = d - E\left(\frac{d}{2} - 1\right) + V_3\left(\frac{d}{2} - 3\right) + V_4(d-4)\,. \tag{5}$$

  where E is the number of external lines. [Hint: Express $E + 2I$ in terms of V_3 and V_4.]

- Take $d = 4$. List all the divergent 1PI diagrams at 1-loop.
- Take $d = 4$. List all the divergent 1PI diagrams at 2-loops.