# MASTER DEGREE COURSE IN MATHEMATICS, A.Y. 2018/19
# ADVANCED GEOMETRY 3 - WORKSHEET 4

To be returned by May 2nd 2019.

1. Let $\varphi : X \to Y$ be a regular map of quasi-projective varieties, let $\varphi^*$ be its comorphism.

a) Prove that the kernel of $\varphi^*$ is the ideal of $\varphi(X)$ in $\mathcal{O}(Y)$.

b) Assume that $K$ is algebraically closed and that $X, Y$ are affine. Deduce that $\varphi$ is dominant if and only if $\varphi^*$ is injective.

2. Let $X$ be an affine algebraic variety, closed in $\mathbb{A}^n$, $F$ a polynomial and $f$ the associated polynomial function on $X$. Prove that $\mathcal{O}(X_F) \simeq \mathcal{O}(X)_f$ (the ring of fractions corresponding to the multiplicative subset $S = \{1, f, f^2, \dots, f^r, \dots\} \subset \mathcal{O}(X)$.

3. Let $v_{1,d} : \mathbb{P}^1 \to \mathbb{P}^d$ be the $d$-tuple Veronese map, such that $v_{1,d}([x_0, x_1]) = [x_0^d, x_0^{d-1}x_1, \dots, x_1^d])$.

a) Check that the image of $v_{1,d}$ is $C_d$, the projective algebraic set defined by the $2 \times 2$ minors of the matrix

$$A = \begin{pmatrix} x_0 & x_1 & \dots & x_{d-1} \\ x_1 & x_2 & \dots & x_d \end{pmatrix}.$$

b) Prove that $v_{1,d} : \mathbb{P}^1 \to C_d$ is an isomorphism, by explicitly constructing its inverse morphism.

c) Prove that any $d+1$ points on $C_d$ are linearly independent in $\mathbb{P}^d$ (Hint: Vandermonde).