

MASTER DEGREE COURSE IN MATHEMATICS, A.Y. 2018/19
ADVANCED GEOMETRY 3 - WORKSHEET 5

To be returned by May 17 2019.

1. Let X, Y be quasi-projective varieties. We identify $X \times Y$ with its image via the Segre map. Prove that the two projection maps

$$X \times Y \xrightarrow{p_1} X, \quad X \times Y \xrightarrow{p_2} Y$$

are regular. (Hint: use the open covering of the Segre variety Σ by $\Sigma \cap U_{ij}$.)

2. Let L, M be the lines in \mathbb{P}^3 defined by the equations:

$$L : x_0 = x_1 = 0; \quad M : x_2 = x_3 = 0.$$

- (1) Find the Plücker coordinates of the points of the Grassmannian $\mathbb{G}(1, 3)$ corresponding to the lines of \mathbb{P}^3 meeting both L and M .
- (2) Prove that these points describe a quadric surface Q of rank 4 contained in $\mathbb{G}(1, 3) \subset \mathbb{P}^5$ and write its equation.
- (3) Describe geometrically the two families of lines contained in Q .

3. Let $X = V_P(F) \subset \mathbb{P}^3$, with $F = x_0x_3^2 - x_1x_2^2$.

- (1) Check that the line $M : x_2 = x_3 = 0$ is contained in X . Let Φ be the pencil of planes containing M .
- (2) Prove that the intersection of X with any plane π of Φ is the union of M with another line L_π .
- (3) Prove that, as π varies in Φ , the points of $\mathbb{G}(1, 3)$ corresponding to the lines L_π describe a skew cubic.