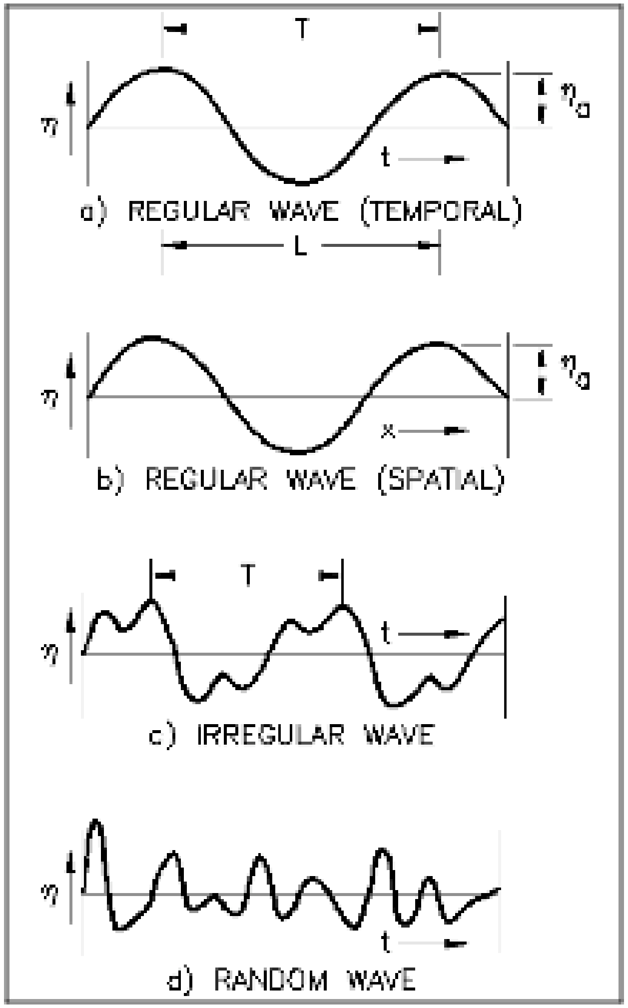
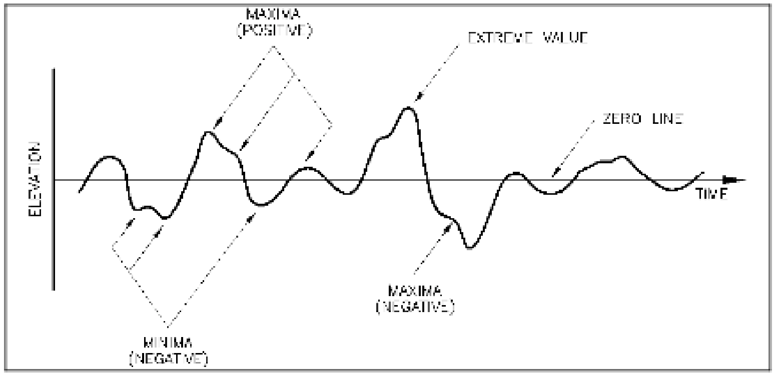


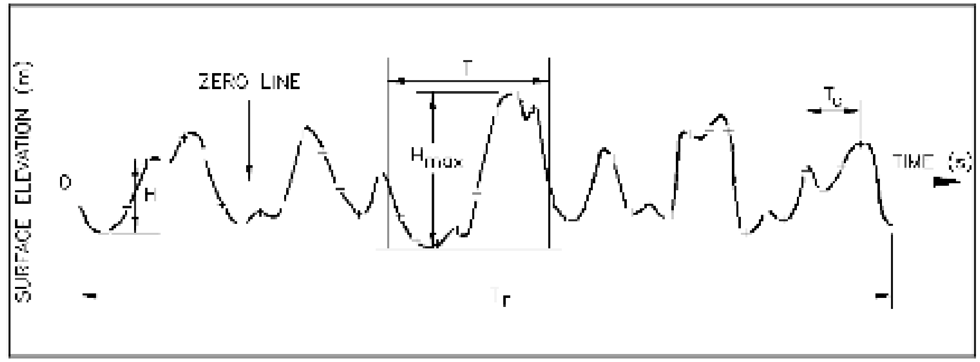
Irregular Waves

Methods of Analysis:

1) Spectral Methods: based on Fourier Transform of the sea surface

2) Wave-by-wave (Wave Train) Analysis: more simplified analysis of the time history of the sea state at a point



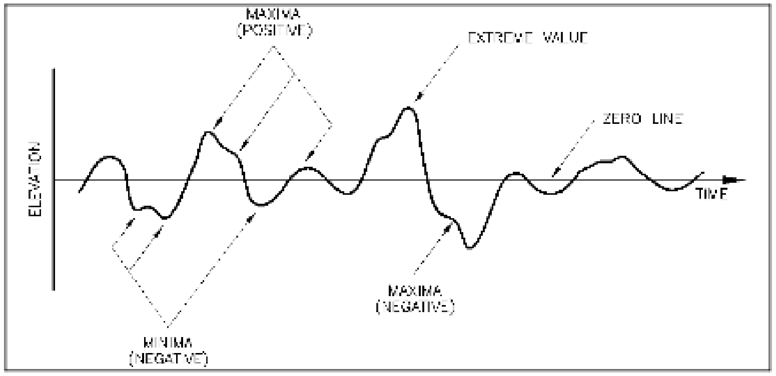


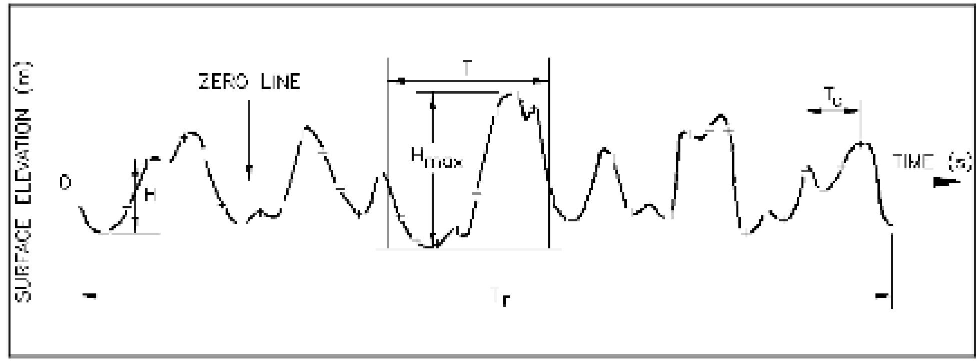
Wave-by-Wave (Wave Train):

Identify local maxima and minima.

Section the record into discrete waves.

Waves are determined by *zero up crossings or down crossings*.





Definition of Wave Parameters:

Hc, Tc: Characteristic wave height and period.

Hmax, Hmean: Maximum and mean wave heights

Hrms: root-mean-square height. $H_{rms} = \sqrt{\frac{1}{N}\sum_{j=1}^{N} H_j^2}$

Hs: significant wave height, average of the 1/3 largest waves in the record.

$$H_s = \frac{1}{\frac{N}{3}} \sum_{i=1}^{N/3} H_i$$

Wave spectra and statistics

A **wave spectrum** is the distribution of wave energy as a function of frequency. It describes the total energy transmitted by a wave-field at a given time. Formally –

$$S(\omega) = 4\int_0^\infty R(\tau)\cos 2\pi\omega\tau d\tau \qquad (1)$$

where ω is the frequency of the waves (defined previously) and $R(\tau)$ is the autocorrelation function of the water-surface time series –

$$R(\tau) = E[x(t)x(t+\tau)] \qquad (2)$$

where τ is the time lag between samples.

Wave spectra are strongly influenced by the wave-producing wind and its statistical/spatial characteristics. The spatial variability is primarily encapsulated into the **fetch.** Fetch is the length over which the wind blows to generate the waves. Virtually all models assume a constant wind speed over the fetch. Unfortunately, this is rarely the case.

Stochastic wave distributions

Another way to assess wave conditions is to describe the water depth (or the perturbation from the mean water level, η) at one point for all time. To do so, the mathematics of probability density functions becomes important.

The most common distribution used is the Rayleigh distribution:

$$p(\eta) = \frac{\eta}{\sigma^2} \exp\left(\frac{\eta^2}{\sigma^2} \right) \tag{3}$$

where η is the perturbation from the mean water surface and σ is the standard deviation of water surface. The standard deviation is defined by

$$\sigma^2 = \int_{-\infty}^{\infty} \eta^2 p(\eta) d\eta \tag{4}$$

Another popular model is the Weibull distribution. The Weibull distribution was developed primarily to describe water flow (and stage) in rivers. It is

$$p(\eta) = \alpha\beta\eta^{\beta-1} \exp\left(-\alpha\eta^{\beta}\right) \tag{5}$$

where α and β are constants to be determined. Massel uses $\alpha = 0.75$ and $\beta = 0.75$ for shallow-water situations.

Occasionally, a log-normal distribution is also assumed.

Common wave-field descriptors

To describe the intensity of the wave-field, it is useful to define **moments**. Moments are defined slightly differently in wave analysis than for turbulent flows. In this case,

$$m_n = \int_0^\infty \omega^n S(\omega) d\omega \tag{6}$$

For instance, you can show that the standard deviation of the water surface $\sigma = \sqrt{m_0}$.

There are several quantities used to describe the strength of a wave field. The most common is the **significant wave height *H_s***. H_s is the average height of the largest 1/3 of the waves. However, it occasionally given the definition

$$H_{m_0} = 4\sqrt{m_0} \tag{7}$$

The other common wave-field descriptor is the root-mean-square wave height H_{rms}. Since the root-mean-square is equivalent to the standard deviation (of a zero-mean process),

$$H_{rms} = 2\sqrt{2m_0} \tag{8}$$

Typical statistical quantities can also be expressed in terms of the zero-moment, if we assume a Rayleigh distribution…

Mean $H = \overline{H} = \sqrt{2\pi m_0}$, Median $H = \sqrt{2\pi m_0}$, Mode $H = 2\sqrt{m_0}$

Of particular interest to sedimentologists is the maximum wave height *Hmax* for a given $\overline{H}$.

The problem is that the Rayleigh distribution is 'flat'.

Assuming a Rayleigh distribution (from Massel) –

$H_{0.1} = 5.09\sqrt{m_0}$, $\quad H_{0.01} = 6.67\sqrt{m_0}$

To counteract this problem, Glukhovskiy (1966) extended the Rayleigh distribution to shallow water and cast the pdf described in (3) to an exceedence pdf. His formulation is

$$P\left(H/\overline{H}\right) = \exp\left[-\frac{\pi}{4}\left(\frac{\varsigma^{2/(1-\varsigma)}}{1+\varsigma/\sqrt{2\pi}}\right)\right] \qquad (9)$$

where $\varsigma = \overline{H}/h$.

Statistics of wave period

The temporal structure of waves (i.e., the period) is more difficult to characterize. There are three different definitions, which have three different results. They are:

Average period between increasing zero-crossings
$\overline{T_z} = 2\pi\sqrt{m_0/m_2}$

Average wave period $\overline{T} = 2\pi m_0/m_1$

Average period between crests $\overline{T_z} = 2\pi\sqrt{m_2/m_4}$

Wind generation of waves

To identify some important factors, we perform a dimensional analysis –

$$H_s, T_s = f(X, U, t, h, g) \tag{10}$$

where H_s and T_s are the significant wave height and period, X is the fetch over which U blows (the most common wind velocity used is obtained at 10 m above the surface), t is time, h is the water depth and g is the gravitational acceleration.

$$\frac{gH_s}{U^2}, \frac{gT_s}{U} = f\left(\frac{gX}{U^2}, \frac{gt}{U}, \frac{gh}{U^2}\right) \tag{11}$$

Common models

SMB – Sverdrup, Munk and Bretschneider

This model was developed during and after WWII from data in the North Atlantic. SMB uses the dimensional analysis above to derive empirical relationships between the dependent and independent variables.

The SMB model is as follows –

$$\frac{gH_s}{U^2} = 0.283 \tanh\left[0.0125\left(\frac{gX}{U^2}\right)^{0.42}\right] \tag{12}$$

$$\frac{gT_s}{2\pi U} = 1.2 \tanh\left[0.077\left(\frac{gX}{U^2}\right)^{0.25}\right] \tag{13}$$

$$\frac{gt}{U} = K \exp\left\{\left[A\left(\ln\left(\frac{gX}{U^2}\right)^2\right) - B\ln\left(\frac{gX}{U^2}\right) + C\right]^{0.5} + D\ln\left(\frac{gX}{U^2}\right)\right\} \tag{14}$$

$K = 6.5882$, $A = 0.0161$, $B = 0.3692$, $C = 2.2024$, $D = 0.8798$

Fully-developed seas (FDS) occur when the fetch no longer controls the development of the waves. In other words, it is at the point when there is no net transport of energy from the wind to the waves.

Of course, the water depth term is negligible (the data was obtained in deep water – mid-North Atlantic).

However, many applications (including sediment transport) require more than H_s. As a result, a second-generation of models was developed that included the entire spectrum.

JONSWAP – Joint North Sea Wave Project

It was noted (first by Phillips, 1958) that at higher frequencies than the peak frequency, the energy in a given wave-field 'saturated'. This saturation produced relatively equivalent energies for a given frequency regardless of virtually all other parameters.

Considerable data taken off the western shore of Denmark was used to produce a model of the wave spectrum (Hasselmann, 1973).

The model is

$$S(\omega) = E(f)\exp\left[-1.25\left(\frac{f_p}{f}\right)^4\right]\gamma^{\Gamma} \qquad (15)$$

where

$$\Gamma = \exp\left[-\frac{(f-f_p)^2}{2\beta^2 f_p^{\ 2}}\right]; \quad E(f) = \frac{\alpha g^2}{(2\pi)^4 f^5}$$

f is the frequency, f_p is the peak frequency (frequency at which $S(f)$ is a maximum), α is the Phillips constant (sometimes called the equilibrium-range parameter), γ is the peak-enhancement factor (usually taken to be 3.3), and $\beta = 0.07$ for $f < f_p$ or $\beta = 0.09$ for $f > f_p$.

There is a slight dependence on the fetch in f_p and α. Hasselmann (1973, 1976) used the nondimensional quantities derived above to create two empirical relations. They are

$$\alpha = 0.076\left(\frac{gX}{U^2}\right)^{-0.22} \tag{16}$$

$$\frac{Uf_p}{g} = 3.5\left(\frac{gX}{U^2}\right)^{-0.33} \tag{17}$$

Donelan

Donelan et al. (1985) proposed a popular alternative which combines the relatively weak effects due to fetch into a single formulation. The formulation also accounts for directionality of the wind.

The model still uses Equation (15) as its basis, though they suggest replacing f^5 in the linear term $E(f)$ with $f^4 f_p^{-1}$. The differences lie in the treatment of α, γ, and f_p. Donelan et al. (1985) suggest

$$\alpha = 0.076\left(\frac{U\cos\theta}{C_p}\right)^{0.55} \qquad \text{for } 0.83 < U/C_p < 5 \qquad (18)$$

where $C_p = g/2\pi f_p$ and θ is the angle between the wind and the waves. The peak-enhancement parameter also changes –

$$\gamma = 1.7 \qquad \text{for } 0.83 < U/C_p < 1$$
$$\gamma = 1.7 + 6.0\ln\left(U/C_p\right) \qquad \text{for } 1 \le U/C_p < 5 \qquad (19)$$

Finally, the directionally dependent, non-dimensional fetch relation is

$$\frac{U\cos\theta}{C_p} = 11.6\left(\frac{gX_\theta}{U^2}\right)^{-0.23} \qquad (20)$$

where X_θ is the fetch in the direction of propagation of the peak waves.

Donelan, like SMB and conventional JONSWAP, does not account for water depth. However, these models are EXTREMELY popular and are often used in shallow-water situations (incorrectly?). For a good review of the pluses and minuses of these deep-water models, see:

Schwab, D. J., et al. 1984. Application of a simple numerical wave model to Lake Erie. *Journal of Geophysical Research*, v. 89, p. 3586-3592.

TMA (Bouws et al., 1985)

To correct for depth-dependent effects, Bouws et al. (1985) also manipulated the linear term $E(f)$ in (15). They wanted it to reflect the loss of energy due to enhanced dissipation of shallow water.

They replace $E(f)$ with $E_k(f,H)$, where

$$E_k(f,H) = \frac{\alpha g^2 \phi_k(\omega_H)}{(2\pi)^4 f^5} \tag{21}$$

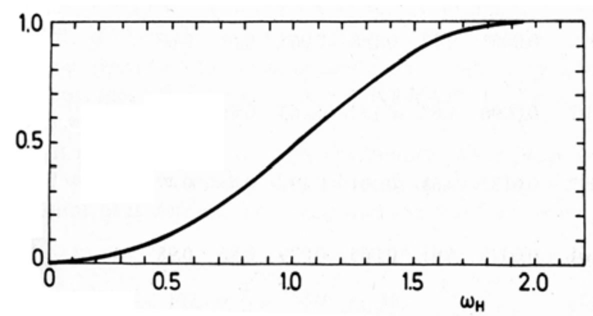
where the factor ϕ_k is defined by

$$\phi_k(\omega_H) = \frac{\left[(k(\omega,h))^{-3} \dfrac{\partial k(\omega,H)}{\partial f}\right]}{\left[(k(\omega,\infty))^{-3} \dfrac{\partial k(\omega,\infty)}{\partial f}\right]} \tag{22}$$

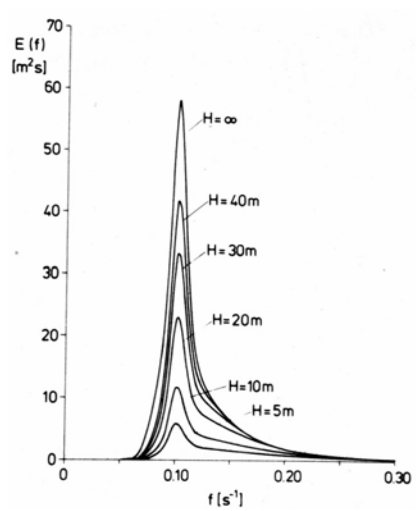
and $\omega_H = 2\pi f \sqrt{h/g}$. To solve for the partial derivatives, consult Kitaigorodskii et al. (1975).

Kitaigorodskii, S. A. et al. 1975. On Phillips theory of equilibrium range in the spectra of wind-generated gravity waves. *Journal of Physical Oceanography*, v. 5, p. 410-420.

Buows et al. (1985), however, provide the graphical solution –



Which Komar uses to make the point that water depth is important.



Third-generation wind-wave models

The second-generation models discussed above are all fetch-limited. Recent research has focused on developing numerical algorithms capable of describing wave growth in two-dimensions. A number of 'canned' codes are available that do this. The WAM (WAve Modeling group) model is one of the most popular of these models. It is described in:

Komen, G. J. et al. 1994. *Dynamics and modeling of ocean waves*. Cambridge University Press.

These models are extremely good at predicting both the temporal and spectral (in wavenumber space) characteristics of wave fields in response to a given wind forcing. However, they require substantial input and are probably too complex for most geological applications.