# Exercises QFT II — 2018/2019

*Problem Sheet 10*

## Problem 19 - Fermion-scalar interation.

Consider the following Lagrangian, describing the interaction between a Dirac fermion with mass $m_\psi$ and a real (pseudo-)scalar $\pi$ with mass $m_\pi$:

$$\mathcal{L} = \bar{\psi}(x)\,(i\gamma^\mu\partial_\mu - m_\psi)\,\psi(x) + \frac{1}{2}\left(\partial_\mu\pi(x)\partial^\mu\pi(x) - m_\pi^2\pi(x)^2\right) + g\,\pi(x)\,\bar{\psi}(x)\gamma^5\psi(x)\,, \tag{1}$$

where $g$ is the coupling constant.

1. Do a preliminary analysis of the renormalizability of this theory. (What is the degree of divergence? What are the possible divergent n-point function? Are there enough parameters to "absorb" the infinities?)
2. Write down the Feynman rules for this theory.
3. Write down the Feynman diagrams contributing to the three-point function (with two fermion and one scalar external lines) at order $g^3$.
4. Is the loop correction divergent?
5. By using dimensional regularization, compute the one-loop amplitude contributing to 1PI three-point function.

## Problem 20: Renormalization of electron field and mass.

We have found that the 1PI two-point function with electron external legs is at order $e^2$

$$\Gamma^{(2\psi)}(\not p) = \not p - m - \Sigma(\not p)\,, \tag{2}$$

where

$$\Sigma(\not p) \;=\; \frac{e^2}{16\pi^2}(4m - \not p)\frac{1}{2-\omega} - \frac{e^2}{8\pi^2}\left[m(2\gamma+1) - \frac{\not p}{2}(\gamma+1)\right] \tag{3}$$

$$-\frac{e^2}{8\pi^2}\int_0^1 dx\,(2m - \not p(1-x))\,\log\left(\frac{m^2x + m_\gamma^2(1-x) - p^2x(1-x)}{4\pi\mu^2}\right)\,, \tag{4}$$

where $\gamma$ is the Euler-Mascheroni constant ($\gamma = 0,5772...$) and $m_\gamma$ is the fictitious photon mass that regulates the IR divergences.

The renormalized two point function is given by

$$\Gamma_r^{(2\psi)}(\not p) = Z_\psi\,\Gamma^{(2\psi)}(\not p) = (1 + \delta Z_\psi)\,\Gamma^{(2\psi)}(\not p)\,. \tag{5}$$

In class, we first defined the generic renormalized couplings and then fixed the arbitrary 'finite' constants by a renormalization prescription. It is equivalent to do it in one step (and only at the end let $\omega \to 2$). In the present case, consider the (on-shell) renormalization prescriptions

$$\Gamma_r^{(2\psi)}(\not p = m_{\rm ph}) = 0 \qquad \text{and} \qquad \left.\frac{d\,\Gamma_r^{(2\psi)}(\not p)}{d\,\not p}\right|_{\not p = m_{\rm ph}} = 1\,. \tag{6}$$

1. Use (6) to compute $m_{\rm ph}$ in terms of the bare parameters $e, m$ and of $\omega$.
2. Write down the expression of $m$ in terms of $m_{\rm ph}$.
3. Use (6) to compute $\delta Z_\psi$. [Remember that $\delta Z_\psi \sim \mathcal{O}(e^2)$].
4. Write down the expression of $\Gamma_r^{(2\psi)}(\not p)$ in terms of $e$ and of the physical mass $m_{\rm ph}$. (Why can we substitute $e$ with the physical coupling $e_{\rm ph}$?)