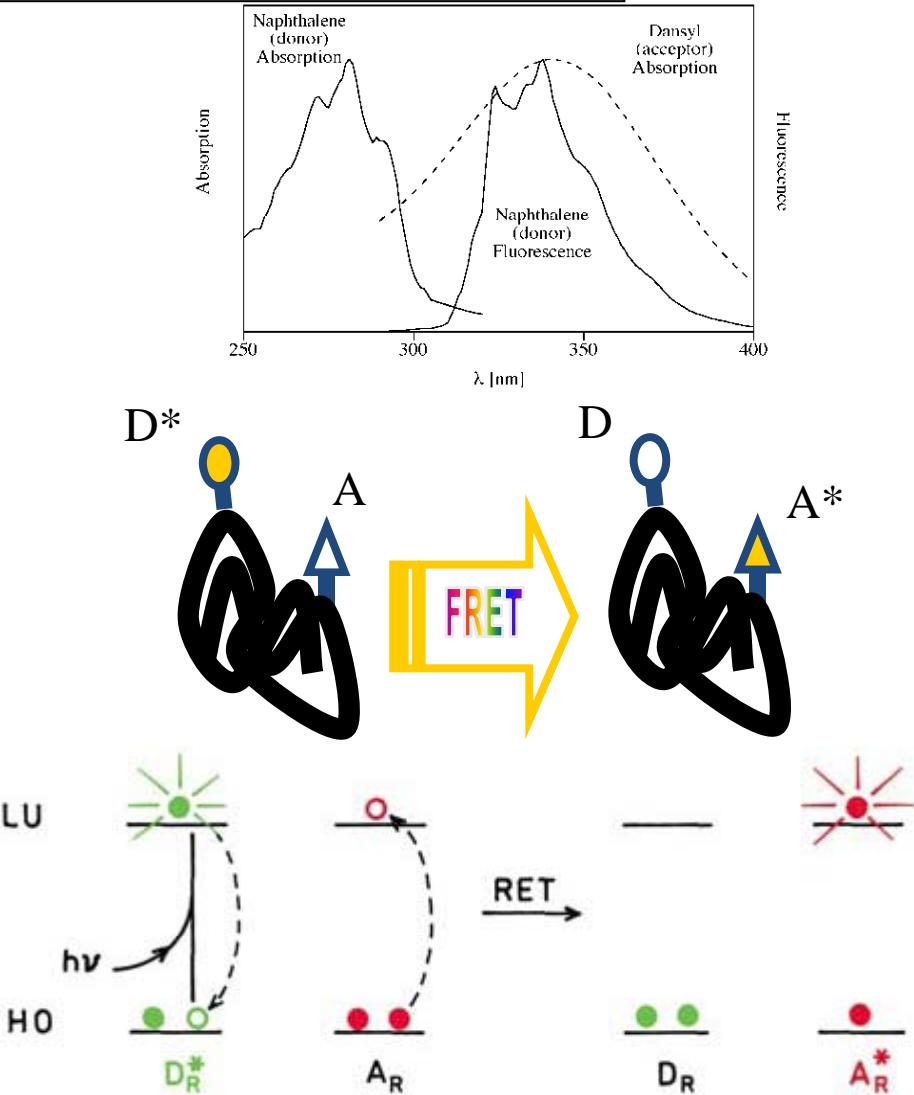


# Trasferimento di energia in risonanza alla Förster (FRET)

# FRET: introduzione

È una tecnica per misurare distanze inter- o intra-molecolari nell'intervallo 1-10 nm

- I due punti di interesse devono essere marcati con due cromofori diversi: il “donatore” (che deve essere fluorescente) e “l'accettore”.
- Lo spettro di emissione del donatore deve sovrapporsi (almeno parzialmente) allo spettro di assorbimento dell'accettore.
- In queste condizioni un donatore eccitato, anziché emettere fluorescenza, può trasferire la propria energia di eccitazione all'accettore, attraverso un fenomeno di risonanza.
- La probabilità di questo fenomeno dipende dalla distanza ed orientazione tra le due sonde, che quindi possono essere misurate.



# FRET: vantaggi

(rispetto ad altre tecniche strutturali, più potenti, come la cristallografia a raggi X o l'NMR)

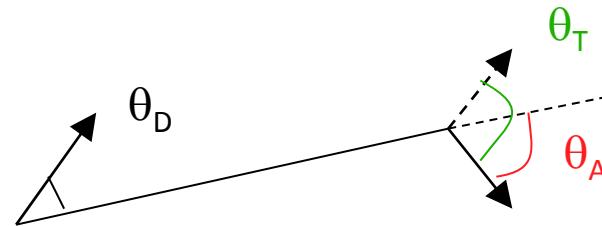
- Strumentazione semplice ed economica.
- Elevata sensibilità (è richiesto poco campione).
- Esperimenti in soluzione, in condizioni fisiologiche.
- Possibilità di effettuare misure in vivo.
- Ampio intervallo di distanze misurabili.
- Risoluzione temporale nell'ordine dei nanosecondi.

# FRET: teoria

- Teoria perturbativa di ordine 0
- Funzione d'onda imperturbate
- Potenziale d'interazione dipolo-dipolo

$$V = \frac{\overrightarrow{\mu_D} \cdot \overrightarrow{\mu_A}}{R^3} - 3 \frac{(\overrightarrow{\mu_D} \cdot \vec{R})(\overrightarrow{\mu_A} \cdot \vec{R})}{R^5} = \\ = k \frac{\mu_D \mu_A}{R^3}$$

$$k = \cos \theta_T - 3 \cos \theta_D \cos \theta_A$$



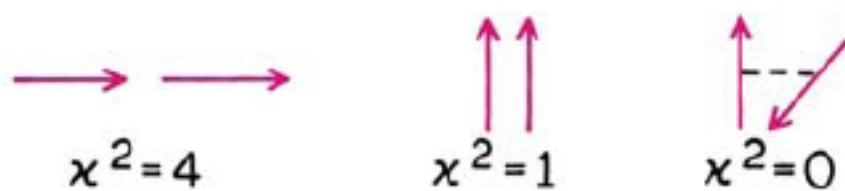
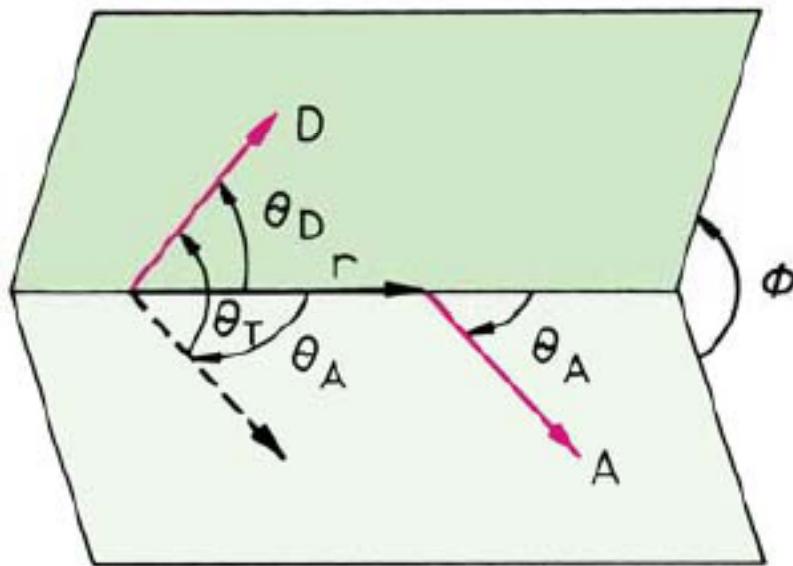
$$k_{FRET} \propto \left| \langle \psi_D \psi_{A*} | V | \psi_{D*} \psi_A \rangle \right|^2 = \frac{k^2}{R^6} \left| \langle \psi_D \psi_{A*} | \mu_D \mu_A | \psi_{D*} \psi_A \rangle \right|^2 = \\ = \frac{k^2}{R^6} \left| \langle \psi_D | \mu_D | \psi_{D*} \rangle \right|^2 \left| \langle \psi_{A*} | \mu_A | \psi_A \rangle \right|^2 \propto \frac{k^2}{R^6} \frac{k_r^D}{\nu^3} \frac{\epsilon^A}{\nu} = \frac{k^2}{R^3} \frac{\phi_0^D}{\tau_0^D \nu^3} \frac{\epsilon^A}{\nu}$$

# FRET: teoria

$$k_{FRET} = \frac{1}{\tau_D^0} \left( \frac{{R_0}^6}{R^6} \right) \left( \frac{3}{2} k^2 \right)$$

$${R_0}^6 = \frac{2}{3} \alpha n^{-4} \Phi_D^0 J$$

$$J = \frac{\int f_D(\nu) \varepsilon_A(\nu) \nu^{-4} d\nu}{\int\limits_0^\infty f_D(\nu) d\nu}$$



$$\chi^2 = (\cos \theta_T - 3 \cos \theta_D \cos \theta_A)^2$$

$$\chi^2 = (\sin \theta_D \sin \theta_A \cos \phi - 2 \cos \theta_D \cos \theta_A)^2$$

**Figure 13.5.** Dependence of the orientation factor  $\kappa^2$  on the direction of the emission dipole of the donor and the absorption dipole of the acceptor.

Per rotazione libera di donatore ed accettore durante il tempo di vita del donatore:

$$\langle k^2 \rangle = \frac{2}{3} \text{ media dinamica}$$

Per fluorofori orientati casualmente, ma immobili durante il tempo di vita (matrice solida o viscosa):

$$\langle k^2 \rangle = 0.476 \text{ media statica}$$

$$\sqrt[6]{2/3} = 0.93 \quad \sqrt[6]{0.476} = 0.87$$

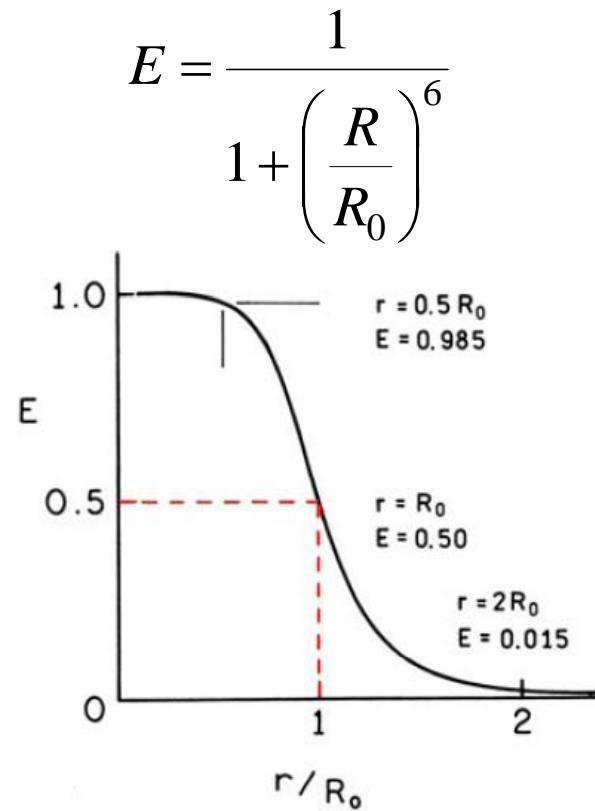
# FRET: efficienza

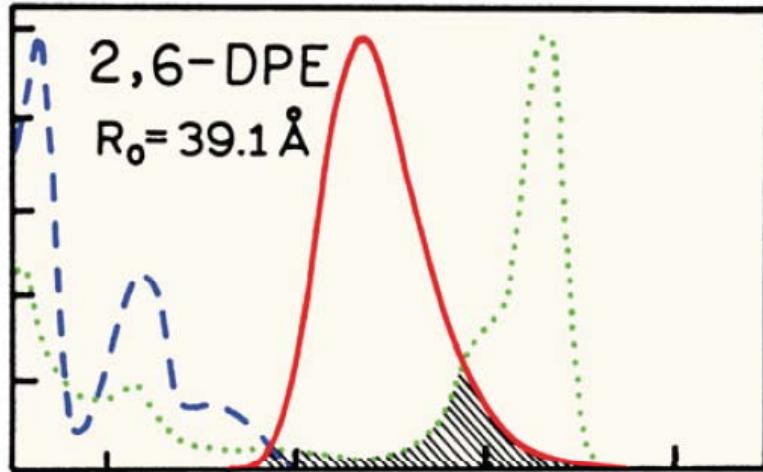
$E$  = probabilità che un donatore eccitato trasferisca la sua energia.

$$E = \frac{k_{FRET}}{k_r^D + k_{nr}^D + k_{FRET}}$$

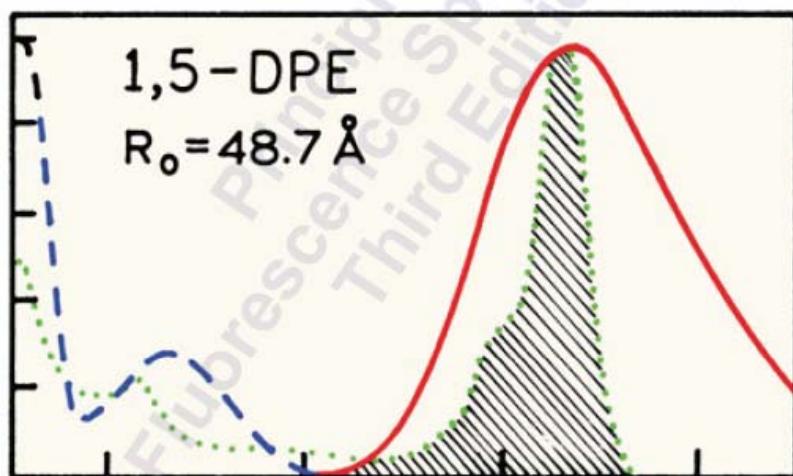
$$E = \frac{1}{1 + \frac{k_r^D + k_{nr}^D}{k_{FRET}}} = \frac{1}{1 + \frac{1}{\tau_0^D k_{FRET}}} = \frac{1}{1 + \left(\frac{R}{R_0}\right)^6 \left(\frac{3}{2} k^2\right)^{-1}}$$

- In generale, l'efficienza dipende dall'orientazione.
  - Se i cromofori ruotano liberamente sulla scala dei tempi della fluorescenza, allora  $k^2=2/3$ .
- L'efficienza dipende dalla sesta potenza della distanza
  - Elevata sensibilità
  - Si misurano bene solo distanze confrontabili con  $R_0$  (da  $R_0/2$  a  $2R_0$ ).
- Si può avere un  $R_0$  ottimale scegliendo opportunamente i cromofori (ossia la loro sovrapposizione spettrale).

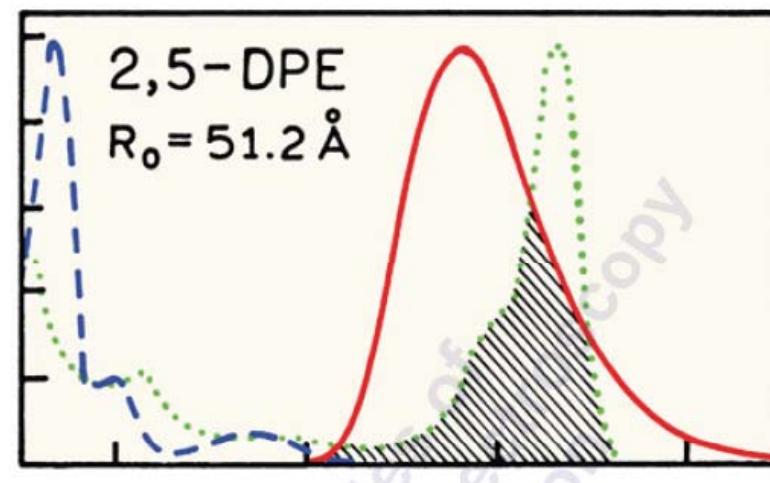




*Dansyl-labeled  
phosphatidylethanolamine*

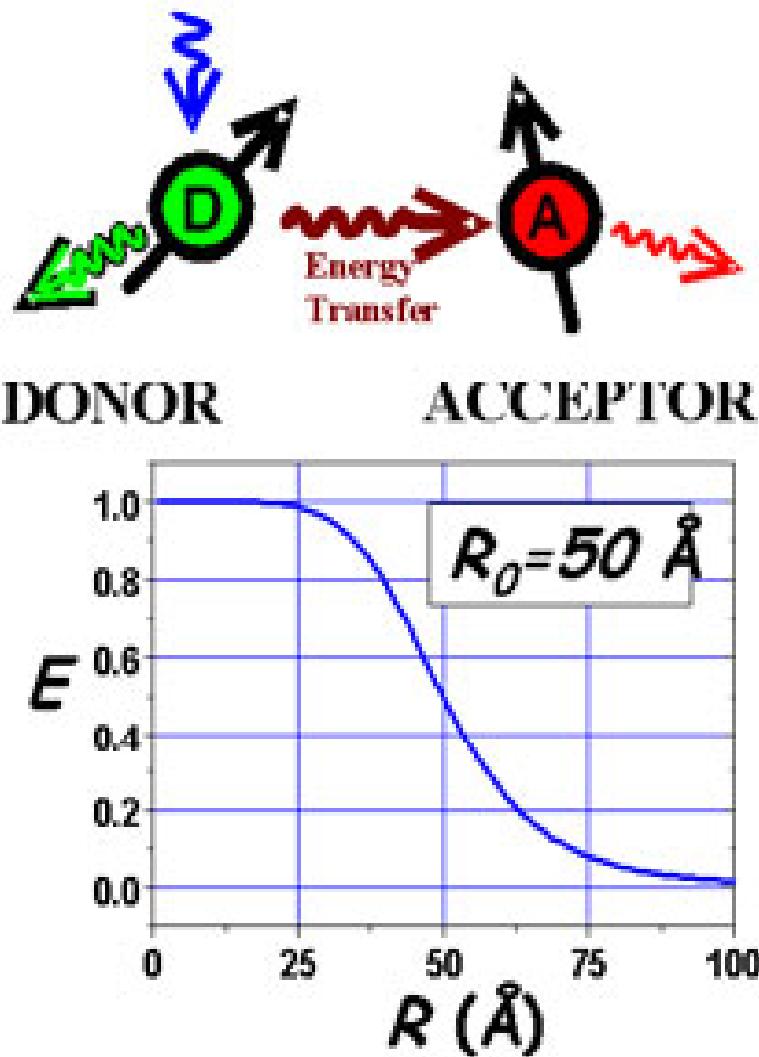


Red = donor emission  
Dotted green = acceptor absorption



<i>Donor</i>	<i>Acceptor</i>	$R_0(\text{\AA})$
Naphthalene	Dansyl	22
Pyrene	Coumarin	39
Terbium	Rhodamine	65
Tryptophan	Dansyl	21-24
Tryptophan	Heme	29
Tryptophan	Pyrene	28
Bodipy	Bodipy	57
Tryptophan	DPH	40
Tryptophan	Tyrosine-NO <sub>2</sub>	26

# A spectroscopic ruler



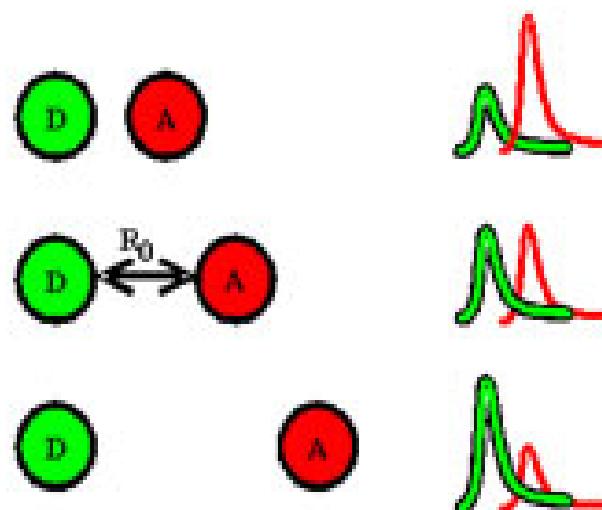
Energy Transfer Efficiency

$$E = \frac{1}{1 + (R/R_0)^6}$$

$R_0$ =50% transfer efficiency distance  
3nm~7nm

$\tau$

"Spectroscopic Ruler"



# FRET: misura dell'efficienza

$$E = \frac{k_{FRET}}{k_r^D + k_{nr}^D + k_{FRET}}$$

L'efficienza si può misurare dalla diminuzione della fluorescenza del donatore

$$\Phi_0^D = \frac{k_r^D}{k_r^D + k_{nr}^D}$$

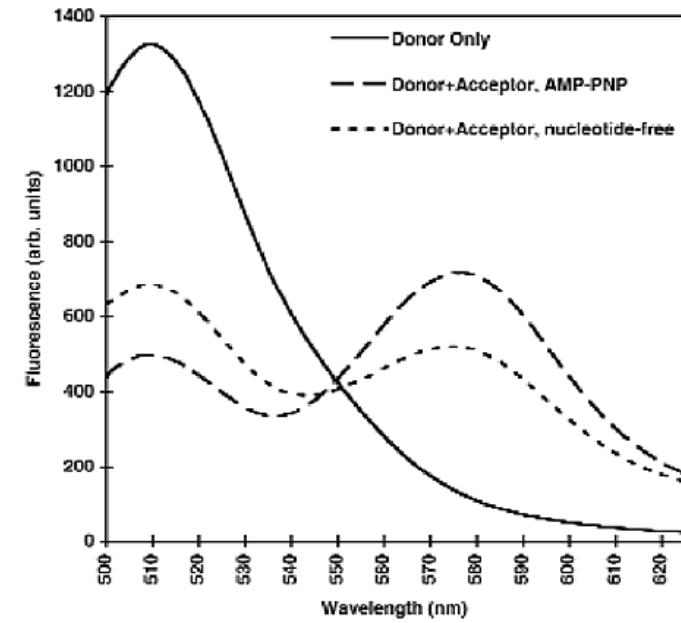
$$\Phi_A^D = \frac{k_r^D}{k_r^D + k_{nr}^D + k_{FRET}}$$

$$\frac{1}{\Phi_A^D} = \frac{k_r^D + k_{nr}^D + k_{FRET}}{k_r^D}$$

$$\frac{1}{\Phi_A^D} - \frac{1}{\Phi_0^D} = \frac{k_r^D + k_{nr}^D + k_{FRET}}{k_r^D} - \frac{k_r^D + k_{nr}^D}{k_r^D} = \frac{k_{FRET}}{k_r^D}$$

$$E = \frac{\frac{1}{\Phi_A^D} - \frac{1}{\Phi_0^D}}{\frac{1}{\Phi_A^D}} = 1 - \frac{\Phi_A^D}{\Phi_0^D}$$

$$E = 1 - \frac{F_A^D}{F_0^D}$$



Oppure dalla diminuzione del tempo di vita del donatore

$$E_{ET} = 1 - \frac{\tau}{\tau_D^0}$$

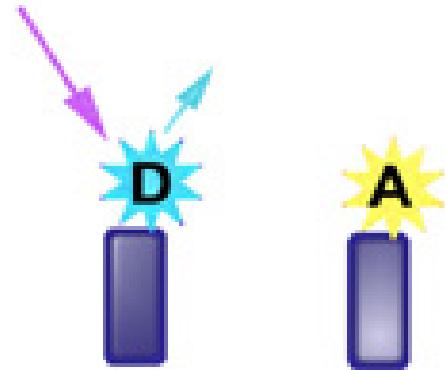
$$\tau_D = \frac{1}{k_r + k_{nr} + k_{ET}}$$

$$\tau_0^D = \frac{1}{k_r + k_{nr}}$$

$$1 - \frac{\tau}{\tau_D^0} = 1 - \frac{k_r + k_{nr}}{k_r + k_{nr} + k_{ET}} = \frac{k_{ET}}{k_r + k_{nr} + k_{ET}} = E_{ET}$$

Oppure dall'aumento della fluorescenza dell'accettore  
(se è fluorescente!)

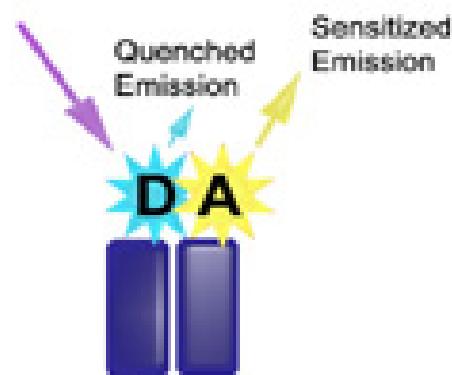
$$F_A(\lambda_A^{em}) \propto \varepsilon_A(\lambda_{ex})\phi_A$$



$$F_{AD}(\lambda_A^{em}) \propto \varepsilon_A(\lambda_{ex})\phi_A + \varepsilon_D(\lambda_{ex})\phi_A E$$

No Energy Transfer

$$\begin{aligned} \frac{F_{AD}(\lambda_A^{em})}{F_A(\lambda_A^{em})} &= \frac{\varepsilon_A(\lambda_{ex})\phi_A + \varepsilon_D(\lambda_{ex})\phi_A E}{\varepsilon_A(\lambda_{ex})\phi_A} = \\ &= 1 + \frac{\varepsilon_D(\lambda_{ex})E}{\varepsilon_A(\lambda_{ex})} \end{aligned}$$

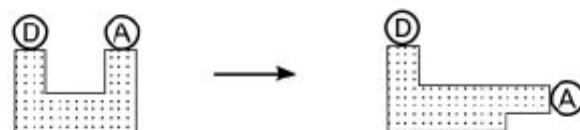


FRET - Energy Transfer

$$E = \frac{\varepsilon_A(\lambda_{ex})}{\varepsilon_D(\lambda_{ex})} \left[ \frac{F_{AD}(\lambda_A^{em})}{F_A(\lambda_A^{em})} - 1 \right]$$

# FRET: applicazioni

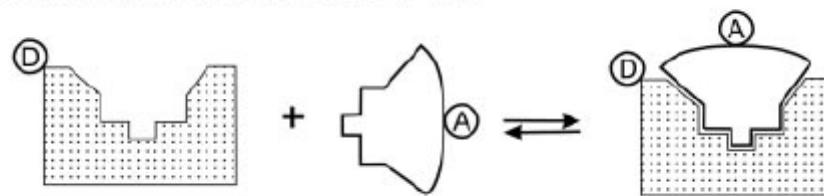
- CONFORMATIONAL CHANGES



- HYDROLYSIS OF A SUBSTRATE



- INTERACTION LIGAND-RECEPTOR



- FUSION OF LIPID VESICLES

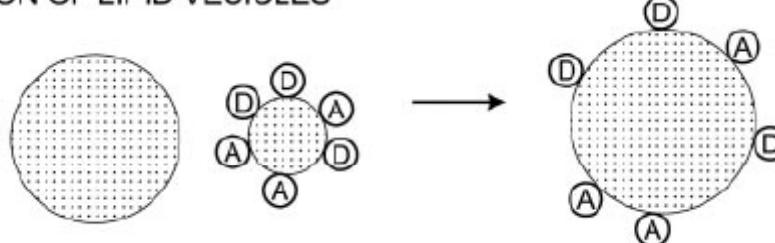
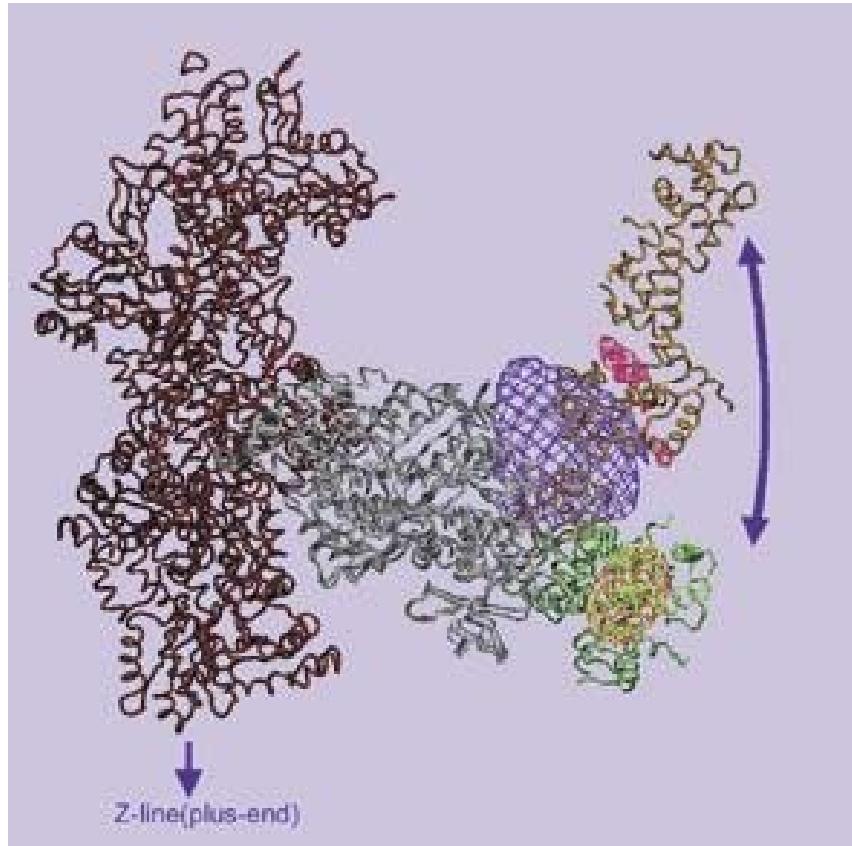
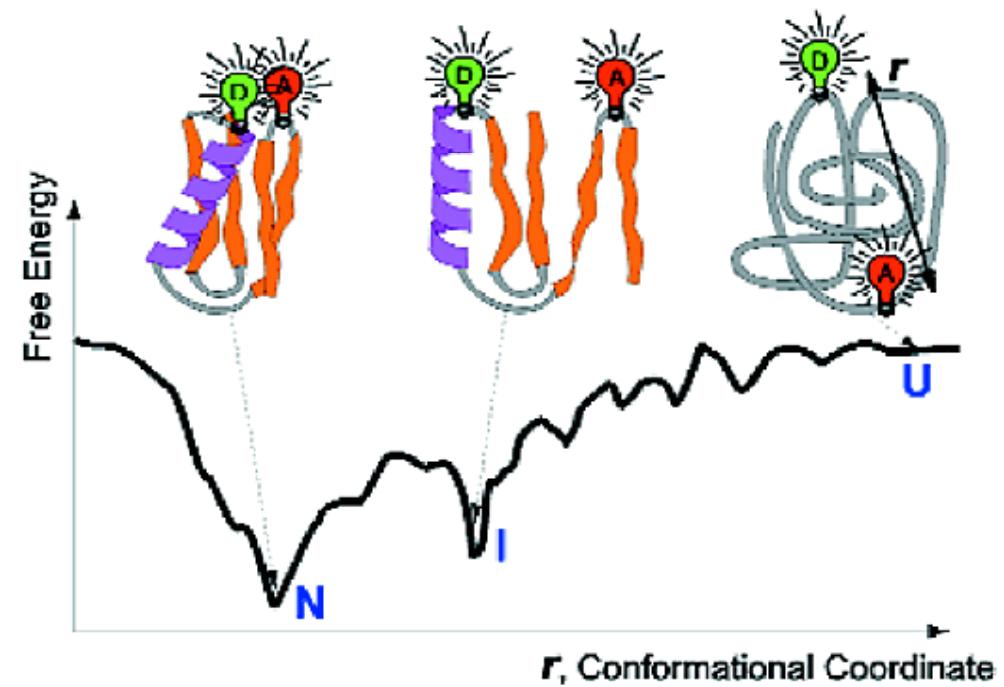


Fig. 9.5. Some applications of RET.

# FRET: applicazioni

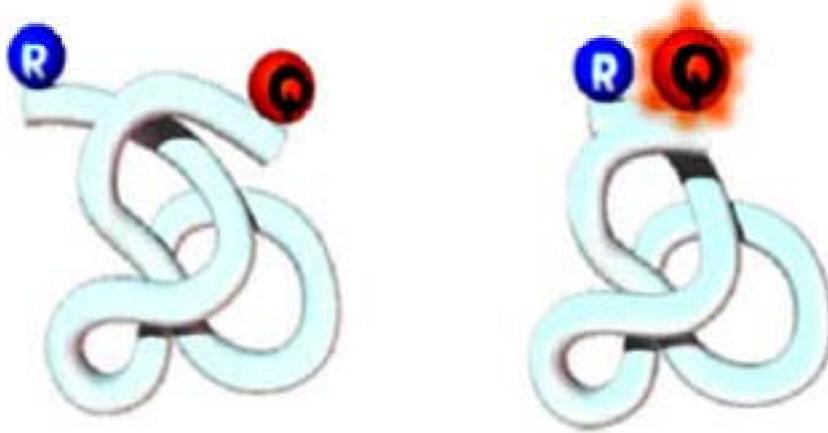


Struttura di macromolecole



# Time-resolved energy transfer

Measuring conformational dynamics:



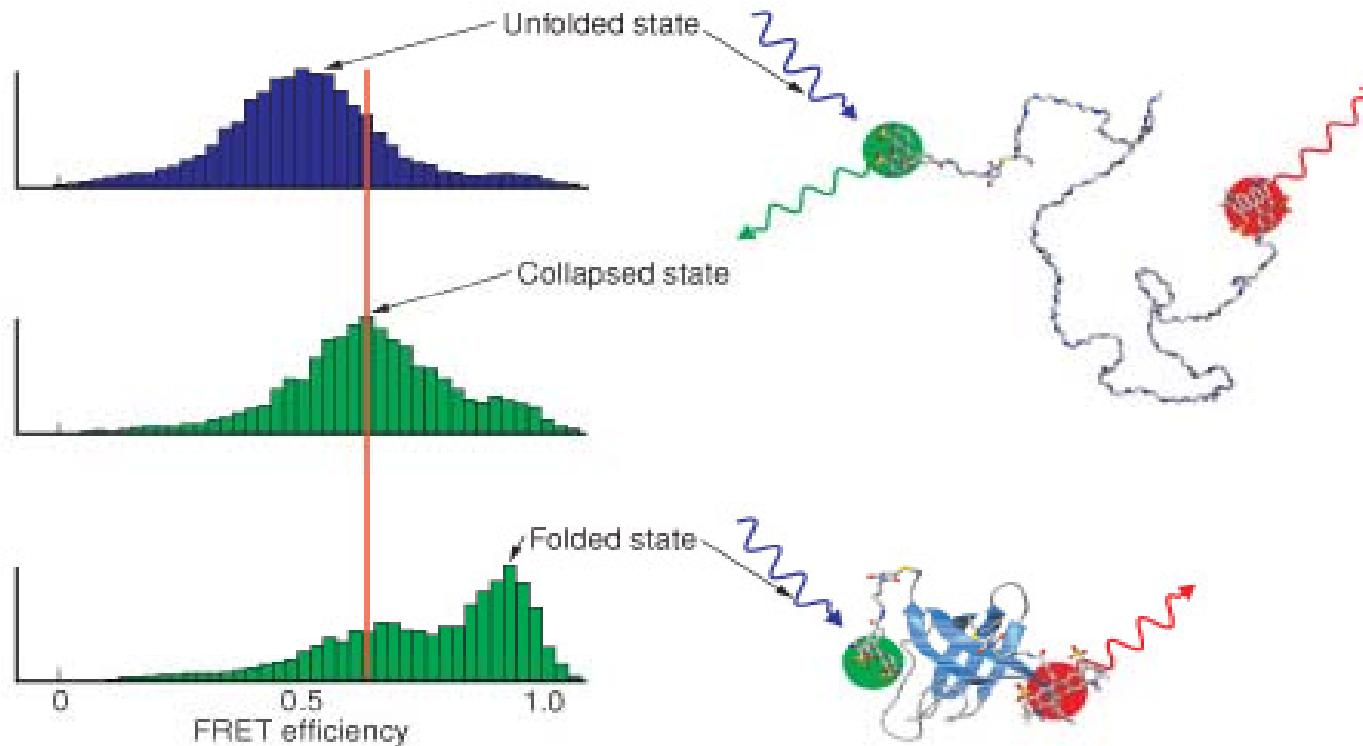
$$k_{ET} = \frac{1}{\tau_D^0} \left( \frac{R_0}{R} \right)^6$$

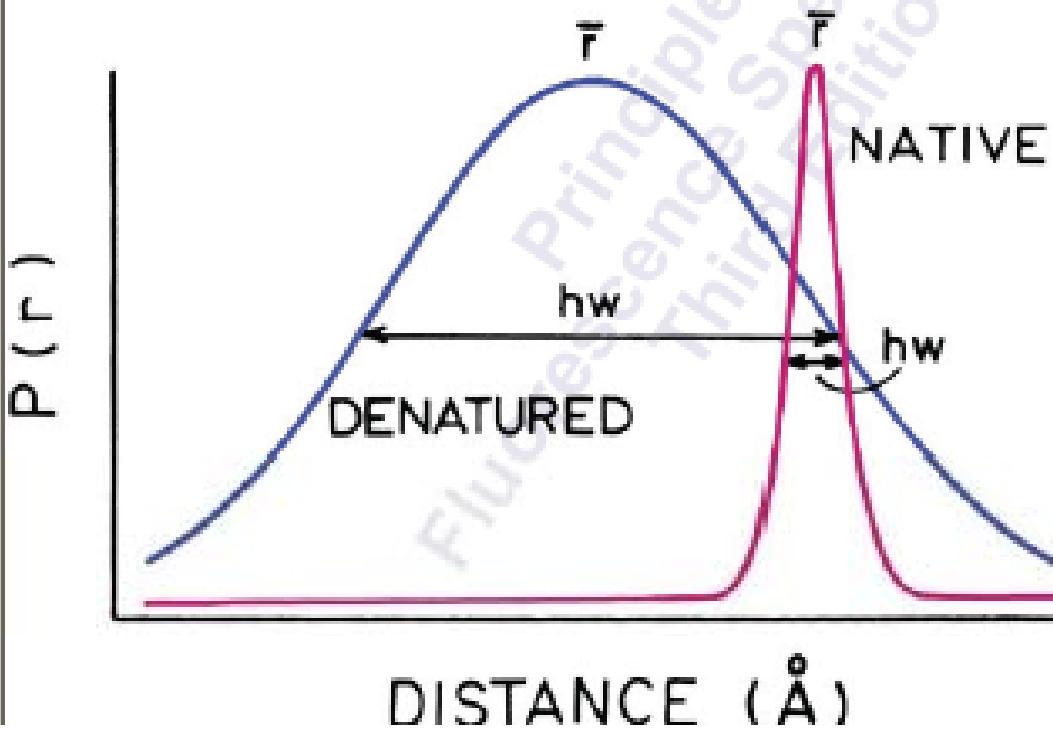
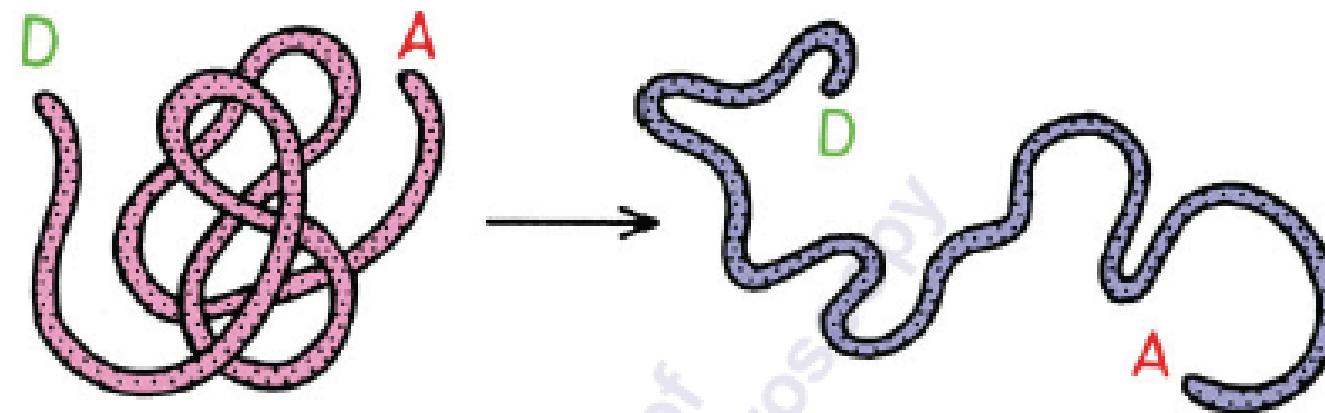
$$I_{DA}(t) = I_{DA}^0 e^{-(k_{ET} + \frac{1}{\tau_D})t}$$

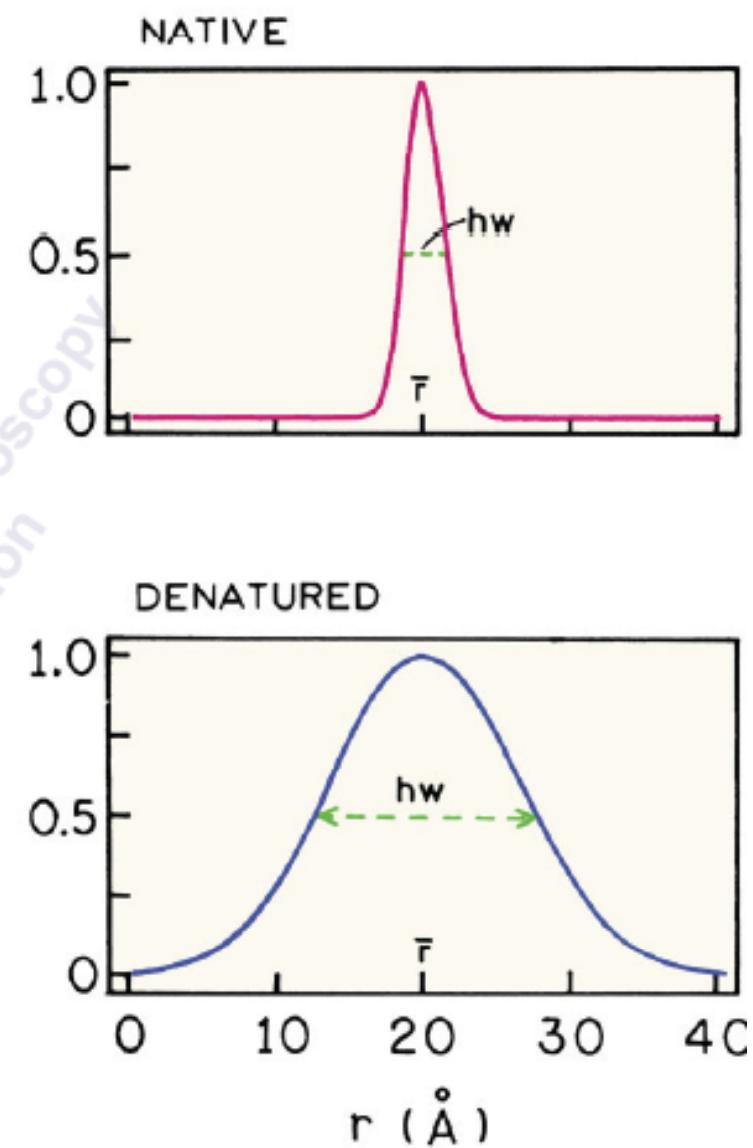
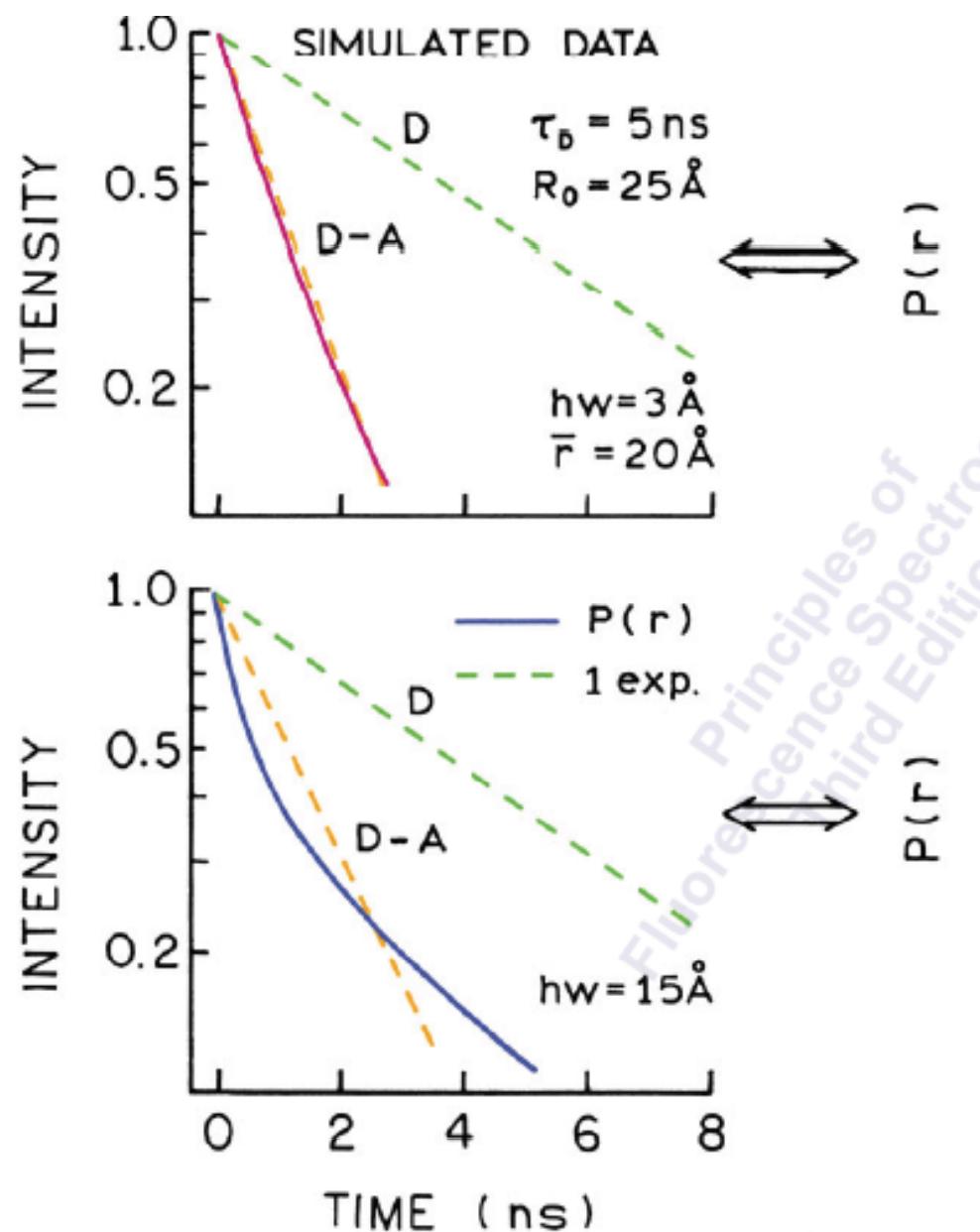
For a gaussian distribution of distances:

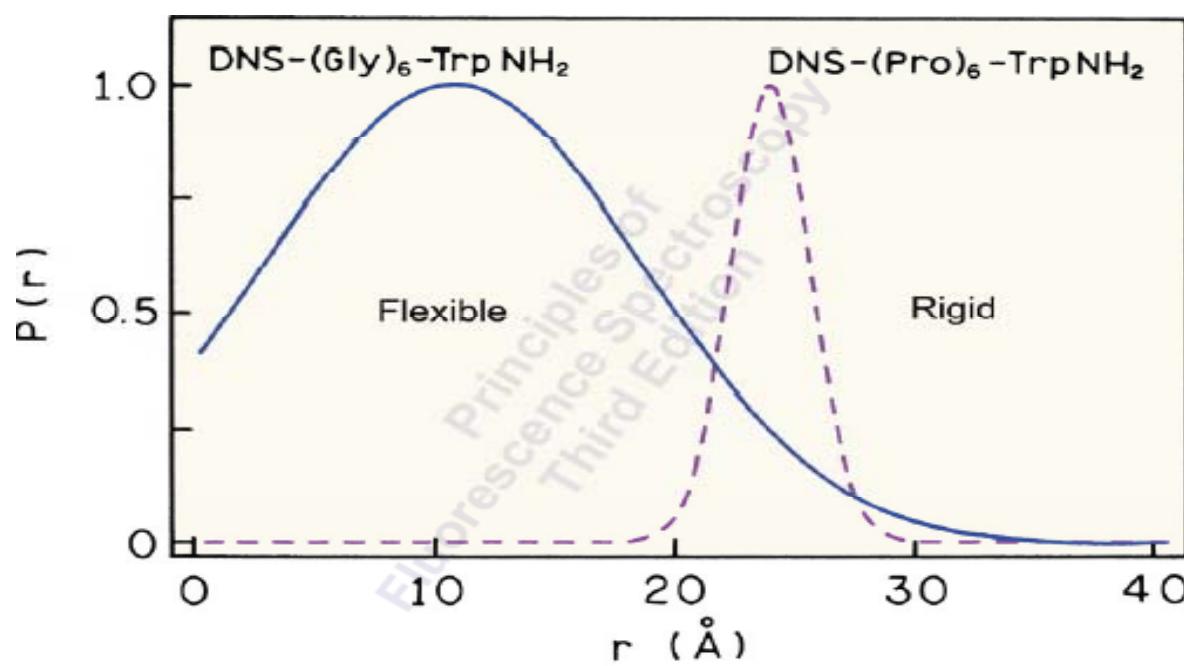
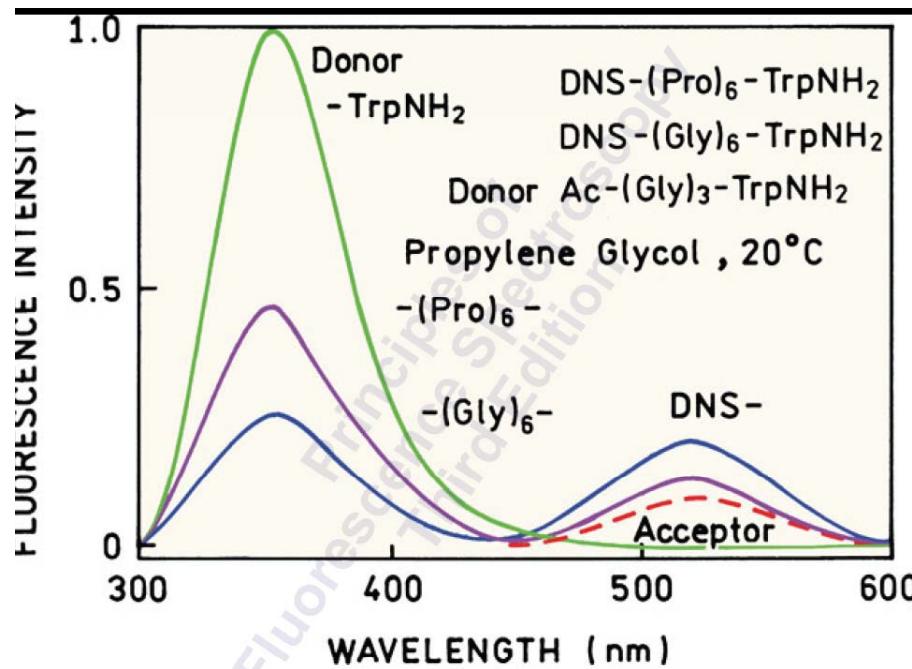
$$P( r ) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{R-\bar{R}}{\sigma}\right)^2}$$

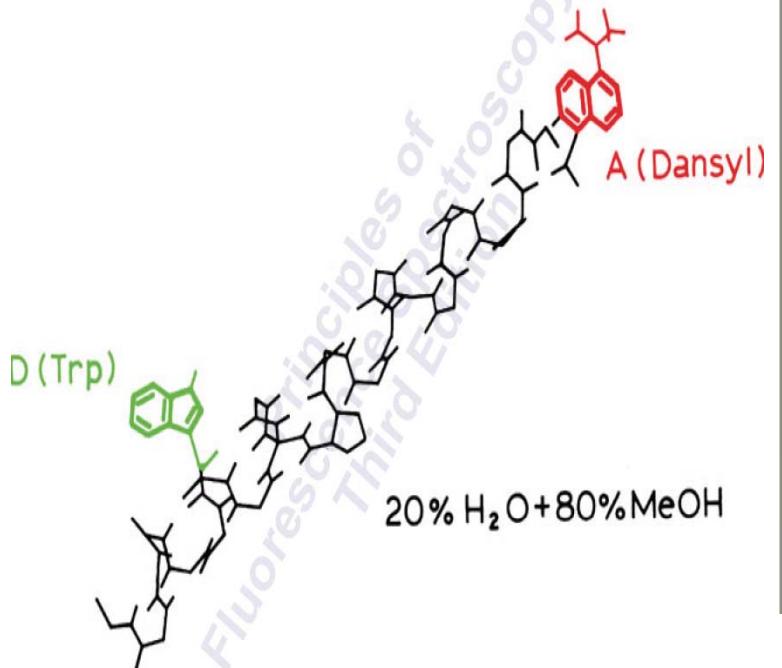
$$I_{DA}(t) = I_{DA}^0 \int_0^{\infty} P(R) e^{-(k_{ET} + \frac{1}{\tau_D})t} dR$$



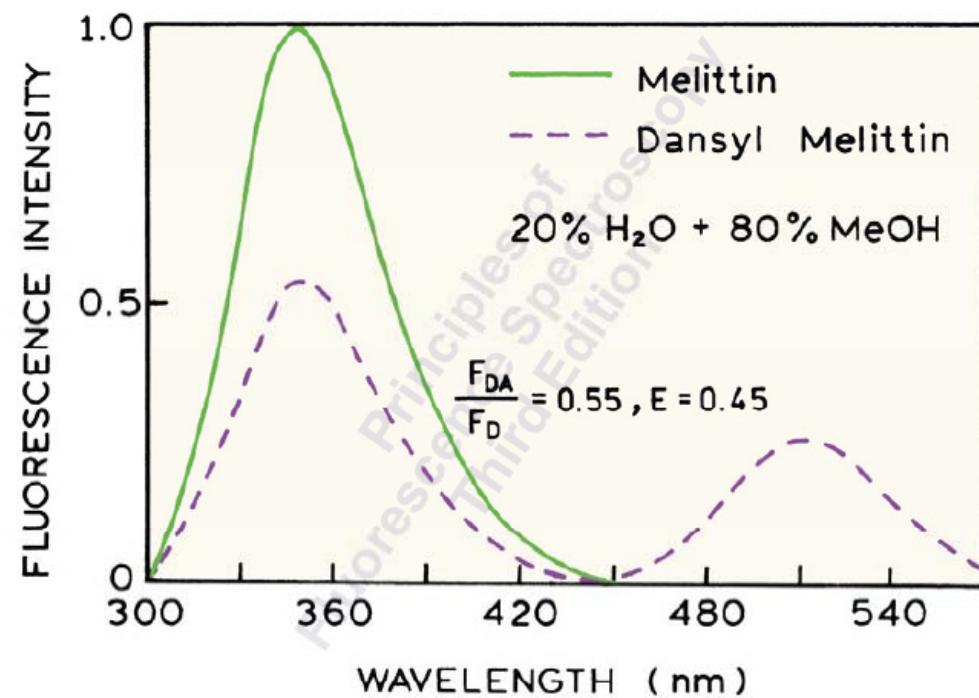
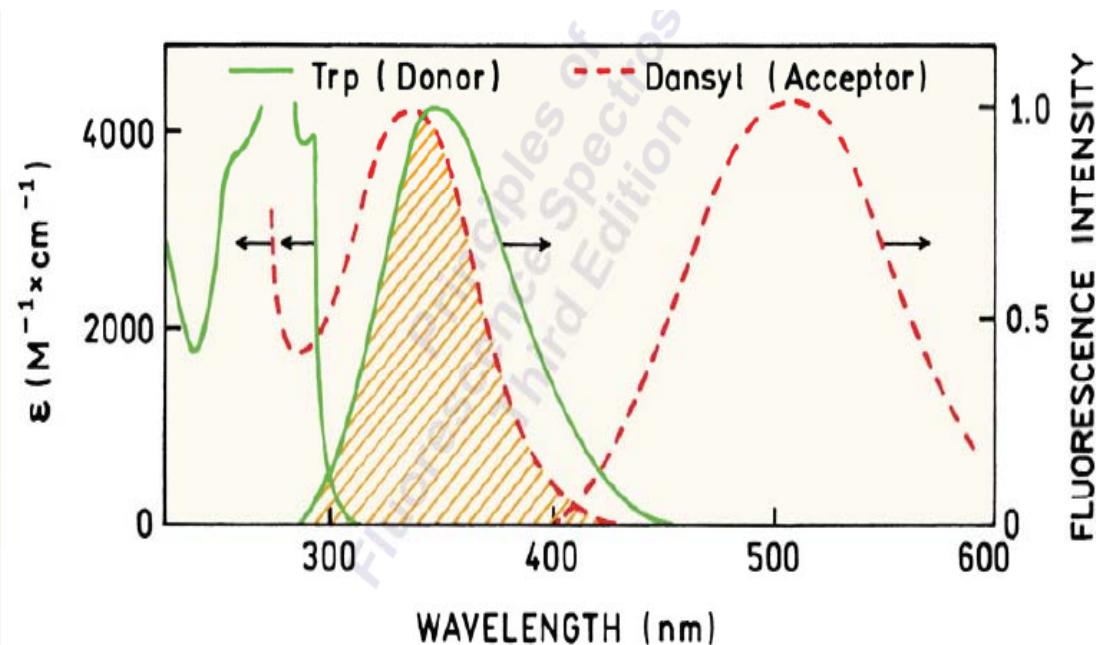




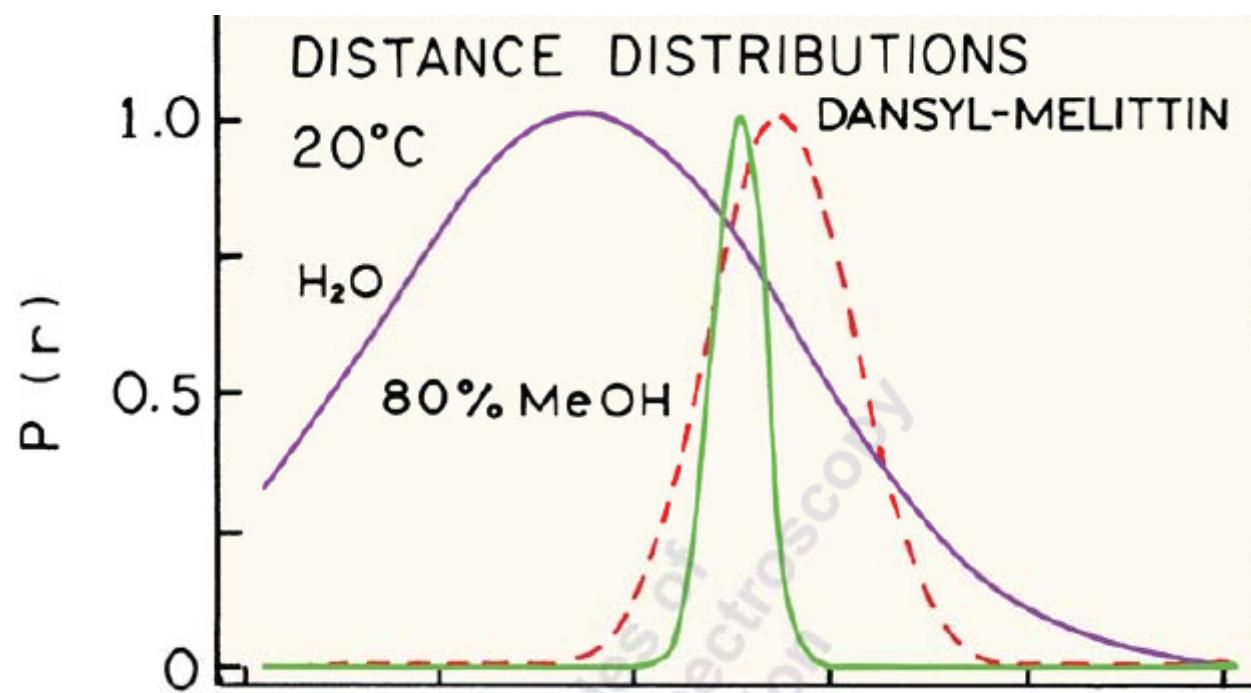
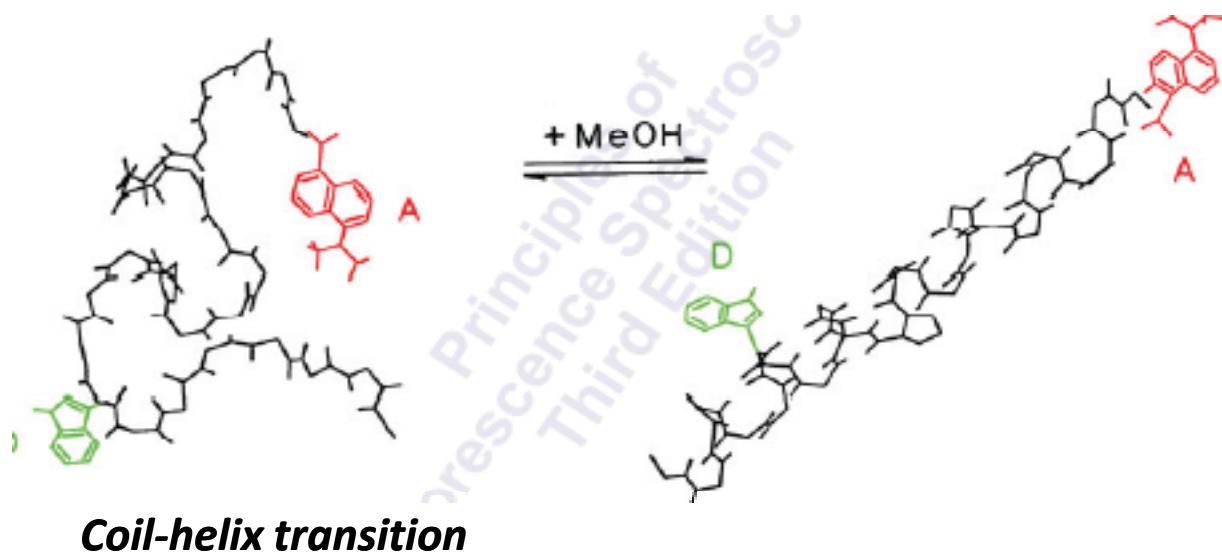




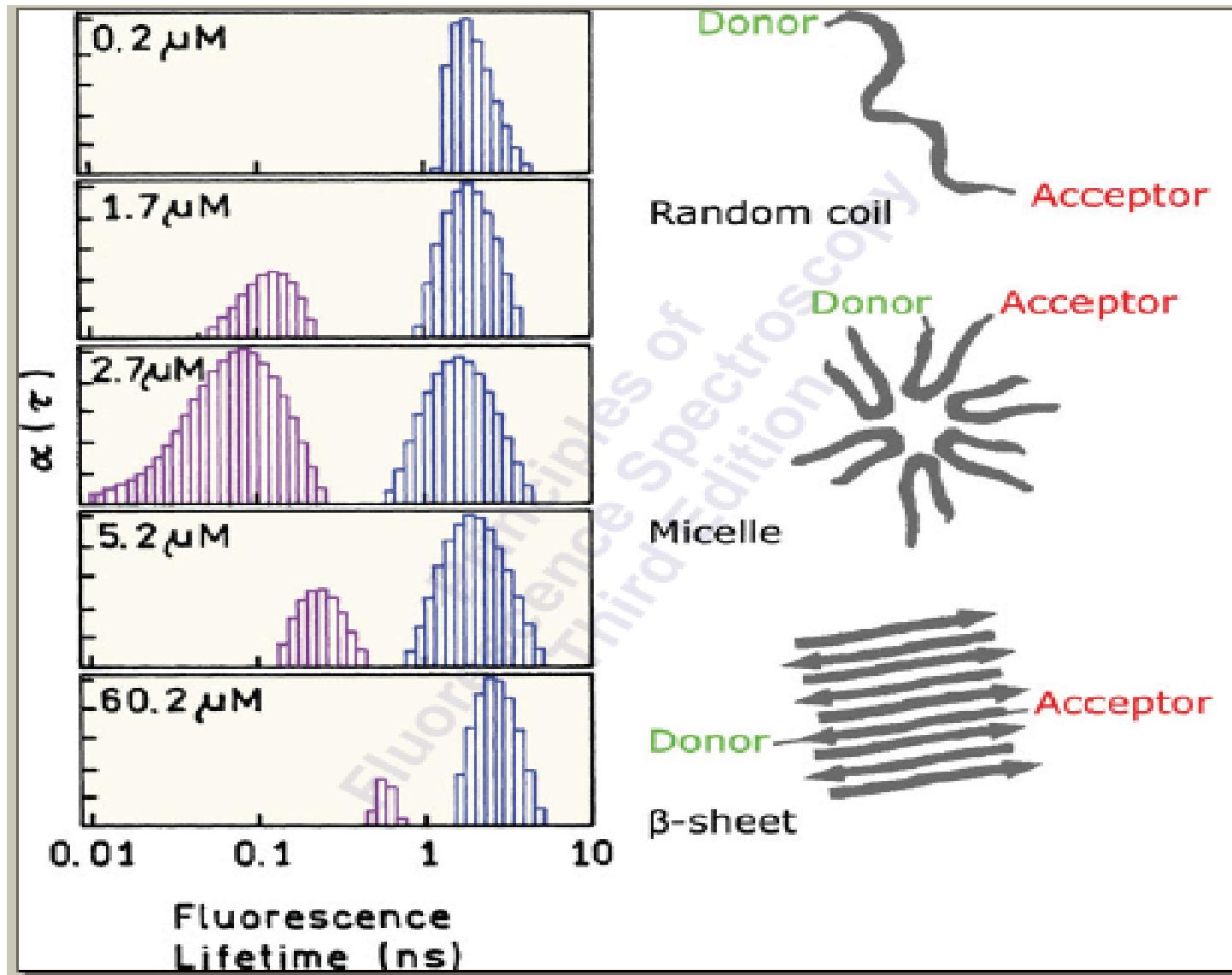
## Dansyl-labeled melittin



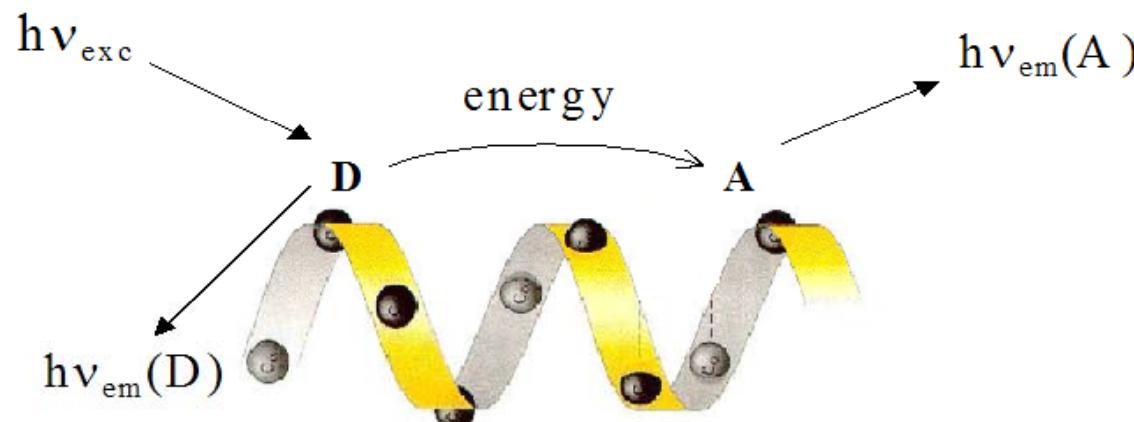
# Melittin



# Aggregation of Amyloid peptides



# *Peptides Foldamers for Energy Transfer*



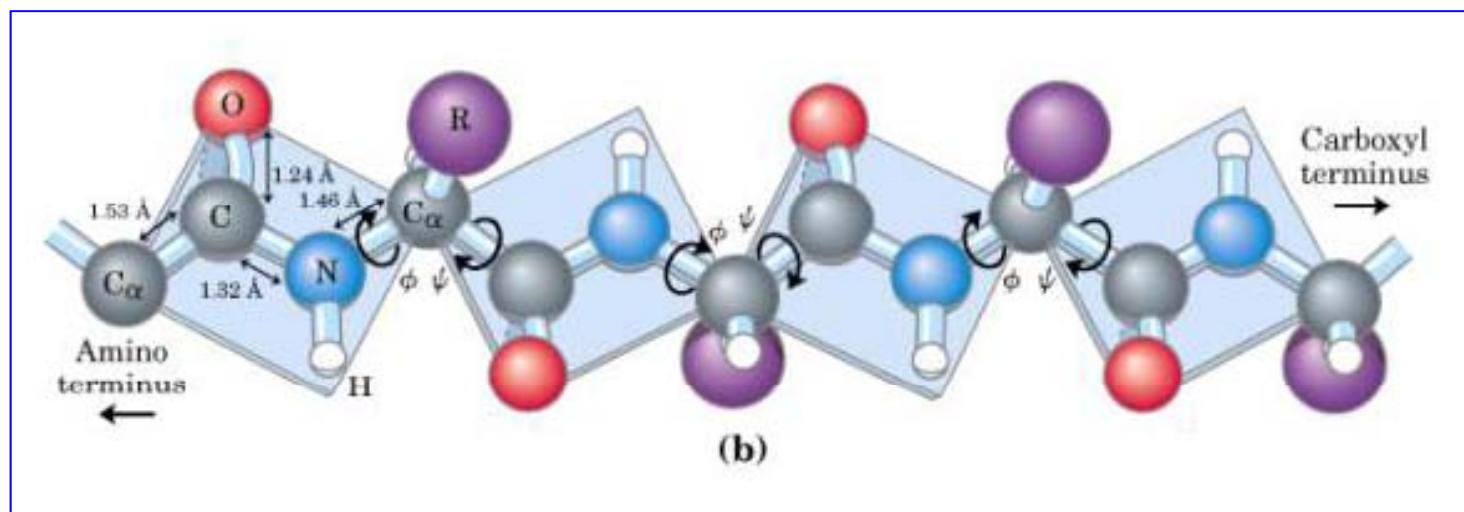
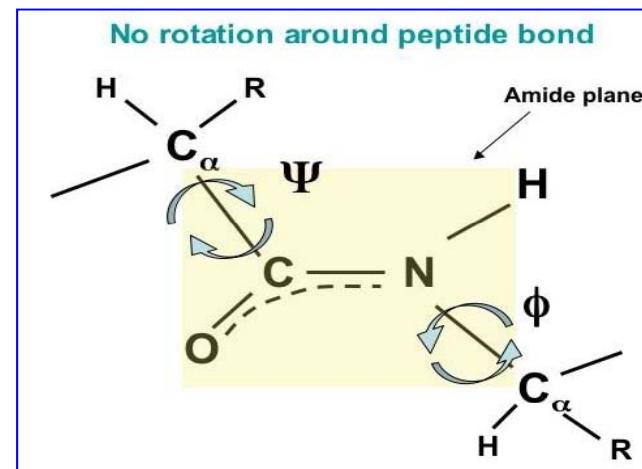
*Förster Energy Transfer  
dipole-dipole interaction*

$$E = \frac{1}{1 + \frac{2}{3k^2} \left( \frac{R}{R_0} \right)^6}$$

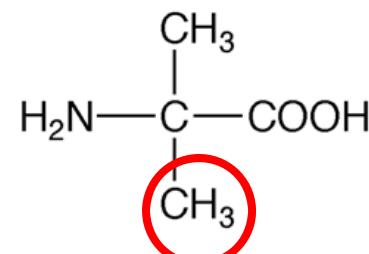
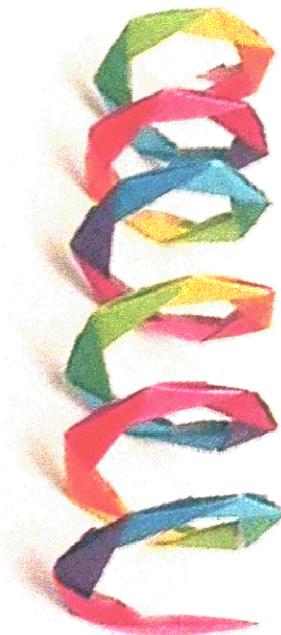
# Peptide foldamers

*'Foldamers are molecules that have well-defined and predictable folding properties in solution.'*

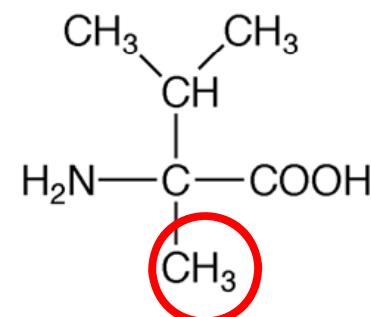
*S. H. Gellman,  
Accounts of Chemical Research 1998, 31, 173.*



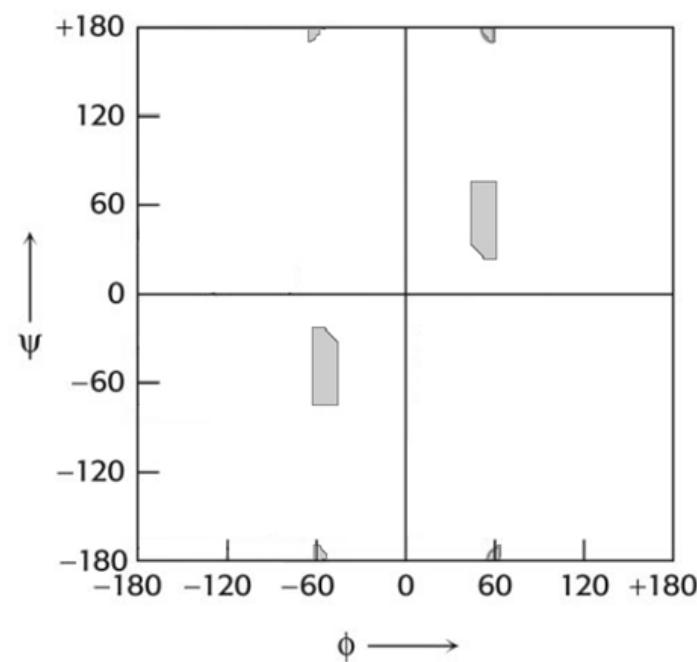
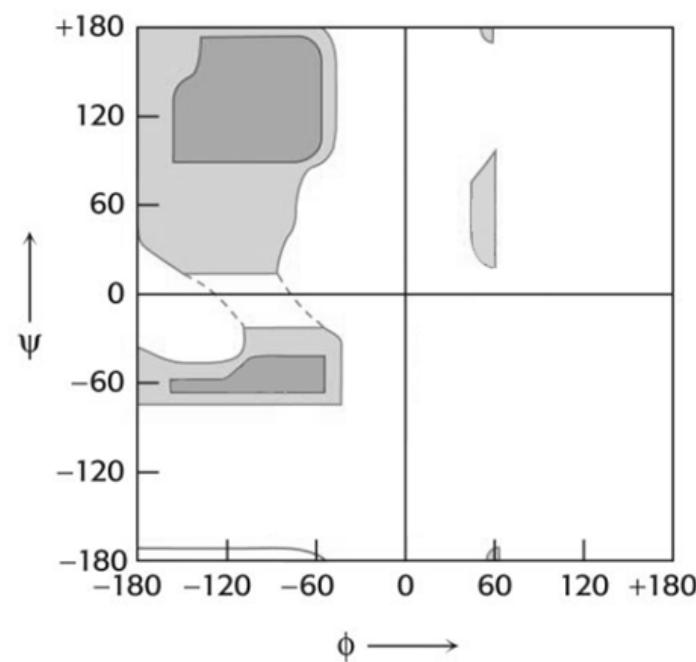
# $C^{\alpha,\alpha}$ disubstituted aminoacids



**Aib**



**( $\alpha$ Me)Val**



P-Aib-Aib-Aib-N

P-Aib-Aib-Aib-Aib-Aib-Aib-N

P-Aib-Aib-Aib-Aib-Aib-Aib-Aib-Aib-Aib - N

P-Aib-Aib-Aib-Aib-Aib-Aib-Aib-Aib-Aib-Aib-Aib-N

P-Aib-Aib-Aib-Aib-Aib-Aib-Aib-Aib-Aib-Aib-Aib-Aib-Aib-N

Fmoc-Toac-( $\alpha$ Me)Val- ( $\alpha$ Me)Val- NHtBu

Fmoc-( $\alpha$ Me)Val-Toac-( $\alpha$ Me)Val- ( $\alpha$ Me)Val- NHtBu

Fmoc-( $\alpha$ Me)Val-( $\alpha$ Me)Val-Toac-( $\alpha$ Me)Val- ( $\alpha$ Me)Val- NHtBu

Fmoc-( $\alpha$ Me)Val-( $\alpha$ Me)Val- ( $\alpha$ Me)Val-Toac-( $\alpha$ Me)Val- ( $\alpha$ Me)Val- NHtBu

Ac-Toac-Trp-Aib-Aib-Aib-Aib-OtBu

Ac-Toac-Aib-Trp-Aib-Aib-Aib-OtBu

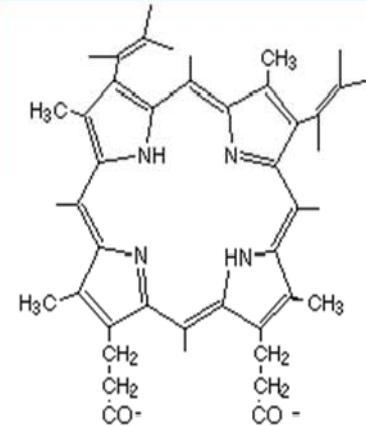
Ac-Toac-Aib-Aib-Trp-Aib-Aib-OtBu

Ac-Toac-Aib-Aib-Aib-Trp-Aib-OtBu

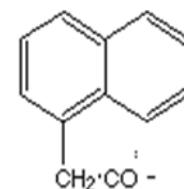
Boc-Aal- Aib-Toac-Ala-Aib-Ala-OtBu

Boc-Aal- Aib-Ala-Toac-Aib-Ala-OtBu

Ciclo{Orn-[Aib-Aib-Trp-Aib-Aib-Z]-Asp-[Aib-Aib-Toac-Aib-Aib-OtBu]}



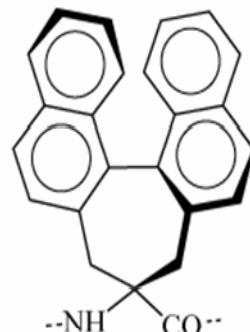
P



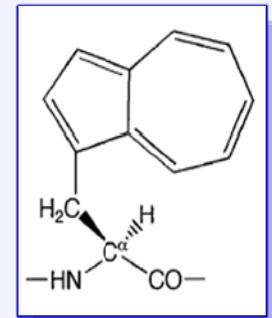
N



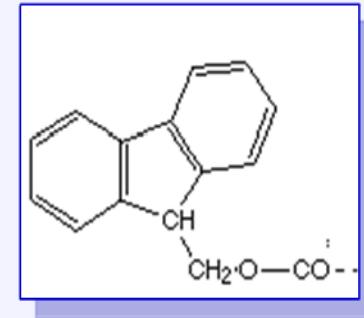
(S)-Bin



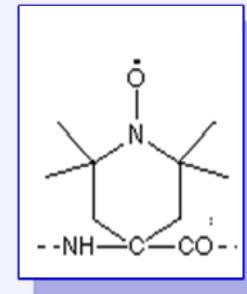
(R)-Bin



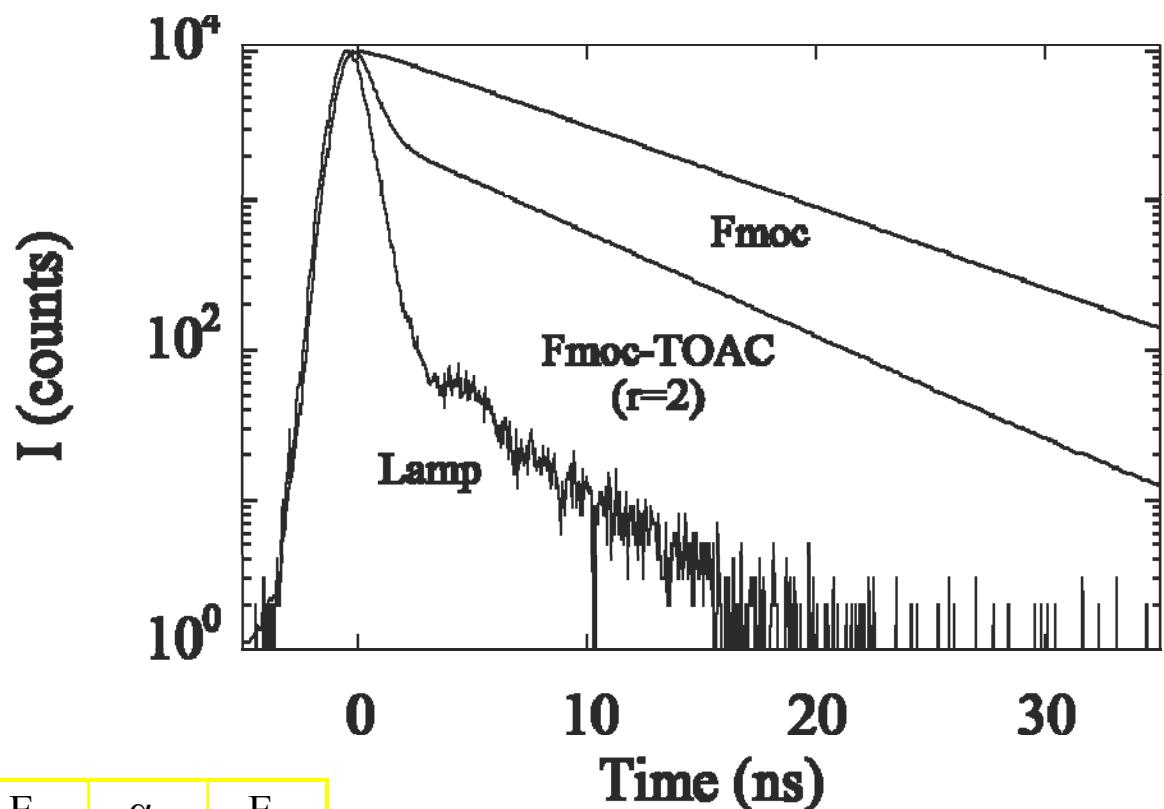
L-Aal



Fmoc

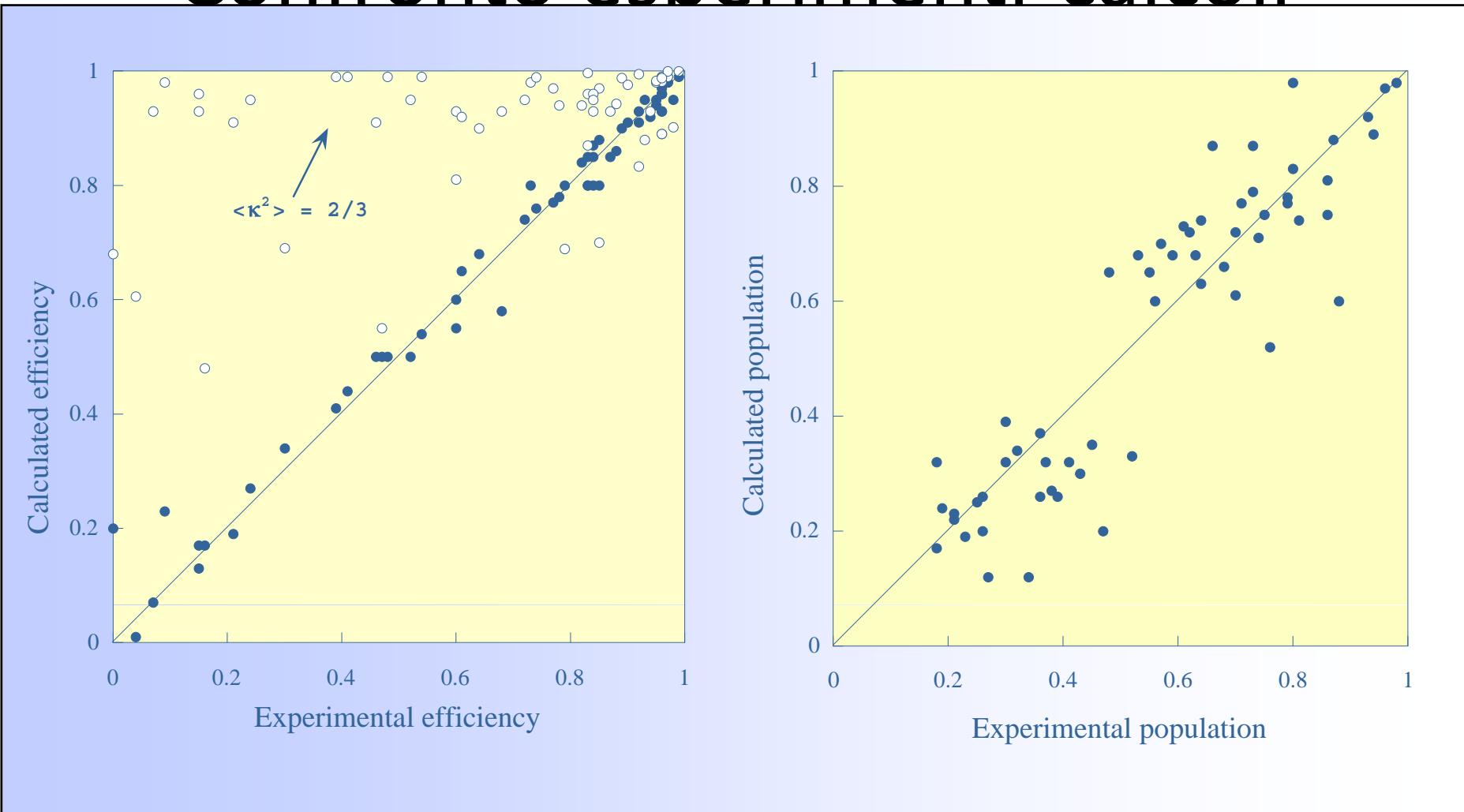


Toac

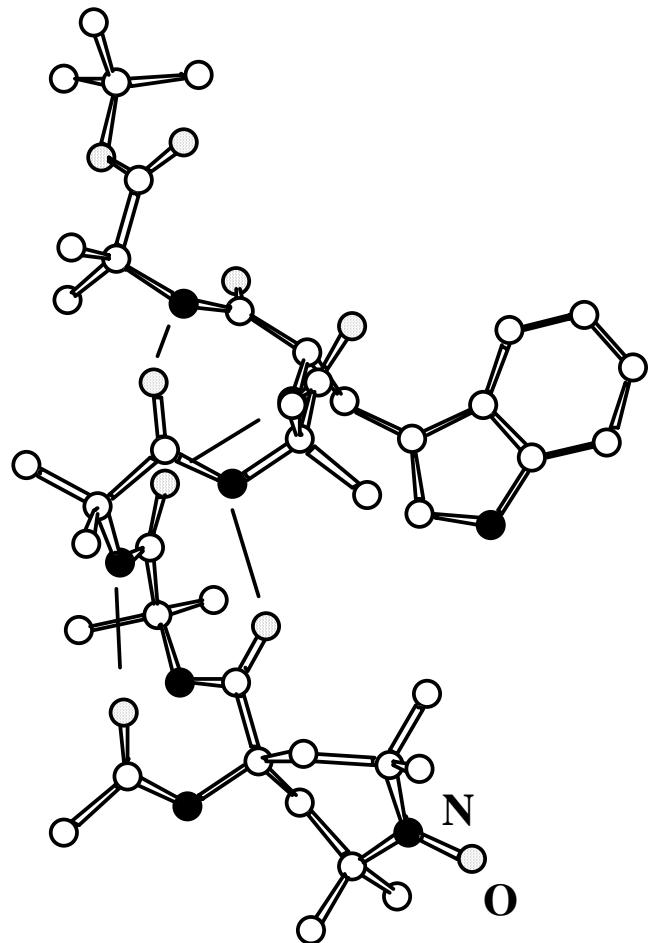


Peptide	r	$\alpha_1$	$E_1$	$\alpha_2$	$E_2$	$\alpha_3$	$E_3$
Trp-TOAC	0	0.86	0.96	0.11	0.86	0.04	0.14
	1	0.86	0.93	0.12	0.83	0.02	0.10
	2	0.56	0.95	0.30	0.68	0.14	0.06
	3	0.80	0.96	0.16	0.71	0.04	0.21
Fmoc-TOAC	0	0.94	0.96	0.06	0.28	-	-
	1	0.98	0.90	0.02	0.24	-	-
	2	0.88	0.98	0.02	0.63	0.10	0.00
	3	0.76	0.92	0.18	0.79	0.06	0.00

# Confronto esperimenti-calcoli

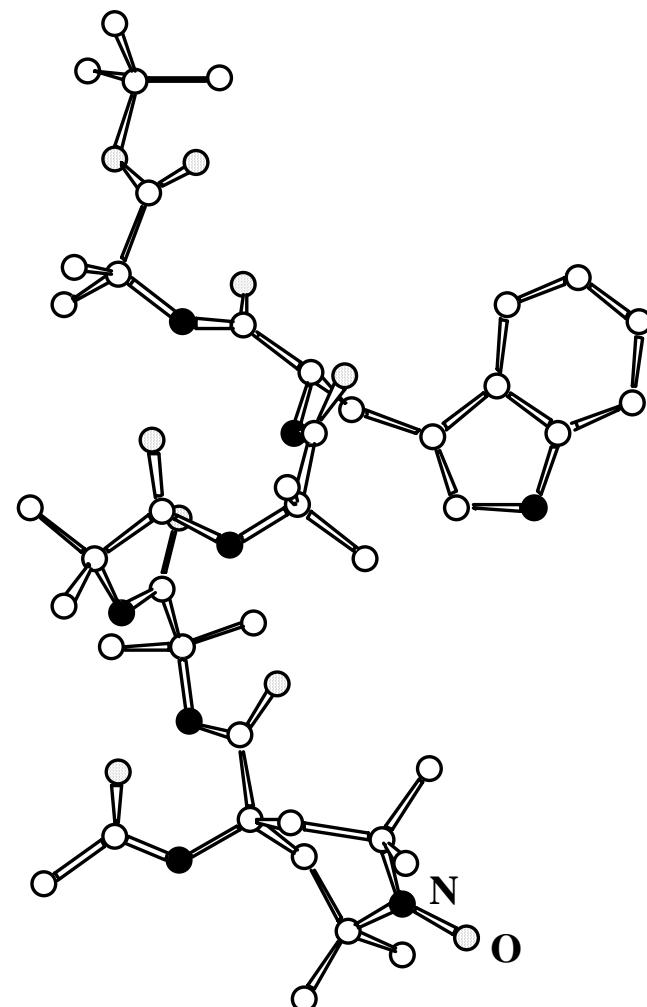


Soluzione

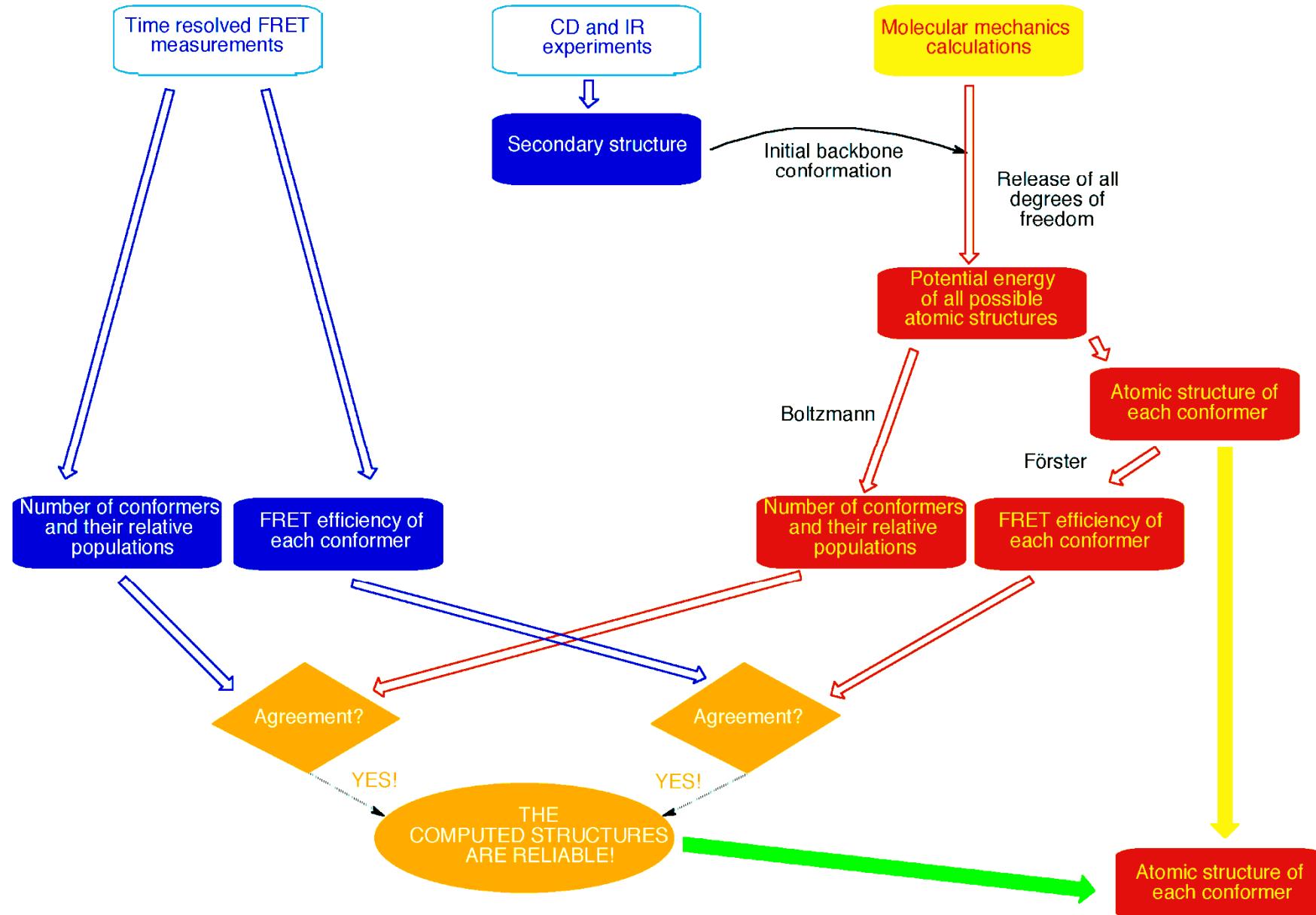


Meccanica molecolare  
(confermata dai dati di fluorescenza)

Cristallo

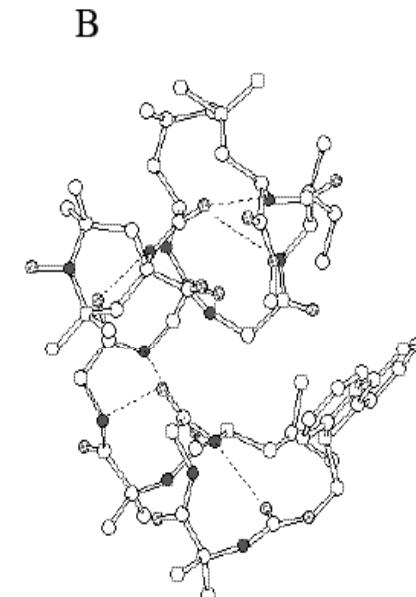
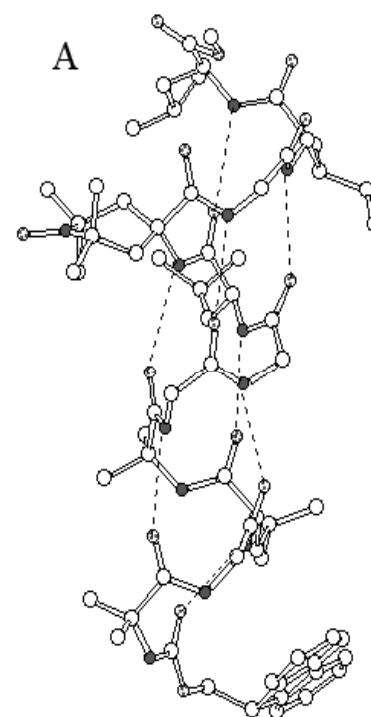
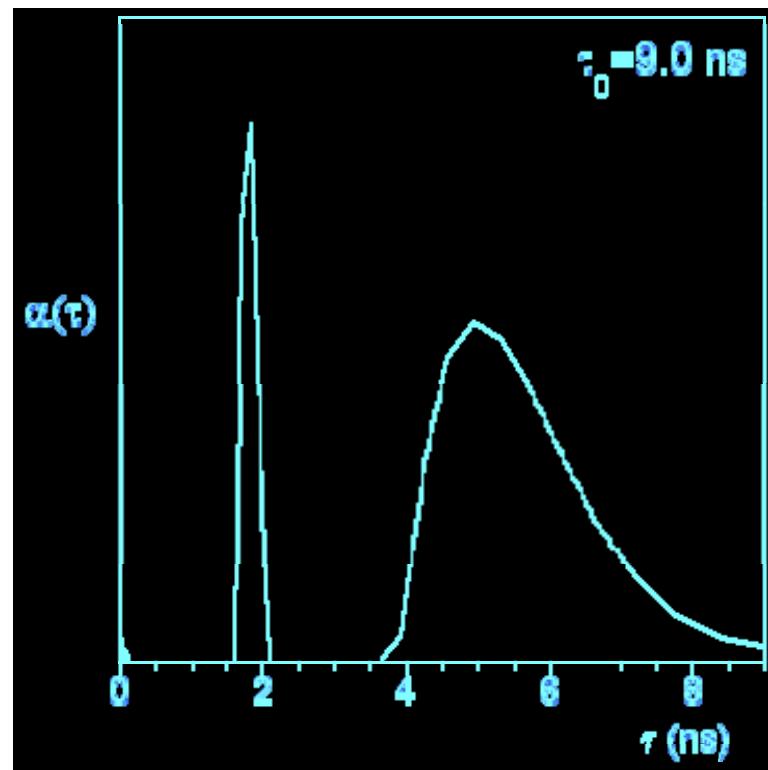


Raggi X



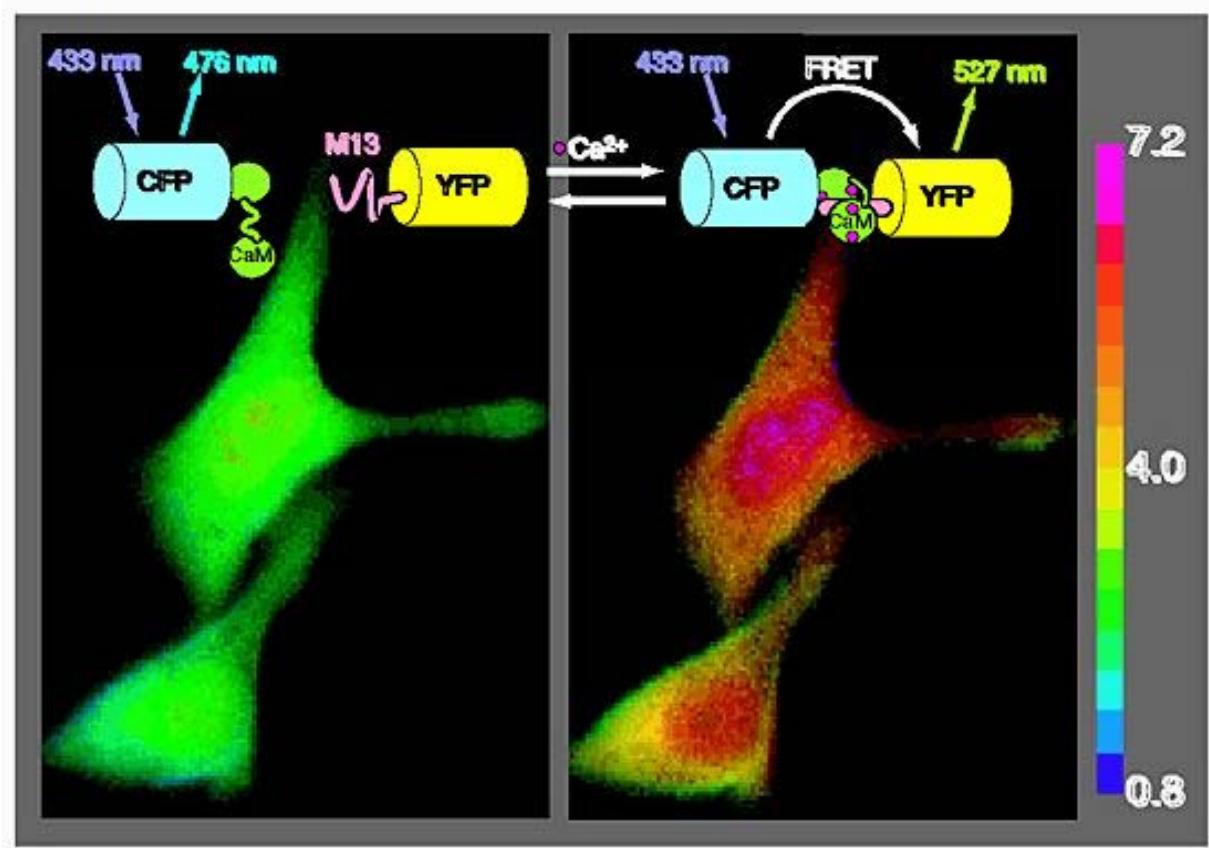
# Fluorescence Energy Transfer

Oct-Aib-Gly-Leu-Aib-Gly-Gly-Leu-Aib-Gly-Ile-Lol



# FRET: applicazioni

Processi di associazione



## In vivo detection of calcium binding

CFP= cyan fluorescent protein

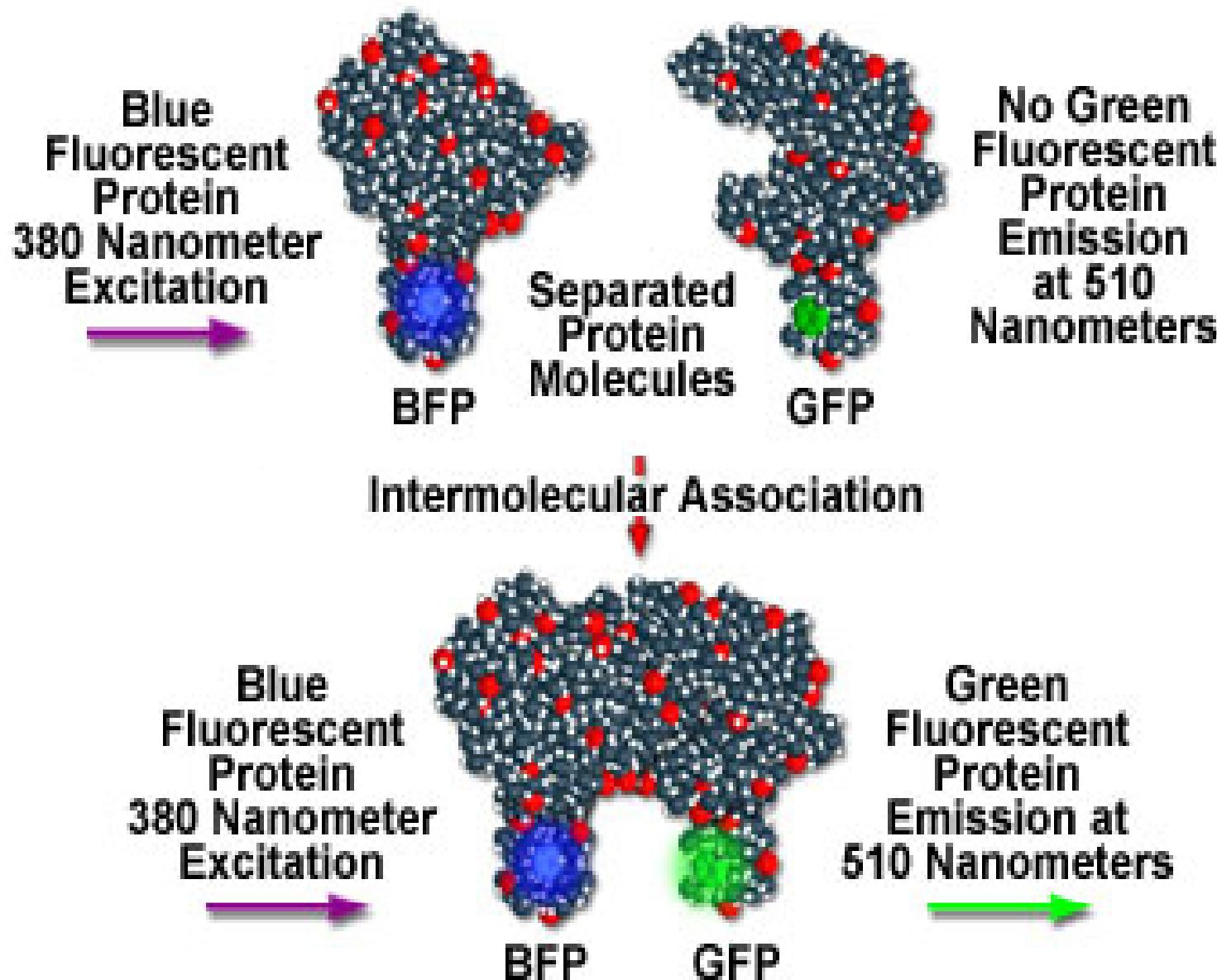
YFP=yellow fluorescent protein

CaM=Calmodulin

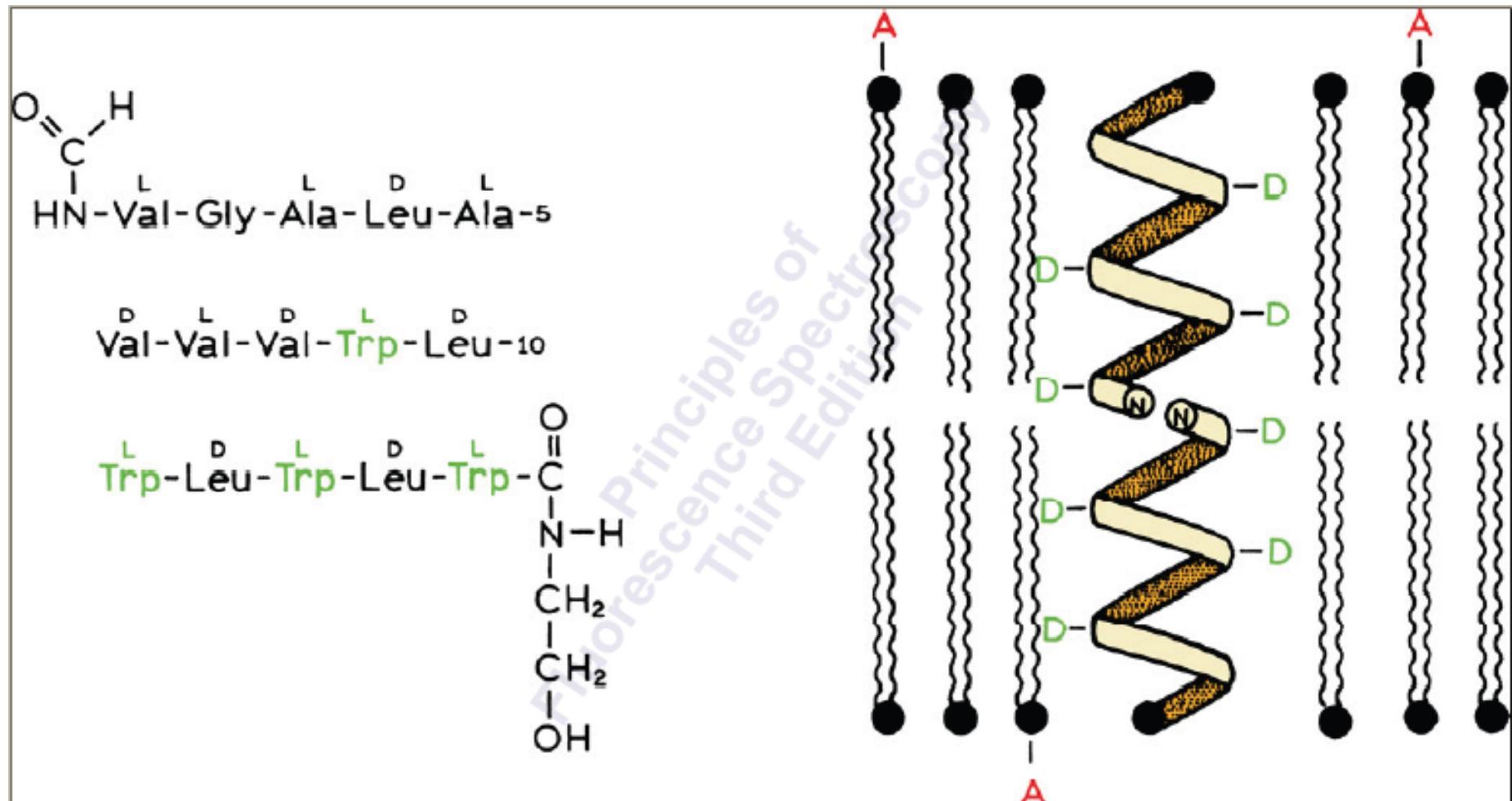
M13=Calmodulin Binding peptide

(Tsien et al. 1998, Science 280:1954)

## FRET Detection of *in vivo* Protein-Protein Interactions

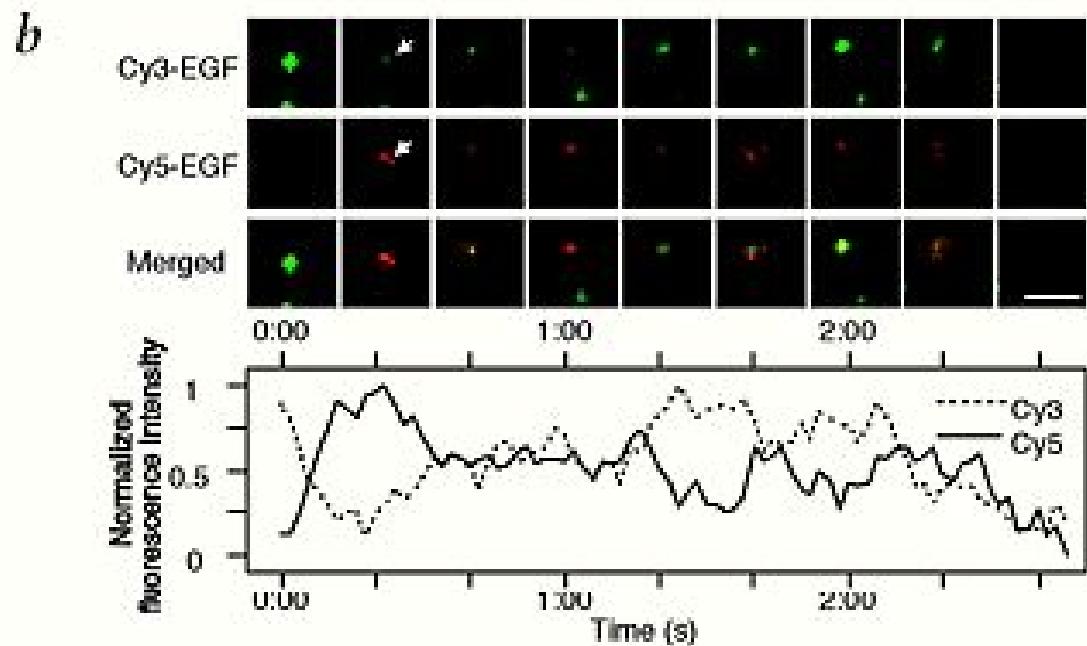
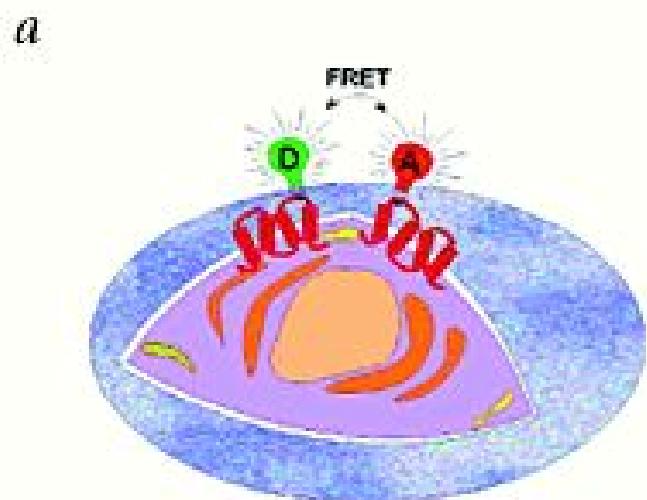


# Membrane permeabilization



# FRET: applicazioni

## Processi di associazione

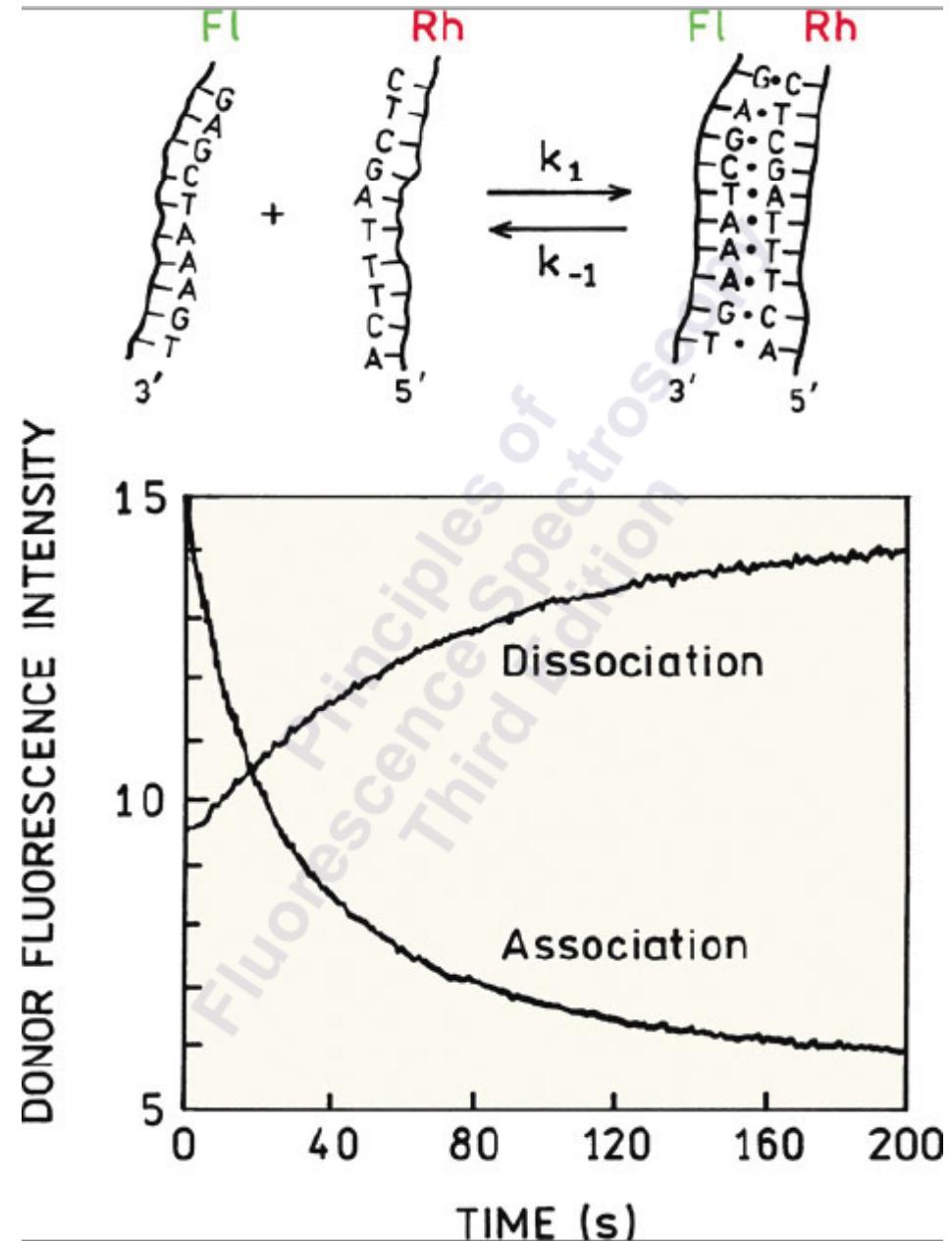


Single molecule detection of association between two epidermal growth factor receptors in the membrane of a living cell

(*Sako et al. 2000, Nature Cell Biol. 2:168*)

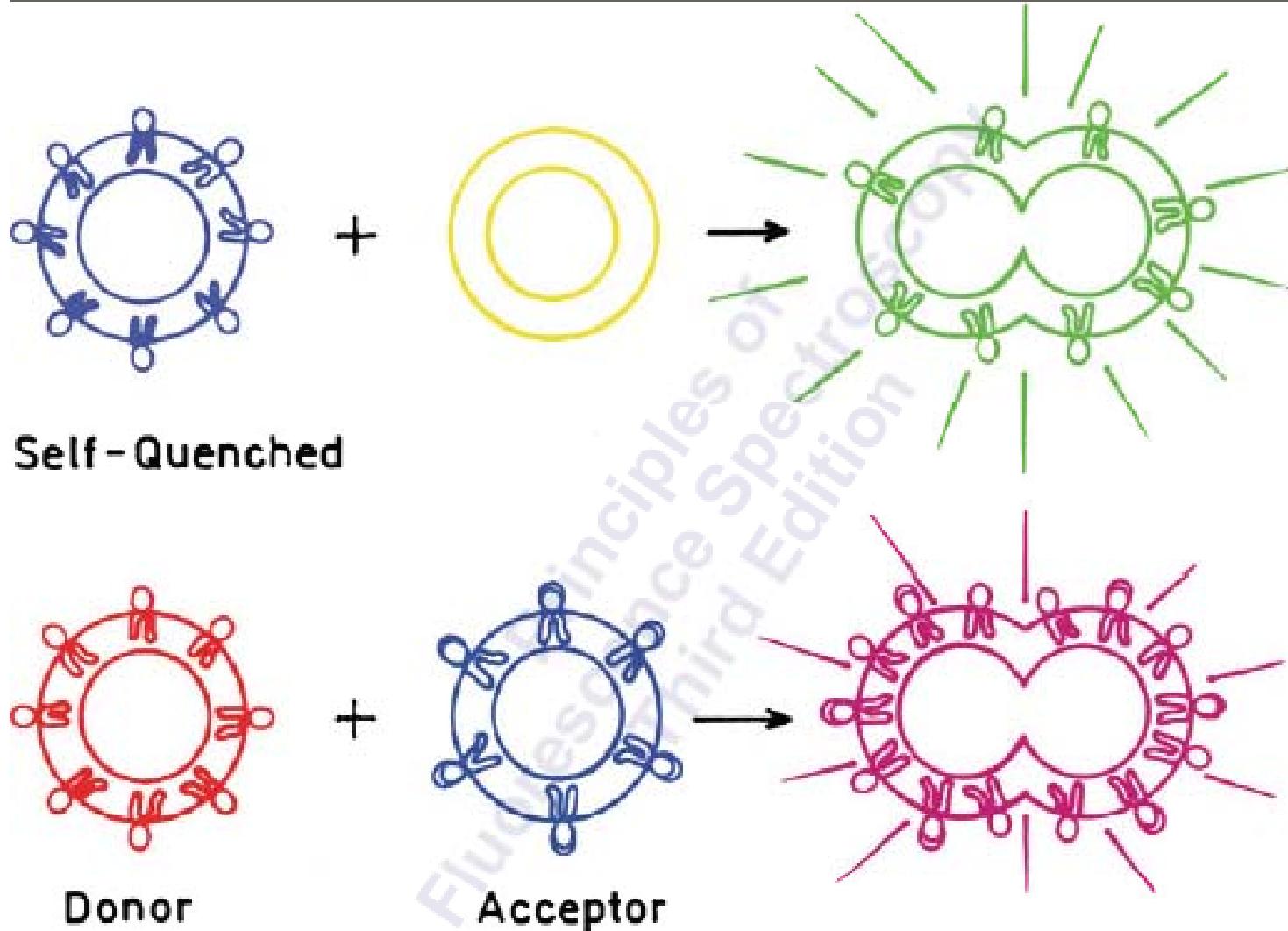
# FRET: applicazioni

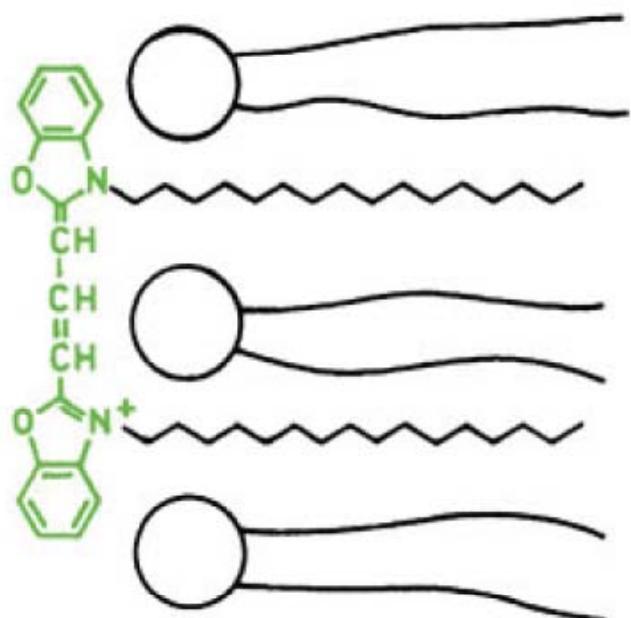
Ibridizzazione



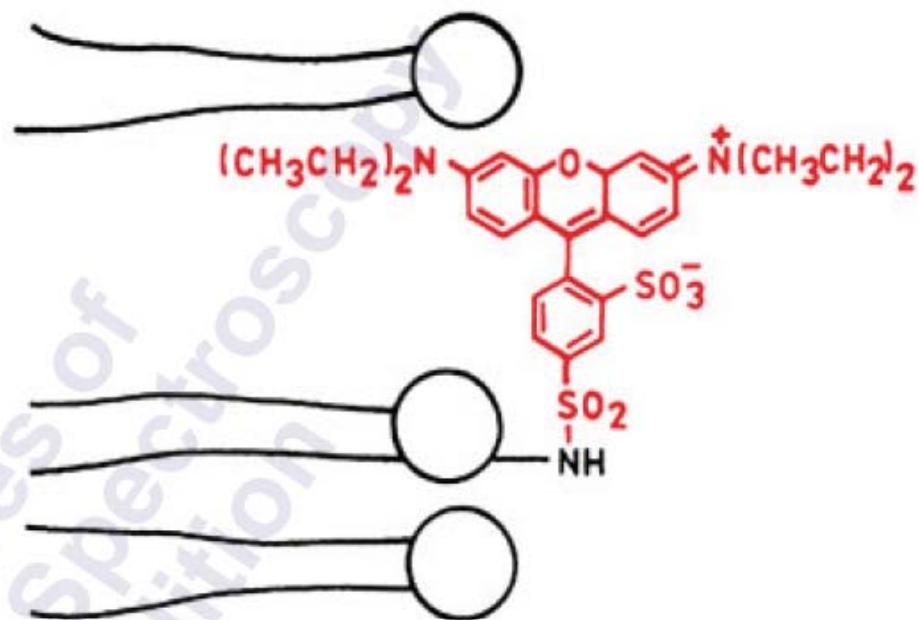
# Membrane fusion

---

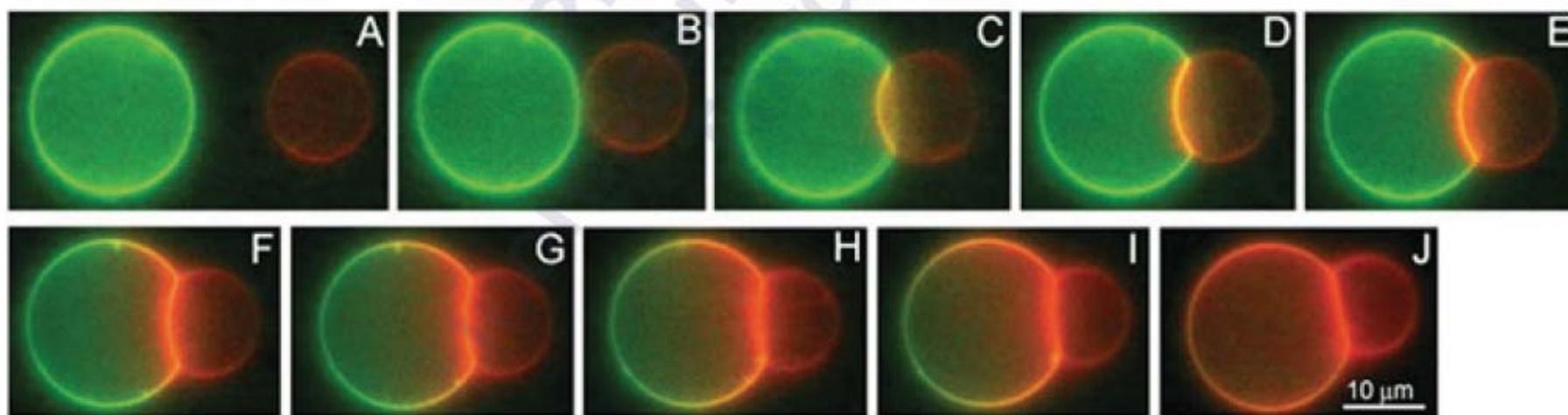




DiO

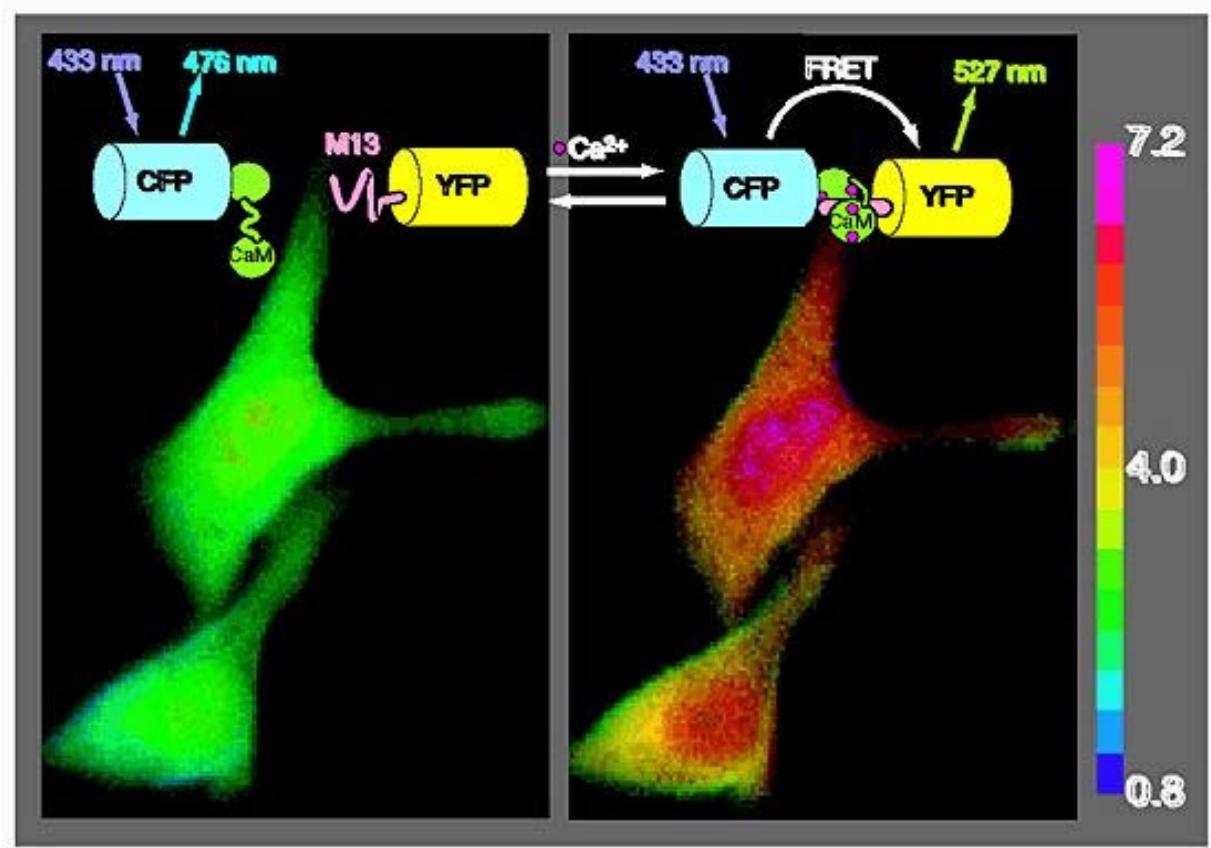


Rh-PE



# FRET: applicazioni

Processi di associazione



## In vivo detection of calcium binding

CFP= cyan fluorescent protein

YFP=yellow fluorescent protein

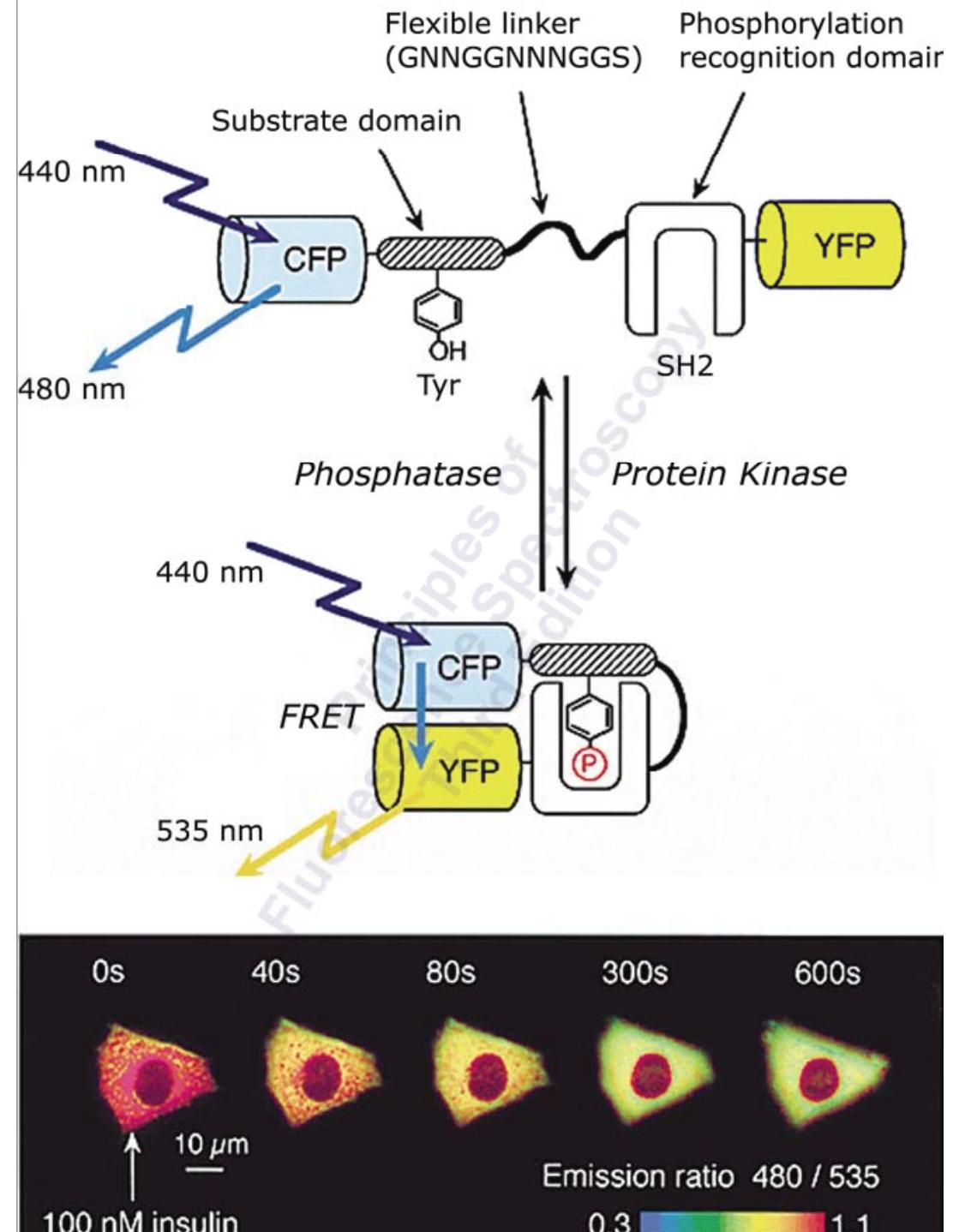
CaM=Calmodulin

M13=Calmodulin Binding peptide

(Tsien et al. 1998, Science 280:1954)

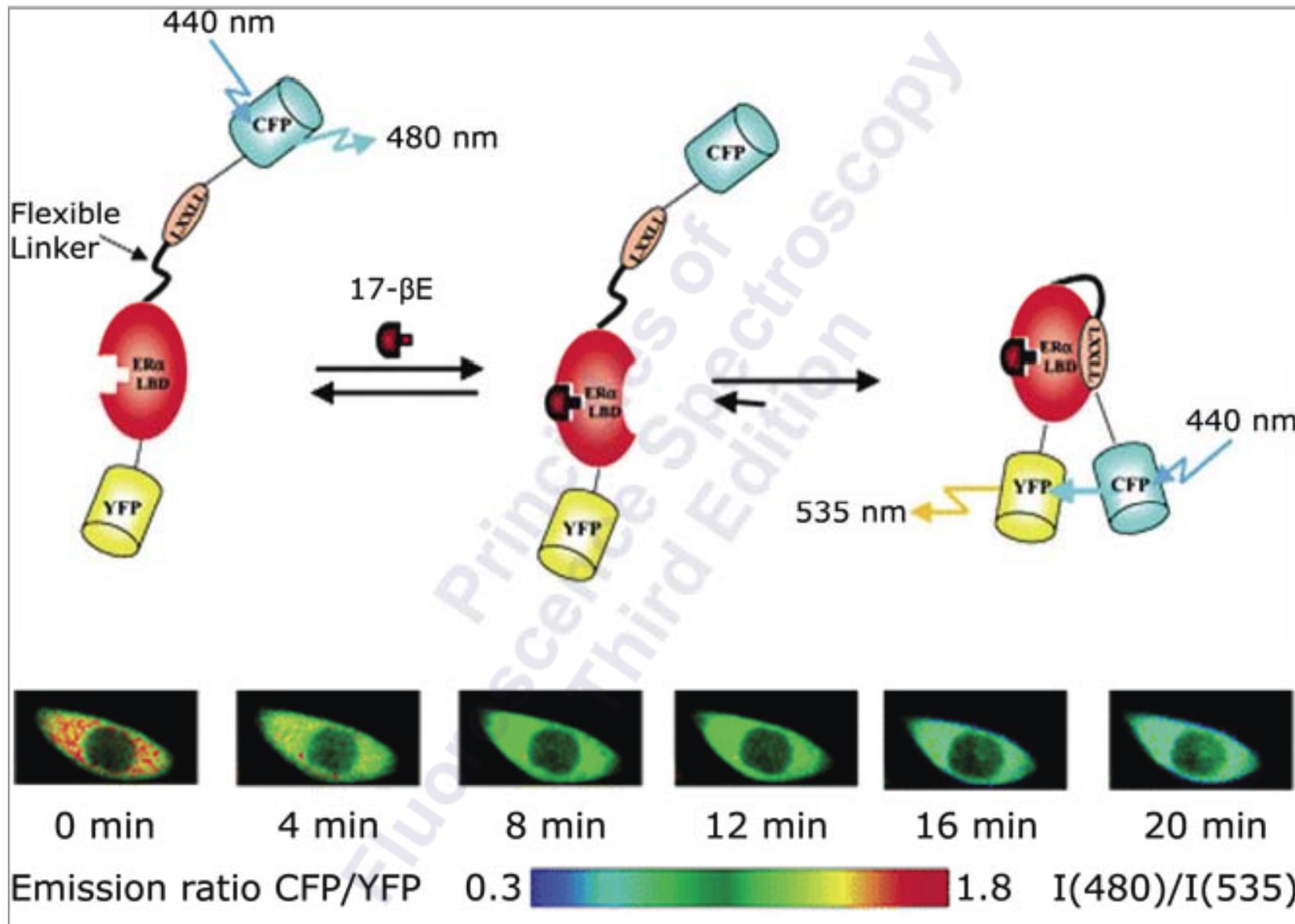
# FRET: applicazioni

## Sensore di fosforilazione



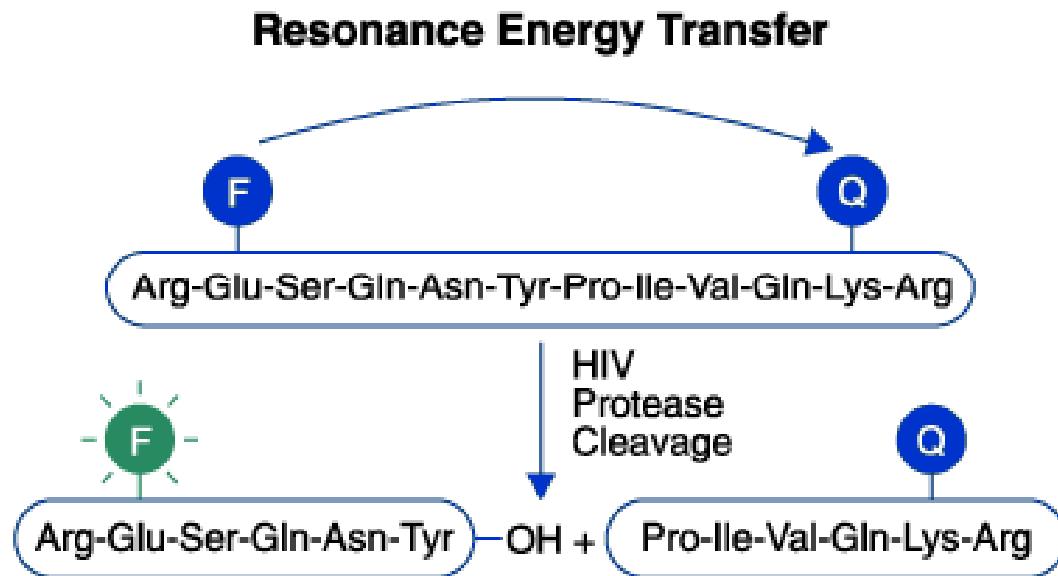
# FRET: applicazioni

## Sensore di estrogeni



# FRET: applicazioni

Attività enzimatica



Nota: il metodo FRET permette la misura di attività enzimatica *in vivo*, utilizzando la microscopia di fluorescenza.