IAGA 13/06

1) Esiste $\{f_{n_k}\}$ s.t. $f_{n_k} \to f$ q.o.

$$|f_{n_k}|^5 \to |f|^5 \quad \text{q.o.}$$

Per Fatou

$$\int |f|^5 \le \liminf \int |f_{n_k}|^5 \le 1$$

$$\Rightarrow f \in L^5.$$

2) $$E_k = \left\{ f > \frac{2k}{k+1} \right\}$$

$\mu(E_k) < \infty \quad \forall k$ per Chebishev.

$\frac{2k}{k+1} = 2 - \frac{1}{k+1}$ è cresc. risp. a k.

Quindi $E_k > E_{k+1} \quad \forall k.$

$$\lim \mu(E_k) = \mu\left(\bigcap_k E_k\right) =$$

$$= \mu\left(\left\{ x \mid f(x) > \frac{2k}{k+1}, \forall k \right\} \right) =$$

$$= \mu\left(\left\{ f \ge 2 \right\} \right).$$

3) $$0 \le \frac{x^{2n}+1}{x^{2n}+x^n+1} \le 1 \quad , \quad \frac{x^{2n}+1}{x^{2n}+x^n+1} \to 1 \quad \forall x > 0.$$

Per il Teor di Lebesgue

$$\lim_{n \to \infty} \int_{[0,1]} \frac{x^{2n}+1}{x^{2n}+x^n+1} \, dx = \underline{1}$$