

ANALISI COMPLESSA
PROVA SCRITTA DEL 02/07/2019

(1) Sia

$$f(z) = \frac{10 - z}{1 - 10z} .$$

Descrivere i seguenti insiemi:

$$f(B_1(0))$$

e

$$f^{-1}(\ell) \text{ dove } \ell = \{w \in \mathbb{C} \mid \Re w = 1\} .$$

(2) Sia $S = \{z \in \mathbb{C} \mid \Re z > 0\}$ e per ogni $z \in S$ si ponga $z = re^{i\vartheta}$, con $r > 0$ e $-\pi/2 < \vartheta < \pi/2$. Si definisca

$$u(z) = r^{\frac{4}{3}} \cos\left(\frac{4}{3}\vartheta\right), \forall z \in S .$$

Stabilire se u è armonica e determinarne, se esiste, l'armonica coniugata.

(3) Calcolare

$$\int_{-\infty}^{+\infty} \frac{t^4}{(t^2 + 1)(t^4 + 1)} dt .$$

English version

(1) Let

$$f(z) = \frac{10 - z}{1 - 10z} .$$

Describe the following sets:

$$f(B_1(0))$$

and

$$f^{-1}(\ell) \text{ where } \ell = \{w \in \mathbb{C} \mid \Re w = 1\} .$$

(2) Let $S = \{z \in \mathbb{C} \mid \Re z > 0\}$ and, for every $z \in S$ we denote $z = re^{i\vartheta}$, where $r > 0$ and $-\pi/2 < \vartheta < \pi/2$. Let us define

$$u(z) = r^{\frac{4}{3}} \cos\left(\frac{4}{3}\vartheta\right), \forall z \in S .$$

Prove or disprove whether u is harmonic and determine its harmonic conjugate, if it exists.

(3) Compute

$$\int_{-\infty}^{+\infty} \frac{t^4}{(t^2 + 1)(t^4 + 1)} dt .$$