# Suggestions

Prof. Dr. Ernst-Erich Doberkat

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**Problem 1.** Given the state space $S$ with a relation $R$, assume that $S' \subseteq S$ and $R' \subseteq R$. On $S$ we have the embedding $id : S' \to S$, which maps $s \in S'$ to $s \in S$. Call $R'$ a *subsystem* of $R$ iff the embedding $id : S' \to S$ is a morphism $R' \to R$.

1. Characterize subsystems in terms of relations.
2. Assume $S$ is a commutative group, written additively, put

$$R := \{\langle x, x + a \rangle \mid x, a \in S\}$$

(this is the universal relation on $S$, connecting each node to each other node). Show that

(a) each subsystem is a subgroup of $S$ (remember: $S_0 \subseteq S$ is a subgroup of $S$ iff $x - y \in S_0$ for each $x, y \in S_0$),

(b) give an example that the converse does not hold (i.e., find a subgroup of a group which is not a subsystem).

**Problem 2.** Show that bisimilarity is transitive: The relations $R, R', R''$ on state spaces $S$, $S'$ resp. $S''$ are given, assume that $R$ is bisimilar to $R'$, and $R'$ is bisimilar to $R''$. Show that $R$ is bisimilar to $R''$.

**Problem 3.** Let $\mathcal{M} = (S, V, R)$ be a Kripke model, and define the equivalence relation $\equiv$ on $S$ by

$$s \equiv s' \text{ iff } s \models \varphi \Leftrightarrow s' \models \varphi \text{ for all formulas } \varphi.$$

Thus $s \equiv s'$ iff $s$ and $s'$ cannot be separated through a formula. Show that $\equiv$ is a congruence for $\mathcal{M}$.